Corri

TD1 Ex2

$a<b$

$$\begin{cases} \min\ x^2+y^2 \\ x+y^2=b \\ x \geq a \end{cases}$$

Méthode 1 : $x = b-y^2 \to f(x,y) = (b-y^2)^2+y^2$

$z = y^2 \to f(x,y) = (b-z)^2+z$

donc on est ramené à $\begin{cases} \min\ \phi(z) = (b-z)^2+z \\ z \geq 0 \\ z \leq b-a \end{cases}$

$\phi'(z) = 2(z-b)+1$

z		$b-\frac{1}{2}$	
ψ'	$-$	0	$+$
ψ	$\searrow$		$\nearrow$

$\hat{z} = b-\frac{1}{2}$

* si $b-\frac{1}{2} \in [0, b-a]$ i.e. $\frac{1}{2} \in [a,b]$
  alors $z^* = b-\frac{1}{2}$, $x^* = \frac{1}{2}$, $y^* = \pm\sqrt{b-\frac{1}{2}}$
* si $b<\frac{1}{2}$, $z^* = 0$, $x^* = b$, $y^* = 0$
* si $a>\frac{1}{2}$, $z^* = b-a$, $x^* = a$, $y^* = \pm\sqrt{b-a}$

Méthode 2 : $y^2 = b-x \to f(x,y) = x^2+b-x$

donc $\begin{cases} \min\ \psi(x) = x^2+b-x \\ a \leq x \leq b \end{cases}$

x		$\frac{1}{2}$	
ψ'	$-$	0	$+$
ψ	$\searrow$		$\nearrow$

$\hat{x} = \frac{1}{2}$

* si $\frac{1}{2} \in [a,b]$, $x^* = \frac{1}{2}$
* si $a > \frac{1}{2}$, $x^* = a$
* si $b < \frac{1}{2}$, $x^* = b$