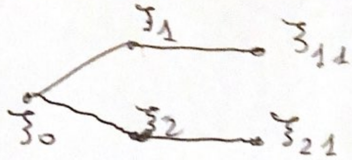
M2 APE - MMMEF

General Equilibrium Theory

Part II: Economics of financial markets

Homework November 2021

1) 


2) $W(q) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & +1 & 0 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

3) $\operatorname{Ker} V = \left\{ (z_1, z_2, z_3) \;\middle|\; \begin{cases} z_1 = 0 \\ -z_1 = 0 \\ -z_1 - z_2 = 0 \\ z_1 + z_3 = 0 \end{cases} \right\} = (0,0,0).$

$\operatorname{Ker} W(q) = \left\{ (z_1, z_2, z_3) \;\middle|\; \begin{cases} z_1 + z_2 = 0 \\ -z_1 - z_3 = 0 \\ -z_1 - z_2 = 0 \\ z_1 + z_3 = 0 \end{cases} \right\} = \{ (z_1, -z_1, -z_1) \mid z_1 \in \mathbb{R} \}$

4) Proof given during the course by a separation argument between the range of $W(q)$ and the simplex in $\mathbb{R}^{D}$.

5) If $\xi(j) = \xi_0 \; \forall j \in J$, $z \in \ker V \Rightarrow$ ~~$-q \cdot z = -\sum_{\xi \in D^+(\xi_0)}$~~

$$-q \cdot z = -\sum_{j \in J} \left( \sum_{\xi \in D^+(\xi_0)} d_\xi v_{j\xi} \right) z_j = -\sum_{\xi \in D^+(\xi_0)} d_\xi \sum_{j \in J} v_{j\xi} z_j$$

$$= -\sum_{\xi \in D^+(\xi_0)} d_\xi v_\xi \cdot z .$$

$v_\xi \cdot z = 0 \; \forall \xi$ since $Vz = 0$ so $-q \cdot z = 0$ and then $z \in W(q)$ since the first row of $W(q)$ is $-q$ and the other rows are the same as the rows of $V$.

Conversely if $W(q) z = 0$ then $Vz = 0$ since the rows of $V$ are a subset of the rows of $W$ plus a null row on the first line.

If $\xi(j) \in D_1 \; \forall j \in J$, we replicate the same argument as above for $\xi \in D_1$. On the row $\xi$, the entries are only the opposite of the prices of the assets issued at node $\xi$ since there is no asset issued at date 0.