

TD – Wednesday, October 18, 2023

Consumer Theory

The following exercises should be submitted on Wednesday, October 18.

Exercise 1. Let $(\mathcal{B}, C(\cdot))$ be a choice structure. We remind that \mathcal{B} is a family of non-empty subsets of $X \subseteq \mathbb{R}^L$, that is $\mathcal{B} = \{B : B \neq \emptyset \text{ and } B \subseteq X\}$, and $C(\cdot)$ is a choice rule that assigns a non-empty set $C(B)$ of elements chosen from B , for every $B \in \mathcal{B}$.

1. Give the general statement of the Weak Axiom of Revealed Preferences (WARP).
2. Let $X = \{x, y, z\}$ and consider the choice structure with $\mathcal{B} = \{\{x, y\}, \{y, z\}, \{x, z\}, X\}$ and $C(\{x, y\}) = \{x\}$, $C(\{y, z\}) = \{y\}$, and $C(\{x, z\}) = \{z\}$. Show that this choice structure must violate WARP.
3. Show that the general statement of WARP is equivalent to the following property:

Suppose that B and B' are two elements of \mathcal{B} such that $\{x, y\} \subseteq B$ and $\{x, y\} \subseteq B'$. Then, if $x \in C(B)$ and $y \in C(B')$, we must have $\{x, y\} \subseteq C(B)$ and $\{x, y\} \subseteq C(B')$.

Exercise 2. Let $L = 2$ be the number of commodities. As usual, $x(p_1, p_2, w) = (x_1(p_1, p_2, w), x_2(p_1, p_2, w))$ denotes the demand of the consumer. For every commodity $\ell = 1, 2$, the demand of commodity ℓ is given by

$$x_\ell(p_1, p_2, w) = \frac{w}{p_1 + p_2}$$

1. Prove that this demand is homogeneous of degree zero.
2. Prove that this demand satisfies Walras' Law.
3. State the Weak Axiom of Revealed Preferences (WARP) in the framework of the demand.
4. Prove that this demand satisfies WARP.