General Equilibrium Theory

Masters M2 MMMEF & APE Elena del Mercato

Homework – Monday, October 9, 2023

Exercise 1 (From EMP to UMP). Assume that $X_i \subseteq \mathbb{R}^L$ is convex and u_i is continuous on X_i. Let $p^* \in \mathbb{R}^L$ be a price system with $p^* \neq 0$. Assume that x_i^* solves the following expenditure minimization problem:

$$(\text{EMP})_i \quad \min_{x_i \in X_i} \quad p^* \cdot x_i \quad \text{subject to} \quad u_i(x_i) \geq u_i(x_i^*)$$

and there exists $\widetilde{x}_i \in X_i$ such that

$$p^* \cdot \widetilde{x}_i < p^* \cdot x_i^*.$$

Prove that x_i^* solves the following utility maximization problem.

$$(\text{UMP})_i \quad \max_{x_i \in X_i} \quad u_i(x_i) \quad \text{subject to} \quad p^* \cdot x_i \leq p^* \cdot x_i^*$$

Exercise 2 (From a quasi-equilibrium to a competitive equilibrium).

- For for all i, assume that $X_i = \mathbb{R}^L_+$, u_i is continuous on X_i, and $e_i \gg 0$ **(strong survival condition)**. Sow that a quasi-equilibrium is a competitive equilibrium.
- For for all i, assume that $X_i = \mathbb{R}^L_+$, $e_i \in \mathbb{R}^L_+$, u_i is continuous, and u_i is **strictly increasing** on X_i (i.e., for all $\bar{x}_i$ and x_i in X_i, $\bar{x}_i > x_i$ implies that $u_i(\bar{x}_i) > u_i(x_i)$).

  Let $(p^*, (x_1^*, ..., x_i^*, ..., x_I^*))$ a quasi-equilibrium of this economy.

  1. Show that if $\sum_{i=1}^{I} e_i \gg 0$ **(survival condition)**, then $p^* \cdot \sum_{i=1}^{I} e_i > 0$.
  2. Deduce that there exists at least an individual h such that x_h^* solves the utility maximization problem $(\text{UMP})_i$ at the price p^* and the wealth $p^* \cdot e_h$ (carefully justify your answer).
  3. Show then that $p^* \gg 0$.
  4. Finally, deduce that $(p^*, (x_1^*, ..., x_i^*, ..., x_I^*))$ a competitive equilibrium of this economy.