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Market Risk measures: A brief introduction

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Outline of the presentation

Abstract and objectives of the course

- *Abstract :* The purpose of this course is to introduce the fundamental concepts for quantitative and statistical analysis of financial risks. The course will cover essential computational aspects of risk measures as well.

The course is divided into several chapters :

1. Modern Portfolio Theory as originally introduced by Harry Markowitz
2. Value-at-Risk and Expected Shortfall risk measures
3. Coherent risk measures

- *Prerequisites :* The course requires advanced knowledge in probability and statistics but many concepts are recalled.

- *References :*
  - McNeil A., Embrechts P. et R. Frey, Quantitative risk management, 2nd ed, Princeton university press, 2015.
  - Föllmer H. et A. Schied, Stochastic Finance, (3rd Ed.) de Gruyter, 2011.
  - Roncalli T., La Gestion des Risques Financières, Editions Economica, 2009.
  - Roncalli T., Handbook of financial risk management, Chapman

Presentation of the course

- Who am I ?
  - Noufel FRIKHA, Full-time professor (professeur d'université), Mathematical Finance Department, Paris 1 Sorbonne.
  - Main research topics : Mathematical finance, Numerical Probability, Machine Learning for finance, Stochastic Analysis and PDEs.

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  - 6 lectures of 3 hours on Monday 9am-12am.
  - Tutorial classes. 2 lectures of 3 hours on Monday 1pm-4pm : 20th November and 4th December.

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- Mark consists in final exam of 2 or 3 hours on Monday 18th December :
  - questions on lectures, test general/basic knowledge of the course
  - 1 or 2 exercices/problems.

- Sorry but...no retake !

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- Financial context : traded volumes on financial markets together with banking supervision have considerably increased over the last decades
  $\rightsquigarrow$ Need for financial risk management and risk assessment.
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- Insurance context : recent evolution of the EU insurance regulation (Solvency II) promotes risk management practices to define capital requirement and reduce risk.
  - Main purpose : reduce the risk that an insurer would be unable to meet claims and promote financial stability of insurance companies/sector.
  - Technical provision is given by the sum of best estimate of the liabilities

- Various financial risks :
  - Market risk : drop of asset's value (stock, bonds, commodities, ...)
  - Credit and counterparty risk : the counterparty to a transaction could default before the final settlement of the transaction's cash flows.
  - Operational risk : computer failure, fraud, cyber-attack, ...
  - Liquidity risk : risk that the investor do not find a counterparty to make a transaction or the asset cannot be traded quickly enough.
  - Model risk : use of a wrong or misspecifed model.
  - Systemic risk : Collapse of the global banking system.
  - Longevity risk : increasing life expectancy of pensioners and policy holders $\rightsquigarrow$ pensions being paid for longer than expected.
  - Climate risk : takes various forms. Physical risks, such as extreme weather events, transition risks, such as uncertainties relating to the shift towards a low-carbon economy.

- Financial risk depends on the considered horizon :
  - very short (intraday) : liquidity risk.
  - short (1 to 10 days) : market risk.
  - medium (1 month to 1 year) : credit and operational risks
  - long (several years) : longevity and climate risks.
- Quantification of the risk consists in returning a number associated to portfolio's loss distribution on a given time horizon.
- Mathematically speaking, given a random variable X (returns or losses of a portfolio), we seek to find a "good" map $\rho : \mathcal{C} \to \mathbb{R}$, where $\mathcal{C}$ is a convex cone of random variables, such that $\rho(X)$ quantifies the risk of our portfolio.
- Difficulties :
  - it should satisfy some nice mathematical properties reflecting reality.
  - practical relevance/meaning.
  - computational/numerical tractability $\rightsquigarrow$ easy to compute.

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- Revolution in 1952 with the introduction of Modern Portfolio Theory (MPT in short) by the American economist Harry Markovitz. He was the first one to formalize the ideas of quantifying risk and diversification : the standard deviation of the portfolio return can be regarded as a measure of portfolio's risk. An investor wants a portfolio whose return has a high expected value and a low standard deviation. These objective implies that the investor should choose a portfolio that maximizes expected return for any given portfolio standard deviation or alternatively, minimizes standard deviation for any given expected return. Despite of its simplicity, MPT was a significant innovation in risk assessment, for which Markovitz won the Nobel prize in 1990.

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- The variance of the Profit and Loss (P&L) distribution became then the most commonly used risk measure in finance.

- However, this risk measure has been subject to several criticisms.
  - It notably requires that the risks are random variables with finite variance excluding heavy tail loss distributions.
  - It also gives the same importance to gains and losses since it is symmetric. In other words, the variance does not distinguish between positive and negative deviations from the mean.

- In parallel, several important events occurred :
  - According to the Federal Reserve Bank of St. Louis, from 1950 to 1981, there were about six bank failures (or bankruptcies) per year in the United States. During the 1980s, bank failures were particularly prominent ”savings and loan crisis”.
  - At the end of the 1980s, the Basel Committee on Banking Supervision was created to lay out the minimum capital requirements for financial institutions with the goal of minimizing credit risk and promoting financial stability.

- A new era of risk assessment can be traced to the introduction of the Value-at-Risk (VaR in short). VaR is a measure of the potential loss in the value of a portfolio.
  - Development was intensively used at JP Morgan which published the methodology (also known as RiskMetrics) and gave free access to estimates of the necessary underlying parameters in 1994.
  - VaR is the market risk measure prescribed by Basel Accord II and III.

- According to JP Morgan, *VaR is a measure of the maximum potential change in value of a portfolio of financial instruments over a pre-set horizon. VaR answers the question : how much can I lose with* α*% probability over a given time horizon ?*

$\rightsquigarrow$ To put it even more simply, VaR is a α-quantile of the profit and loss distribution of a portfolio over a specified horizon. A 95%-VaR is the size of the loss that will be exceeded with only 5% probability ; a 99%-VaR is a loss that will be exceeded with only 1% probability.

- Axiomatic approach of risk measures : a line of research started by an international group of scholars, P. Artzner, F. Delbaen, J.-M Eber, D. Heath, in the late 1990's. They proposed the first axioms that a good risk measure should satisfy. Risk measures that satisfy these axioms are called *coherent risk measures*.

- With respect to these axioms, VaR was no longer considered as an adequate risk measure. In response to a coherent equivalent to VaR, a variety of risk measures were proposed : Expected Shortfall, Expectiles, ...

- The Basel Committee on Banking Supervision also recommends replacing VaR by Expected Shortfall in internal market risk models.