MASTER
Economics & Psychology

PROGRAMME	COURSE TITLE	PROFESSOR	TUTORIAL
M1. Economics & Psychology	Introduction to Economics	Lise Rochaix	Lily Savey

Final exam 2022

Part 2 [12/20]:

General advice: You can receive points for partial answers. Therefore, if you cannot solve a question, you should move on and come back to it later.

Exercise 1. [6 points]

A consumer has the following utility function: $u(x_1; x_2) = \frac{1}{4}\ln(7x_1) + \ln(3x_2)$.

1. Find the optimal bundle $(x_1^*; x_2^*)$ for this consumer. Is good 1 a regular good? What about good 2? **[1.5 points]**

Now the government wants to tax the income of this consumer at rate t.

2. Explain the new budget constraint: $p_1x_1 + p_2x_2 = m(1-t)$. **[0.5 point]**
3. Find the new demand $(x_1'; x_2')$ after the tax, as a function of m, p_1, p_2, and t. **[2 points]**
4. The consumer declares that their income is equal to 400. Show that if the government wants to achieve a tax revenue of 40, the tax rate should be $\bar{t} = 0.1$. **[0.5 point]**
5. Consider that $m = 400$, $t = \bar{t}$, $p_1 = 4$ and $p_2 = 8$. Compute the numerical values of the optimal bundle before the tax $(x_1^*; x_2^*)$ and after the tax $(x_1'; x_2')$. **[0.5 point]**
6. Show mathematically that the consumer was better off before the implementation of the tax. **[1 point]**

Exercise 2. [6 points]

A firm has the following Cobb-Douglas production function: $Q(L, K) = AL^{\beta}K^{1-\beta}$ with $0 < \beta < 1$, $A > 0$ a constant, Q the output, L the quantity of labour and K the quantity of capital. The costs of production are given by the cost of labour $c_L > 0$ and the cost of capital $c_K > 0$.

1. Show that this function has constant returns to scale. In other words, prove that multiplying both inputs L and K by the same amount $\alpha > 0$ results in multiplying the output Q by the same amount α. *Hint: try to write $Q(\alpha L, \alpha K)$* **[0.5 point]**
2. Consider that $A = 10$, $\beta = \frac{1}{2}$, $c_L = 4$ and $c_K = 16$. Knowing that the firm wants to produce 20 units for a total cost $TC = 32$, draw the isoquant and isocost curves. Verify graphically (no computations required) that the optimal quantities of inputs are $L^* = 4$ and $K^* = 1$. **[2 points]**

3. For any $A > 0$, $0 < \beta < 1$, $c_L > 0$ and $c_K > 0$, compute the optimal capital per labour ratio $\frac{K^*}{L^*}$. **[1.5 points]**

The firm got bigger and is now producing $Q = 840$ with $A = 7$. The inputs are now much more costly, such that $c_L = 40$ and $c_K = 720$.

4. Considering that $\beta = \frac{2}{3}$, compute the optimal quantity of capital K^* and the optimal quantity of labour L^* for this firm. **[2 points]**