"Pure" Abstract Theory (structure of a well defined world)

"Applied" Theory (build and analyze a model with a particular problem)

Overview of a "complete" model of an economy or an equilibrium model

- General Formulation
- Specific Formulation (competitive or Walrasian model of pure exchange, i.e., distribution of privately owned commodities though competitive markets)

General Formulation	Specific Formulation
agents $i \in \{1, \ldots, m\}$ $m < +\infty$ (finite) / alternatives: infinite — $i \in \mathbb{N}$ (countable); $i \in [0,1]$ (continuum)	**consumers / households** $i \in \{1, \ldots, m\}$ $m < +\infty$
actions $x_i \in X_i \subseteq \mathbb{R}^{n_i}$ (or $x_i \in S_i$ abstract space) $X := \prod_{i=1}^{m} X_i$	**consumptions of goods** $x_i \in X_i = \mathbb{R}^L_{++}$ $(\mathbb{R}^L_{+})$ consumption set $L < +\infty$

(continued)

General Formulation

- **environmental / institutional parameters:**

- individual parameters

$\alpha_i \in A_i \subseteq \mathbb{R}^{k_i}$ (or T_i abstract space)

- social parameters

$\beta \in B \subseteq \mathbb{R}^{k}$ (or T abstract space)

- **payoffs:**

$\varphi_i : X_i \times A_i \times B \longrightarrow \mathbb{R}$

(or X "externalities")

- **environmental / institutional constraints:**

- individual constraint

$x_i \in F_i(\alpha_i, \beta)$

correspondence

or $f_i(x_i, \alpha_i, \beta) \geqslant 0$ ($=$)

- social constraint

$$h(x, \alpha, \beta) = 0$$

$\alpha = (\alpha_1, \dots, \alpha_i, \dots, \alpha_m)$

$x = (x_1, \dots, x_i, \dots, x_m)$

Specific Formulation

- individual

endowment of goods

$e_i \in \mathbb{R}^L_{++}$ ($\mathbb{R}^L_+$)

- social parameters

market prices $p \in \mathbb{R}^L_+ \setminus \{0\}$

(or $p \in \mathbb{R}^L_{++}$)

- **utility functions:**

$u_i : X_i \longrightarrow \mathbb{R}$

(or X "consumption externalities")

- **Consumption constraints:**

- individual

budget constraint

$$p \cdot x_i \leq p \cdot e_i$$

$$\Updownarrow$$

$$p \cdot e_i - p \cdot x_i \geqslant 0$$

- social

$$\sum_{i=1}^{m} x_i = \sum_{i=1}^{m} e_i$$

$$\Updownarrow$$

$$\sum_{i=1}^{m} x_i - \sum_{i=1}^{m} e_i = 0$$

Market Clearing Conditions

(continued)

General Formulation

EQUILIBRIUM

- Individual behavior

x_i^* is a (The) solution To

$$\max \ \varphi_i(x_i, \alpha_i, \beta)$$
$$\text{subject To} \begin{cases} x_i \in X_i \text{ and} \\ f_i(x_i, \alpha_i, \beta) \geq 0 \ (=) \end{cases}$$

- Social consistency

$$h(x^*, \alpha, \beta) = 0$$

Specific Formulation

COMPETITIVE EQUILIBRIUM

- consumer's behavior

x_i^* is a (The) solution To

$$\max \ u_i(x_i)$$
$$\text{subject To} \begin{cases} x_i \in \mathbb{R}^L_{++} \\ p^* \cdot e_i - p^* \cdot x_i \geq 0 \end{cases}$$

- Market Clearing Conditions

$$\sum_{i=1}^{m} x_i^* - \sum_{i=1}^{m} e_i = 0$$

Remarks

- When we introduce firms in a competitive / Walrasian model, a consumer has To Take into account firms' profit, firms' behavior (somewhat "Nash equilibrium" flavor).
- When we include "externalities" (for instance, by modifying $u_i : X \longrightarrow \mathbb{R}$) we have To have a different equilibrium concept "Nash-competitive" equilibrium. That is, each consumer Takes The other consumptions as given when he makes his decision, and in equilibrium, consumer's conjecture must be fulfilled. The point here is That we can consider both cases as specific formulations of The general case.