Microeconomics 1 – Part A: Individual decision making
Masters M1 IMMAEF & MAEF

TD – Wednesday, October 11, 2023

Consumer Theory

The following exercises should be submitted on Wednesday, October 11. A particular attention will be given to your presentation.

Exercise 1 (Cobb-Douglas utility function). For all $x = (x_1, x_2) \in \mathbb{R}^2_+$,

$$u(x_1, x_2) = (x_1)^\alpha (x_2)^{1-\alpha} \text{ with } 0 < \alpha < 1$$

1. For every $\overline{x} \in \mathbb{R}^2_+$, determine and draw the indifference curve $I(\overline{x})$ and the upper contour set $U(\overline{x})$.
2. Determine the following properties of u: continuity, differentiability, (strictly) increasing, (strictly) (quasi-)concavity.

Exercise 2. Let $p = (p_1, p_2) \gg 0$ be a price system and $w > 0$ be the wealth of the consumer. Consider the Cobb-Douglas preferences given above.

1. Show that if $x^* = (x_1^*, x_2^*)$ belongs to the demand of the consumer, then $x^* \gg 0$.
2. Verify that the following utility function represents the Cobb-Douglas preferences on the interior of $\mathbb{R}^2_+$:
$$\widetilde{u}(x_1, x_2) = \alpha \ln x_1 + (1 - \alpha) \ln x_2$$
3. Determine the following properties of $\widetilde{u}$: differentiability, (strictly) increasing, (strictly) (quasi-) concavity.
4. Determine the demand of the consumer.