

Ch

TD1 Ex 2

$$\left\{ \begin{array}{l} \text{Min } x^2 + y^2 \\ x + y^2 = b \\ x > a \end{array} \right.$$

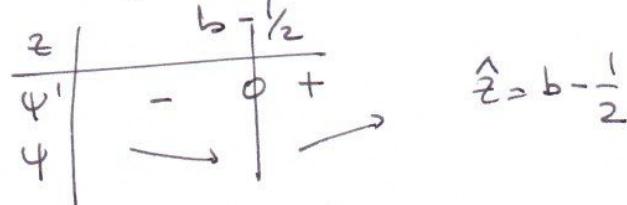
$a < b$

Méthode 1 : $x = b - y^2 \rightarrow f(x, y) = (b - y^2)^2 + y^2$
 $z = y^2 \rightarrow f(x, y) = (b - z)^2 + z$

donc on est ramené à

$$\left\{ \begin{array}{l} \text{Min } \phi(z) = (b - z)^2 + z \\ z \geq 0 \\ z \leq b - a \end{array} \right.$$

$$\phi'(z) = 2(z - b) + 1$$



$$\hat{z} = b - \frac{1}{2}$$

* si $b - \frac{1}{2} \in [0, b-a]$ i.e. $\frac{1}{2} \in [a, b]$

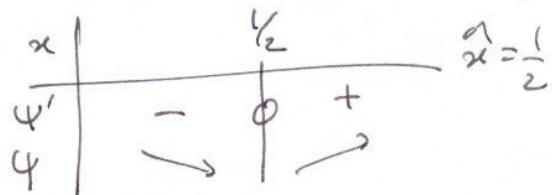
$$\text{alors } z^* = b - \frac{1}{2}, \underline{x^* = \frac{1}{2}}, \underline{y^* = \pm \sqrt{b - \frac{1}{2}}}$$

$$* \text{ si } b < \frac{1}{2}, z^* = 0, \underline{x^* = b}, \underline{y^* = 0}$$

$$* \text{ si } a > \frac{1}{2}, z^* = b-a, \underline{x^* = a}, \underline{y^* = \pm \sqrt{b-a}}$$

Méthode 2 : $y^2 = b - x \rightarrow f(x, y) = x^2 + b - x$

$$\text{donc} \left\{ \begin{array}{l} \text{Min } \psi(x) = x^2 + b - x \\ a \leq x \leq b \end{array} \right.$$



$$\hat{x} = \frac{1}{2}$$

$$* \text{ si } \frac{1}{2} \in [a, b], x^* = \frac{1}{2}$$

$$* \text{ si } a > \frac{1}{2}, x^* = a$$

$$* \text{ si } b < \frac{1}{2}, x^* = b$$