Microeconomics 1 – Part A: Individual decision making
Masters M1 IMMAEF & MAEF

TD – Wednesday, October 25, 2023

Producer Theory

The following exercises should be submitted on Wednesday, November 15.

Exercise 1. Let L be the finite number of commodities. A firm produces commodity L using the other $L-1$ commodities as inputs. $z := (z_1, ..., z_l, ..., z_{L-1}) \in \mathbb{R}_+^{L-1}$ denotes a generic bundle of inputs. Show that if the production function $f : \mathbb{R}_+^{L-1} \longrightarrow \mathbb{R}_+$ is **concave**, then the transformation function defined by

$$t_f(y) = y_L - f(z)$$

is **quasi-convex** on the convex set $A = \{y = (-z, y_L) \in \mathbb{R}^L \colon z \geq 0 \text{ and } y_L \geq 0\}$.

Exercise 2. Let L be the finite number of commodities. Assume that the production set Y of the firm is represented by a transformation function $t : \mathbb{R}^L \to \mathbb{R}$, so that $Y = \{y \in \mathbb{R}^L \colon t(y) \leq 0\}$.

1. State the profit maximization problem (PMP) of the firm.
2. Let t be continuous and strictly quasi-convex. Show that if PMP has a solution for $p \gg 0$, then it must be unique.

Exercise 3. $L = 2$ is the number of commodities. The firm produces commodity 2 by using commodity 1 as an input. The production function is

$$f(z) = \alpha\sqrt{z}$$

with $\alpha > 0$ and $z \geq 0$.

1. Show that if $\bar{y} = (\bar{y}_1, \bar{y}_2)$ belongs to the supply of the firm, then $\bar{y}_1 < 0$ and $\bar{y}_2 > 0$.
2. Write the first order conditions associated with (PMP), and determine if these conditions are necessary and/or sufficient to solve (PMP).
3. Compute the supply and the profit function of the firm.