

FINAL EXAM of MICROECONOMICS 1 (1 hour and 50 minutes) – Master M1 IMMAEF

Mobile phones, class notes and problem sets are strictly prohibited

Read and think before you write, and try to be both concise and precise

Exercise 1 (25 minutes). We consider an expected-utility decision-maker facing the following possible professional occupations: working in the financial industry (A), working in the movie industry (B), working in the car industry (C). We further assume that the decision-maker can apply to three different schools.

- After School 1, he is certain to find a job in the car industry.
- After School 2, he is certain to find a job in the financial industry.
- After School 3, he would find a job in the movie industry with probability 0.1 and in the financial industry with probability 0.9

We assume that School 2 is the least preferred option of the decision-maker and that he is indifferent between school 1 and school 3.

- 1) Give a representation of the utility function of this decision-maker.
- 2) We assume that a new school (school 4) opens, after which a student would find a job in each industry with probability $1/3$. How would the new school rank compared to schools 1, 2, 3 ?
- 3) There are too many application in school 4. The ministry of education decides that rather than applying to School 4 directly, the decision-maker must apply to a lottery that leads to admission in School 4 with probability $\alpha \in [0, 1]$ and to admission in School 2 with probability $1 - \alpha$. For which value of α does the decision-maker prefers to apply to school 1 than to school 4 ?

Exercise 2 (25 minutes). We consider a setting with two commodities and one firm. The firm produces commodity 2 using commodity 1 as an input. The firm has two possible technologies. The production function of the first technology is $f_1(z) = \sqrt{z}$ and the production function of the second technology is $f_2(z) = z$, where $z \geq 0$.

- 1) If the firm has 0.5 units of good 1, which production technology shall it use to produce the maximum quantity of output ?
- 2) If the firm has 2 units of good 1, which production technology shall it use to produce the maximum quantity of output ?
- 3) In general, determine as a function of $z \in \mathbb{R}_+$, the maximal quantity of output the firm can produce using z units of input ?
- 4) Determine the production set Y of the firm and represent it graphically
- 5) Prove that Y satisfies free-disposal.

Exercise 3 (40 minutes). We consider a production economy with two commodities, one consumer and one firm. The firm produces commodity 2 by using commodity 1 as an input. The production function of the firm is

$$f(z) = \alpha\sqrt{z} \quad \text{with } z \geq 0$$

and $\alpha > 0$ is a parameter of the production technology. The preferences of the consumer are represented by the utility function $u(x_1, x_2) = x_1x_2$ defined on \mathbb{R}_+^2 . His initial endowment is $e = (12, 0)$ and the consumer is the sole owner of the firm.

- 1) Write the production set Y associated with the production function f .

- 2) Remind the definitions of supply and profit function.
- 3) Give the definition of a competitive equilibrium of this economy.

Without loss of generality, from now on, normalize to 1 the price of commodity 2.

- 4) Compute the supply and the profit function of the firm, depending on the price of commodity 1 and the parameter α .
- 5) Compute the demand of the consumer, depending on the price of commodity 1 and the parameter α .
- 6) Compute the unique competitive equilibrium of this economy, depending on the parameter α .
- 7) Show that, at equilibrium, the demand of commodity 1 does not depend on the parameter α . Finally, compute the utility level at equilibrium, and show that it is strictly increasing in α .

Exercise 4 (20 minutes). We consider a pure exchange economy with two commodities and two consumers. The individual utility functions are

$$u_1(x_{11}, x_{12}) = x_{11} \quad \text{and} \quad u_2(x_{21}, x_{22}) = x_{22}$$

The total resources of the economy are $r = (r_1, r_2) = (3, 4)$.

- 1) Represent the indifference curves and the upper contour sets of both consumers in the Edgeworth box.
- 2) Write the definition of a Pareto optimal allocation for this economy.
- 3) Determine graphically the set of Pareto optimal allocations of this economy.