Python for finance and optimization
Monte-Carlo

The goal of this lecture is to code Monte-Carlo methods and the classical variance reduction techniques.

A bit of option pricing

Let us consider a European call with payoff $(S_T - K)_+$.

We assume that

$$dS_t = rS_t dt + \sigma S_t dW_t$$

i.e.

$$S_T = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\epsilon\right), \qquad \epsilon \sim \mathcal{N}(0,1).$$

The price of the option can be computed as

$$\mathbb{E}[e^{-rT}(S_T - K)_+].$$

For information and benchmarking, the Black and Scholes formula is

$$S_0 N(d_1) - Ke^{-rT}N(d_2)$$

with $d_1 = \frac{1}{\sigma\sqrt{T}}\left(\log\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T\right)$ and $d_2 = d_1 - \sigma\sqrt{T}$.

A basic Monte-Carlo algorithm

- Code in Python a Monte-Carlo method to approximate the price of the call option (no loop please). Give a confidence interval.

- Test the code with 2^{15} samples for a 1-year ATM call option with $S_0 = 10$, $\sigma = 20\%$ and $r = 3\%$. Compare with the Black and Scholes formula.

Antithetic variables

- Code in Python a Monte-Carlo method to approximate the price of the call option (no loop please) with the same sample of Gaussian variables and their symmetric around 0. Give a confidence interval.

- Test the code with 2^{15} samples for a 1-year ATM call option with $S_0 = 10$, $\sigma = 20\%$ and $r = 3\%$.

Control variates / Call-Put parity

- Code in Python a Monte-Carlo method to approximate the price of put options (no loop please) with the same sample of Gaussian variables (with and without their symmetric around 0). Deduce the price of call options. Give a confidence interval.

- Test the code with 2^{15} samples for a 1-year ATM call option with $S_0 = 10$, $\sigma = 20\%$ and $r = 3\%$.

Out-of-the-money call

- Test the above codes with 2^{15} samples for a 1-year call option with $K = 15$, $S_0 = 10$, $\sigma = 20\%$ and $r = 3\%$. Compare with the Black and Scholes formula.

- Use importance sampling (seen in class) to price the above call using a sample of prices that go up by 50% per year on average.