# Logic and Sets
# Mid-term exam 2021 (1h)

Name: QEM/MMEF

## Exercise 1 (9pts)

Indicate for each of the following assertions if they are true (T) or false (F).

1. If 2+2=5, then Paris is in France
2. London is in France if and only if 2+2=5
3. The negation of $(p \rightarrow \neg q) \vee r$ is $p \wedge q \wedge (\neg r)$
4. The negation of $p \leftrightarrow q$ is $\neg q \leftrightarrow \neg p$
5. The negation of $[\forall x \in \mathbb{R}, \forall y \in \mathbb{N}, \exists z \in \mathbb{R}, x + y + z \geq 2$ or $x - y - z \leq 0]$ is $[\exists x \in \mathbb{R}, \exists y \in \mathbb{N}, \forall z \in \mathbb{R}, x + y + z < 2$ or $x - y - z > 0]$
6. The negation of $[\forall x \in \mathbb{Z}_+, \exists y \in \mathbb{Q}, y = \frac{1}{x}]$ is $[\exists x \in \mathbb{Z}_-, \forall y \in \mathbb{Q}, y \neq \frac{1}{x}]$.
7. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 + y^2 = 2$.
8. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 + y^2 = 2$
9. $(p \rightarrow q) \rightarrow r \Leftrightarrow \neg r \rightarrow (\neg q \rightarrow \neg p)$
10. $(A \setminus B) \setminus A = \emptyset$
11. $A \times \emptyset = A$
12. $\{1\} \in \mathcal{P}(\{0, 1\})$
13. $\emptyset \subseteq A \times B$
14. $A \cup B = B \Rightarrow A \cap B = \emptyset$
15. $\{1\} \in \{1, \{2\}\}$
16. $\mathcal{P}(\emptyset) = \{\emptyset\}$
17. If $C = \{\emptyset, \{1, 2\}, \{\{1\}\}\}$ then $\bigcup C = \{1, 2, \{1\}\}$
18. $A \times A = \{(a, a) : a \in A\}$

## Exercise 2 (8pts)

1. Show by contraposition that for any integer $n \geq 1$, $n^4 - 2n^2 + 1$ even implies $n$ odd.

2. Show by contradiction that there is no integer $a, b \in \mathbb{Z}$ such that $16a + 4b = 3$.

3. Show by induction that for any $n \geq 1$, $1 \times 1! + 2 \times 2! + \cdots + n \times n! = (n+1)! - 1$.

4. Show that $A \cap B = \emptyset \Leftrightarrow A \cup B^c = B^c$.

## Exercise 3 (3pts)

Write the truth tables of $(p \to q) \wedge \neg(q \to \neg p)$ and $((p \to q) \to r) \to (p \vee r)$.