Macroeconomics: Economic Growth (Licence 3)
Lesson 9: Schumpeterian model

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Lesson 7: Summary

- Romer model: endogenous technological progress
  - (1) Technological progress is viewed as an increase in the **number of intermediate goods ("horizontal innovation")**
  - (2) It is a function of the **number of researchers** (certain).
- Once an intermediate good has been invented, it continues to be used forever.
- The Romer model thought of innovation as adding new intermediate goods to the market (cars, robots, iPhones), but once invented, each good was fixed in **quality**.

Lesson 9: Schumpeterian model

- An alternative way to model endogenous technological progress:
- **Creative destruction**: is to allow innovations to replace existing intermediate good with a more productive (or better quality) goods
- The innovation can be **uncertain** (a probability of being able to find an innovation)
- This alternative approach is based on **Schumpeter's (1939, 1942) original idea of creative destruction** (see Aghion and Howitt, 1988 and Grossman and Helpman (1991)

Lesson 9: Schumpeterian model

- **Creative destruction implies that**:
- Economic growth required the continual obsolescence of old techniques when new ones were invented
- Implying an improvement of quality of the techniques and of the productivity at each step

Schumpeterian model

- **Schumpeterian growth changes the mechanical details of growth, but not the general conclusions**
  - The long-run trend growth rate depends on population growth as in Romer (1990)
  - The allocation of workers to research may not be optimal as in Romer (1990)
- One advantage of the Schumpeterian model is that it explicitly allows us to think about alert dynamics, or the creation and destruction of firms over time.

Schumpeterian model

Aggregate production function

$$Y = K^{\alpha}(A_i L_Y)^{1-\alpha} \quad (1)$$

- where i is the index of ideas (larger i, then larger A)

$$A_{i+1} = (1+\gamma)A_i \quad (2)$$

- where γ is called the "step size" that corresponds to the **amount of productivity growth if innovation occurs**
- If innovation occurs, the growth rate of A is γ
- Therefore γ is not the growth rate of A over time, it depends on **the probability of innovation**

Schumpeterian model

- In this model economic growth depends on innovation: $\rightarrow$
- **Growth occurs when we innovate, but that doesn't always happen.**
- The growth rate of A_i from innovation to innovation is

$$\frac{A_{i+1} - A_i}{A_i} = \gamma \tag{3}$$

- but this is not how fast A_i grows over *time*.

Schumpeterian model

- **The Speed of Innovation or probability to innovate**

- The chance that any given researcher will produce an innovation at any given moment is
$$\overline{\mu} = \theta \frac{L_A^{\lambda-1}}{A_i^{1-\phi}} \tag{4}$$

- or **the probability of innovating** μ depends on the same forces as before: amount of other researchers (L_A) and the spillovers of A_i (stock of ideas)on research.

- For the economy the probability of innovation is the individual probability times the number of researcher $= \overline{\mu} L_A$

Schumpeterian model

- For the economy as a whole, the probability of making an innovation depends on how many researchers are working, so

$$P(Innovation) = \overline{\mu} L_A = \theta \frac{L_A^{\lambda}}{A_i^{1-\phi}}. \quad (5)$$

- As in the Romer model we have two opposite effects of A on innovation:
- (1) "**standing on the shoulders of the giants**" : increasing A increases the chances of findings a new innovation
- (2) "**congestion of researchers**" effects: the probability of finding new innovations is lower as A increases. (new possibilities are far away)
- However, here these factors affect the probability (not the size) of innovation

Schumpeterian model

- **Capital and labor accumulation**
- **The capital stock K evolves in the same way as in the Solow model**

$$\dot{K} = S_K Y - \delta K \tag{6}$$

- Where $S_K Y$ is the investment and δ stands for the depreciation rate of K
- **Population grows at a constant rate n**
- $L = L_A + L_Y$, where L_A are the people involved in research activities and L_Y the workers in the production sector

Schumpeterian model

- **Economic Growth**
- Since innovations occur randomly, we cannot specify the precise path of the income per capita, but
- **we can only look at the growth over long periods of time**
- At a given point in time, we have an expected growth of innovation, which is given by

$$E\left[\frac{\dot{A}}{A}\right] = \gamma\overline{\mu}L_A = \gamma\theta\frac{L_A^{\lambda}}{A_i^{1-\phi}}. \qquad (7)$$

  - γ tells us how much A jumps when an innovation occurs
  - $\overline{\mu}L_A$ tells us the expected value of the number of jumps (probability of innovation)

Schumpeterian model

- **Economic Growth**

- For the law of large numbers, in the very long period the actual average growth approaches the expected average growth:

$$E\left[\frac{\dot{A}}{A}\right] = g_A = g_Y = g_K \tag{8}$$

Schumpeterian model

- **Growth along the BGP**
- In equilibrium along the BGP the expected growth rate of A **will be constant.**

$$E\left[\frac{\dot{A}}{A}\right] = \gamma\overline{\mu}L_A = \gamma\theta\frac{L_A^{\lambda}}{A_i^{1-\phi}}. \tag{9}$$

- Using the above and taking logs and derivative respect to time of the right hand side and set it equal to zero:

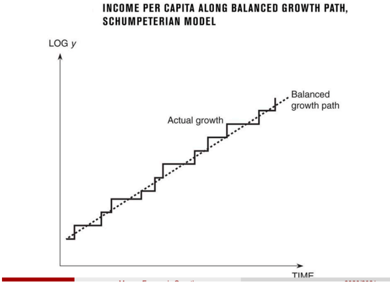
$$0 = \lambda\frac{\dot{L_A}}{L_A} - (1-\phi)E\left[\frac{\dot{A}}{A}\right] \tag{10}$$

- where A is replaced with its expected value
- Recall that in the long run $\dot{L}/L = \dot{L_A}/L_A = n$ along the BGP, means that

$$E\left[\frac{\dot{A}}{A}\right] = g_A = \frac{\lambda}{1-\phi}n \tag{11}$$

- **which is identical to what we got in the Romer model for growth on the BGP**

Schumpeterian model



- The bold line shows how log income per capita actually changes over time:
- There are flat sections → no innovations have been made
- When someone innovates, A_i jumps by γ, so growth is very rapid in that moment
- **On average income per capita is growing along the line called "BGP" which depends on population**

Schumpeterian model

A first comparison between Romer and Schumpeterian models

- The Schumpeterian model does not change our conclusion about the long run trend growth rate.
- **Not relevant whether A is a larger variety of goods or better goods**
- It is interesting to note that γ does not affect the growth rate along the BGP

$$g_A = \frac{\lambda}{1-\phi} n \tag{12}$$

  - on the one hand, a larger γ boosts the size of jumps in A, which is good for growth
  - while, on the other hand, larger γ , though, raises A, making it harder to find the next innovation
- **The Schumpeterian model differs in the underlying economics, and will differ in the equilibrium value of S_R**

Schumpeterian model

- **Assumptions of the Schumpeterian model**
  - **Imperfect competition:** Necessary to generate profits to compensate the researchers work
  - **3 sectors in the Economy:**
    - **Final Goods Sector:** perfect competition
    - **Intermediate Goods Sector:** only a single input produce by one monopolist that owns the patent producing capital good used by final good sector
    - **Research sector:** individuals trying to generate a new version of the capital good more productive for the final good sector.
  - **Difference with Romer (1990)** $\rightarrow$ Creative destruction: The research sector can sell the patent to a new intermediate good firm that monopolize the market and replace the old one.

Schumpeterian model

- **Main differences between Schumpeterian model and Romer model**
  - How intermediate goods are used
  - Nature of innovation
- **Different solution of the Schumpeterian model for:**
  - The proportion of researchers
  - Implications for the role of competition

Final Goods Sector

- Final goods produced under **perfect competition and CRS** using

$$Y = L_Y^{1-\alpha} A_i^{1-\alpha} x_i^{\alpha} \tag{13}$$

- where x_i is a **single intermediate (or capital) good** $x_i = K$ used in the final goods sector. (Different Romer)

- x_i is indexed by i because each intermediate good (units of machine used) has a specific productivity level, A_i (efficiency) associated with it.

- Similar to before, final good firms will maximize profits,
  - Choose how many units of x_i to use
  - Choose which version of x_i to use (latest, or older less productive version)
  - **Will turn out that all versions cost the same, so they will pick best one (the latest version) that gives the highest productivity level**
  - Nobody uses the oldest version that becomes obsolet

Schumpeterian model

Final Goods Sector

- Final Good Sector Profits:

$$max_{L_Y, x_i} L_Y^{1-\alpha} A_i^{1-\alpha} x_i^{\alpha} - wL_Y - p_i x_i \tag{14}$$

- giving first-order conditions of

$$w = (1-\alpha)\frac{Y}{L_Y} \tag{15}$$

$$p_i = \alpha L_Y^{1-\alpha} A_i^{1-\alpha} x_i^{\alpha-1} \tag{16}$$

- **which again is just that the firm sets marginal cost equal to marginal product.**
- As in the Romer model, the elasticity of demand for the intermediate good is $\alpha - 1$

Intermediate Good Sector

- **Intermediate good firms are monopolists** at producing their version of the int. good.
- They have bought the **design from the research sector and there is patent protection**
- As before, they transform one unit of capital into one unit of the int. good.
- Their profits are
$$\pi_i = p_i(x_i)x_i - rx_i \tag{17}$$
- The first-order condition is
$$p_i'(x_i)x_i + p_i(x_i) = r \tag{18}$$
- or set marginal revenue to marginal cost.

Schumpeterian model

Intermediate Good Sector

- As before, we can transform this FOC into

$$p_i = \frac{1}{1 + \frac{p_i'(x_i)x_i}{p_i}} r \tag{19}$$

- which given the elasticity $1 - \alpha$, we found that they charge a constant markup over MC:

$$p_i = \frac{r}{\alpha} \tag{20}$$

- Since the intermediate good firm will always charge the same price for a unit of input, buying the old version of machine costs the same as the new version
- Since the productivity of the new version if higher, **all final good producers will buy the latest version.**
- The economy operates at its technological frontier

Schumpeterian model

Markup Pricing

- Similar to the Romer model, int. good firms charge a markup over marginal cost.
  - This generates the **profits that will motivate innovation**
  - The price they charge does *not* depend on the version of the int. good they produce
  - So all versions of x_i sell for the same price, final good firms only buy the best one (highest A_i)
- **This set-up ensures the creative destruction in the economy.**

Aggregate Output

- Given that only one int. good firm operates at a time, it must be that

$$x_i = K \tag{21}$$

- meaning that aggregate output is as in Romer (1990)

$$Y = K^{\alpha}(A_i L_Y)^{1-\alpha} \tag{22}$$

- which is the same as the standard production function we always use.

Schumpeterian model

Research Sector

- **Innovation is the main difference with Romer**

- Inventing a new version x_{i+1} gives you a patent on that good you can sell (or use to be the monopolist).

- Researchers have a constant probability of finding the new version $\overline{\mu}$

- The one that discovered the new version gets the patent and sell it to the intermediate good firm

- We use an arbitrage condition to get the value of the patent

$$rP_A = \pi + \dot{P}_A - (\overline{\mu} L_A) P_A \tag{23}$$

- rP_A is again the value of putting your money in the bank instead
- $\pi + \dot{P}_A$ is the value of the patent: profits plus capital gains
- With L_A people doing research each with a probability μ of innovating then:
- $(\overline{\mu} L_A) P_A$ captures the fact that with probability $\overline{\mu} L_A$ you will be replaced

Schumpeterian model

Research Sector

- Re-arrange to

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} - \overline{\mu} L_A \tag{24}$$

- and for convenience let $\mu = \overline{\mu} L_A$ so

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} - \mu \tag{25}$$

- $(\overline{\mu} L_A)$ is the probability of a new innovation and it is constant in BGP

Schumpeterian model

Patents Along BGP

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} - \mu \qquad (26)$$

- As in the Romer model, we want to consider the value of a patent along the BGP, **where r is constant.**
- This implies π and P_A must grow at the same rate.
  - We know that profits are $\pi = \alpha(1-\alpha)Y$, so profits grow at the rate $g_Y + n$
  - We know that $g_Y = \gamma\bar{\mu}L_A = \gamma\mu$ along the BGP since $E\left[\frac{\dot{A}}{A}\right] = \gamma\mu = g_A = g_Y = g_K$
  - $\rightarrow$ Patents should grow at the same rate as profits $\frac{\dot{P}_A}{P_A} = \gamma\mu + n$
- so we have that

$$r = \frac{\pi}{P_A} + \gamma\mu + n - \mu \qquad (27)$$

Patents Along BGP

- so we have that

$$r = \frac{\pi}{P_A} + \gamma\mu + n - \mu \tag{28}$$

- which solves to

$$P_A = \frac{\pi}{r - n + \mu(1-\gamma)}. \tag{29}$$

- Again, patents are the present discounted value of profits.

Schumpeterian model

Patents Along BGP

$$P_A = \frac{\pi}{r - n + \mu(1-\gamma)}. \qquad (30)$$

- Again, patents are the present discounted value of profits.
- Relative to Romer model, **the discount rate is higher because of μ, which captures the chance of being replaced.**
- A higher probability of innovation means that the current capital good is more likely to be replaced quickly making the value of the patent for the current capital good lower
- **As the size of innovation increases (*gamma*), the value of patents increases**

Equilibrium for Labor

- Again, individuals can either do research to get the next idea, or work in the final goods sector.
- They move back and forth until the returns to these two activities are identical, or

$$(1-\alpha)\frac{Y}{L_Y} = \overline{\mu} P_A \quad (31)$$

- where $\overline{\mu}$ is the chance that an individual will innovate, and P_A is the value of that innovation to them.

Schumpeterian model

Equilibrium for Labor

$$(1-\alpha)\frac{Y}{L_Y} = \overline{\mu}\frac{\pi}{r-n+\mu(1-\gamma)} \quad (32)$$

$$(1-\alpha)\frac{Y}{L_Y} = \overline{\mu}\frac{\alpha(1-\alpha)Y}{r-n+\mu(1-\gamma)} \quad (33)$$

$$\frac{1}{L_Y} = \frac{\mu}{L_A}\frac{\alpha}{r-n+\mu(1-\gamma)} \quad (34)$$

$$\frac{L_A}{L_Y} = \mu\frac{\alpha}{r-n+\mu(1-\gamma)} \quad (35)$$

$$\frac{s_R}{1-s_R} = \mu\frac{\alpha}{r-n+\mu(1-\gamma)} \quad (36)$$

- which solves to

$$s_R = \frac{1}{1+\frac{r-n+\mu(1-\gamma)}{\alpha\mu}}. \quad (37)$$

Schumpeterian model

Equilibrium s_R

- Found

$$s_R = \frac{1}{1 + \frac{r - n + \mu(1-\gamma)}{\alpha\mu}} \tag{38}$$

  - Same discount factor $r - n$. If that goes up, value of patents goes down, lower s_R
  - **Two opposite effects of the probability of innovation μ**

Equilibrium s_R

- Found

$$s_R = \frac{1}{1 + \frac{r-n+\mu(1-\gamma)}{\alpha\mu}} \tag{39}$$

  - Two opposite effects of the probability of innovation μ:
  - **(1) First effect of μ: from $\mu(1-\gamma)$ captures the fact that as the probability of innovation goes up, the value of patents declines due to replacement effects**
  - **(2) Second effect of μ: from $\alpha\mu$ captures the fact that as the probability of innovation goes up**, you are more likely to get a patent in the first place
  - On net, the second effect "wins". You get a patent now, and will only be replace later, so if μ goes up, s_R goes up

Schumpeterian model

Comparing Schumpeter and Romer

- For realistic values of $\phi < 1$, the long-run growth rate depends on n and is identical $g = \lambda n/(1-\phi)$.
- So whether innovation takes the form of inventing more and new intermediate goods or replacing the existent ones is not essential for economic growth.
- **The main contribution of the Schumpeterian model is to connect growth theory to firm dynamics**

Comparing Schumpeter and Romer

- **The difference is in the level of income along the BGP implied by the value of s_R.**
  - Schumpeterian model has higher s_R if $g < r - n$. In this case the discount rate is very large, and so I care most about profits in the immediate future and little about the fact that I might be replaced some day $\rightarrow$ **So more people do research than in Romer.**
  - Schumpeterian model has lower s_R if $g > r - n$. In this case the discount rate is low, so people do care about the future replacement a lot. $\rightarrow$ **Hence s_R is low compared to Romer.**

Comparing Schumpeter and Romer

- Remember that higher s_R is not necessarily optimal. $y(t)$ along the BGP depends both positively and negatively on s_R.
- There is no sense in which Romer or Schumpeter is "better". They are different ways of conceiving of the growth process.