METHODOLOGICAL ASPECTS OF THE CONSTRUCTION OF NUPTIALITY TABLES

WALTER MERTENS
Université de Montréal

RESUMEN

Los modelos de matrimonio constituyen una rama importante del estudio demográfico, especialmente en relación a los estudios de fertilidad. La falta de información no ha permitido el desarrollo de estudios comparativos ya sea a nivel internacional o nacional entre regiones. Este artículo describe procedimientos para desarrollar tasas de nupcialidad, y por tanto cuadros de nupcialidad, de los censos o del estado marital. La primera parte del artículo desarrolla la metodología. La segunda ilustra la metodología y presenta un programa para el computador, que permite calcular los cuadros de nupcialidad, después de hacer los ajustes necesarios a los datos censales. El autor conduce actualmente un estudio internacional sobre modelos de matrimonio y las tendencias de los mismos, en el que hace uso de los procedimientos aquí analizados.

Nuptiality is the great unknown behind many demographic phenomena. There is no doubt that fertility and family formation would be much enlightened by a better knowledge of nuptiality patterns. Anyone eager to study nuptiality has to cope with two big disadvantages. There is less accurate information from the registration of marriage on the whole subject of nuptiality than there is for either mortality or fertility. Second, techniques of analysis are not as well developed. This paper makes a preliminary effort to investigate the theoretical as well as the practical ways to get at the phenomenon of nuptiality with the help of nuptiality tables. It is the first progress report of a project presently under way at the Community and Family Study Center of the University of Chicago.[1] The methodology of the construction of nuptiality tables may be divided into two aspects, each of which is considered separately.

[1] This article finds its inspiration in an unpublished paper of Professor Donald J. Bogue. The computations and methodological experimentations that underlie the present article were financed by a grant from the Rockefeller Foundation to the University of Chicago for research in basic demography. Professor Bogue is the principal investigator. A comparative analysis of differences in nuptiality patterns for more than 60 nations for which census data are available, and for changes over time 1900–60 in these patterns, is being completed as a doctoral dissertation.

I. THEORETICAL ASPECTS

1. *The nuptiality rate as inferred from a vital statistics system.*—This article concerns mainly the derivation of primary nuptiality rates from census data and hence the possibility of constructing nuptiality tables based on census data.[2] If, however, there should exist a good registration system for mortality and nuptiality, the nuptiality rate could be inferred from information based on vital statistics and on census data. (To show how this is done will give at the same time a definition of the nuptiality rate.)

To infer nuptiality and mortality rates from a vital statistics system, the following quantities are supposed to be available from a vital statistics system:

$D_x^{(t)}$ = Total number dying between ages x and $x + 1$.

$D_x^{(s)}$ = Number single dying between ages x and $x + 1$.

$D_x^{(mm)}$ = Number ever married at age x and dying between ages x and $x + 1$.

[2] Although this is not common practice, it is proposed that the term "nuptiality rate" be used for the probability of marriage, while the term "marriage rate" would be used in the sense of a central rate. This use has a parallel in the terminology used to distinguish between mortality and death rate. It should be stressed that both marriage rate and nuptiality rate refer to the population single. Following N. Ryder, it is called primary because it refers to first marriage.

$D_x^{(sm)}$ = Number single at beginning of year of age x and dying as married between ages x and $x+1$.

$D_x^{(m)}$ = Total number ever married dying between ages x and $x+1$ $D_x^{(m)} = D_x^{(mm)} + D_x^{(sm)}$

N_x = Number single marrying between ages x and $x+1$.

Besides the above data obtained from a registration system, the following quantities are supposed to have been obtained by enumeration:[3]

P_x = Total number of population alive at exact age x.

S_x = Total number single in population alive at exact age x.

M_x = Total number ever married in population alive at exact age x.

Evidently $P_x = S_x + M_x$. The argument to be developed makes use of data for proportions single and ever-married persons of each age. A prime[4] will be used to designate proportions: S_x' will be used to refer to the proportion single in the population and M_x' for the proportion ever married in the population.

Two kinds of rates (and this is as well for nuptiality as for mortality) can be derived from these quantities—"independent" and "dependent" rates.[5] An independent rate always indicates the probability of exit during any year of age in a single decrement table. It represents the rate of decrement from the given cause with no other causes of decrement operating. The independent rates of nuptiality and of mortality each shows separately the influence of nuptiality or of mortality without taking account of their mutual interaction.

Dependent rates of decrement for nuptiality and mortality reflect the simultaneous influence of both factors. Suppose there are S_x single persons (or single lives, to use actuarial terminology), aged x, subject to decrement by death and first marriage. The dependent rates may then be expressed as follows:

$$q_x'^{(s)} = \frac{D_x^{(s)}}{S_x} \tag{1}$$

for the mortality rate of the single population;

$$n_x' = \frac{N_x}{S_x} \tag{2}$$

for the nuptiality rate of the single population.

Assuming that both deaths and marriages are evenly distributed over the year of age among the population single, the independent rates are given by the relations:

$$q_x^{(s)} = \frac{D_x^{(s)}}{S_x - \frac{1}{2}N_x} \tag{3}$$

for the mortality rate,

$$n_x = \frac{N_x}{S_x - \frac{1}{2}D_x}. \tag{4}$$

for the nuptiality rate.

At first sight relations (1) and (2) for the dependent rates as compared with relations (3) and (4) may seem confusing. The dependent rate of mortality does not include a term for N_x, and the dependent rate of nuptiality does not include a term in $D_x^{(s)}$. Hence the former is sometimes mistaken for the independent and the latter for the dependent. To get a clearer insight, one should consider, for example, the effect upon the mortality rate of an increase in the number of withdrawals due to primary nuptiality. The population of single persons exposed to risk of death, $S_x - \frac{1}{2}N_x$, would be reduced by marriage and there would be a proportionate reduction in $D_x^{(s)}$, provided that the persons

[3] The census figures refer to values which are on the average for exact age $x + \frac{1}{2}$. In the theoretical part it is assumed that the values for exact age x have already been obtained. The method used to obtain the values for exact age x is explained in the second part.

[4] In this paper a prime is used to designate proportions used in conjunction with a capital letter. If used with a lower-case letter, it indicates a dependent rate of probability. It is never used in this paper to indicate a derivative.

[5] See J. L. Anderson and J. B. Dow, *Actuarial Statistics*, II. *Construction of Mortality and Other Tables* (Cambridge: Cambridge University Press, 1952), pp. 251–65, and C. W. Jordan, Jr., *Life Contingencies* (Chicago: The Society of Actuaries, 1952), pp. 251–65.

marrying did not experience an extremely different rate of mortality for the rest of the age interval. The value of $q_x^{(s)}$ would not therefore be affected. In the case of the dependent rate $q_x'^{(s)}$, however, the reduction in $D_x^{(s)}$ due to the attrition of marriage would not be balanced by any change in the denominator, and the value of $q_x'^{(s)}$ would therefore be reduced.[6]

The independent and dependent rates of mortality and nuptiality may be expressed in terms of each other.

Dividing both numerator and denominator by S_x in (3) and (4), the independent rates can be expressed in terms of the dependent rates:

$$q_x^{(s)} = \frac{q_x'^{(s)}}{1 - \frac{1}{2}n_x'}; \qquad (5)$$

$$n_x = \frac{n_x'}{1 - \frac{1}{2}q_x'^{(s)}}. \qquad (6)$$

This procedure can be used when vital statistics are available. First, dependent rates will be computed; and, if independent rates are asked for to build single decrement tables, the above conversion formulae may be used.

If, however, one has to start from independent rates, as will be shown to be the case for nuptiality rates obtained from census data, dependent rates may be obtained from the independent rates.

From (5) and (6),

$$q_x^{(s)}(1 - \tfrac{1}{2}n_x') = q_x'^{(s)} ; \qquad (7)$$

$$n_x[1 - \tfrac{1}{2}q_x'^{(s)}] = n_x' . \qquad (8)$$

Replacing n_x' in (7) by its value in (8),

$$q_x^{(s)}[\![1 - \tfrac{1}{2}\{n_x[1 - \tfrac{1}{2}q_x'^{(s)}]\}]\!] = q_x'^{(s)} ;$$

$$q_x^{(s)} - \tfrac{1}{2}n_x q_x^{(s)} + \tfrac{1}{4}q_x^{(s)} n_x q_x'^{(s)} = q_x'^{(s)} ; \qquad (9)$$

$$q_x^{(s)}(1 - \tfrac{1}{2}n_x) = q_x'^{(s)}[1 - \tfrac{1}{4}q_x^{(s)}n_x];$$

$$q_x'^{(s)} = \frac{q_x^{(s)}(1 - \frac{1}{2}n_x)}{1 - \frac{1}{4}q_x^{(s)}n_x}.$$

The above conversion formula expresses the dependent rate of mortality for single people in terms of the independent rates of mortality and nuptiality.

Similarly, by replacing $q_x'^{(s)}$ in (8) by its value in (7), the dependent rate of nuptiality may be expressed in terms of the independent rates of mortality and nuptiality:

$$n_x' = \frac{n_x[1 - \frac{1}{2}q_x^{(s)}]}{1 - \frac{1}{4}q_x^{(s)}n_x}. \qquad (10)$$

As in many cases mortality rates specific for single persons will not exist; they can be replaced by mortality rates for the total population.

Very often a slightly different relationship is assumed to exist between the dependent and independent rates. The dependent rates in terms of the independent rates can then be written:

$$q_x'^{(s)} = q^{(s)}(1 - \tfrac{1}{2}n_x) ; \qquad (11)$$

$$n_x' = n_x[1 - \tfrac{1}{2}q_x^{(s)}] . \qquad (12)$$

These relations can be proved to be more consistent.[7] They have, however, a disadvantage. To give the independent rates in terms of the dependents, the following quadratic equations have to be solved:

$$q_x^{(s)^2} + q_x^{(s)}[n_x' - q_x'^{(s)} - 2] + 2q_x'^{(s)} = 0 , \qquad (13)$$

to have the independent rate of mortality in terms of the dependent rates,

$$n_x^2 + n_x[q_x'^{(s)} - n_x' - 2] + 2n_x' = 0 , \qquad (14)$$

to have the independent rate of nuptiality in terms of the dependent rates.

From the above quantities, the rate of mortality for the total population can be inferred too:

$$q_x^{(t)} = \frac{D_x^{(t)}}{P_x}. \qquad (15)$$

This value gives the probability of death between ages x and $x + 1$ for the total population at exact age x. As for the

[6] This is clearly indicated in Anderson and Dow, *op. cit.*, pp. 209–10.

[7] See Anderson and Dow, *op. cit.*, pp. 213–15.

total population, no other factor of decrement is assumed to exist; this is by nature an independent rate.

To obtain the independent rate of mortality for the ever-married population, it should be taken into account that the population ever married is subject to increase from the people single marrying through that year of age. Assuming, again, that marriages are equally distributed over the interval, its value is given by

$$q_x{}^{(m)} = \frac{D_x{}^{(m)}}{M_x + \frac{1}{2}N_x}. \qquad (16)$$

2. *The mechanism of a nuptiality table.*—Once independent rates of nuptiality have been obtained, gross nuptiality tables can be constructed. These gross nuptiality tables show the impact of primary nuptiality neutralizing the effect of mortality. They are the most adequate tools with which to study comparative nuptiality patterns among nations with differential mortality patterns.

Anyone familiar with the ordinary life table should not have difficulty understanding the mechanism of a gross nuptiality table, the only difference being that the factor of decrement is primary nuptiality and not mortality. As a concrete

Table 1.—NUPTIALITY TABLE FOR NORWAY, 1900—FEMALES

Age	MAR(X) (1)	NUP(X) (2)	SIG(X) (3)	M(X) (4)	E(X) (5)	SL(X) (6)	STA2(X) (7)	PRE(X) (8)	ADS(X) (9)
15	1.38	0.0014	100000.	138.	83082.	99931.	1551998.	0.8308	15.5
16	3.49	0.0035	99862.	348.	82944.	99688.	1452067.	0.8306	14.5
17	19.26	0.0191	99514.	1899.	82596.	98565.	1352379.	0.8300	13.6
18	39.06	0.0383	97616.	3740.	80697.	95746.	1253814.	0.8267	12.8
19	55.49	0.0540	93876.	5069.	76958.	91342.	1158068.	0.8198	12.3
20	68.78	0.0665	88807.	5905.	71889.	85855.	1066726.	0.8095	12.0
21	79.16	0.0761	82902.	6313.	65984.	79746.	980872.	0.7959	11.8
22	86.85	0.0832	76589.	6375.	59671.	73402.	901126.	0.7791	11.8
23	92.09	0.0880	70214.	6181.	53296.	67123.	827725.	0.7590	11.8
24	95.08	0.0908	64033.	5812.	47115.	61127.	760601.	0.7358	11.9
25	96.08	0.0917	58221.	5338.	41303.	55552.	699474.	0.7094	12.0
26	95.32	0.0910	52883.	4812.	35965.	50478.	643922.	0.6801	12.2
27	92.97	0.0888	48072.	4271.	31154.	45936.	593444.	0.6481	12.3
28	89.24	0.0854	43801.	3742.	26883.	41930.	547508.	0.6137	12.5
29	84.38	0.0810	40059.	3243.	23141.	38438.	505578.	0.5777	12.6
30	78.67	0.0757	36816.	2787.	19896.	35423.	467140.	0.5405	12.7
31	72.34	0.0698	34029.	2376.	17111.	32841.	431718.	0.5028	12.7
32	65.66	0.0636	31653.	2012.	14735.	30647.	398877.	0.4655	12.6
33	58.89	0.0572	29641.	1696.	12723.	28793.	368229.	0.4292	12.4
34	52.24	0.0509	27946.	1423.	11027.	27234.	339436.	0.3946	12.1
35	45.42	0.0444	26523.	1178.	9605.	25934.	312201.	0.3621	11.8
36	38.24	0.0375	25345.	951.	8427.	24869.	286267.	0.3325	11.3
37	33.16	0.0326	24394.	796.	7476.	23996.	261398.	0.3065	10.7
38*	31.28	0.0308	23598.	727.	6680.	23235.	237402.	0.2831	10.1
39	31.51	0.0310	22871.	710.	5953.	22517.	214167.	0.2603	9.4
40*	31.39	0.0309	22162.	685.	5244.	21819.	191651.	0.2366	8.6
41	31.16	0.0307	21477.	659.	4559.	21147.	169832.	0.2123	7.9
42*	31.03	0.0306	20818.	636.	3900.	20500.	148684.	0.1873	7.1
43	30.79	0.0303	20182.	612.	3264.	19876.	128185.	0.1617	6.4
44*	30.21	0.0298	19570.	582.	2652.	19279.	108309.	0.1355	5.5
45	29.08	0.0287	18987.	544.	2069.	18715.	89030.	0.1090	4.7
46	27.18	0.0268	18443.	495.	1525.	18196.	70315.	0.0827	3.8
47*	24.29	0.0240	17949.	431.	1030.	17733.	52119.	0.0574	2.9
48*	20.19	0.0200	17518.	350.	600.	17343.	34386.	0.0342	2.0
49	14.65	0.0145	17168.	250.	250.	17043.	17043.	0.0145	1.0
50*	0.	0.	16918.	0.	0.	0.	0.	0.	0.

(1) Marriage rate
(2) Nuptiality rate
(3) Proportion single at exact age X
(4) Persons marrying for the first time during year of age X
(5) Persons marrying during year of age X and all further ages
(6) Number of single-persons lived during each age interim
(7) Total number of single-person years lived after age X
(8) Probability of ever marrying at age X
(9) Average expectancy of remaining single

example, the Nuptiality Table for Norway, 1900—Females is reproduced in Table 1.

In column (1), MAR (X), m_x gives the marriage rate, which is a central rate. It refers to a ratio of first marriages to single-person years.

Column (2), NUP (X), refers to n_x, the independent rate of nuptiality. It is based on the notion of risk and expresses the impact of primary nuptiality independent of mortality (eq. (4) above). The same relation that exists in the ordinary life table between the death rate and the mortality rate is assumed to exist between the marriage rate and the nuptiality rate:

$$n_x = \frac{2m_x}{2 + m_x}. \qquad (17)$$

Column (3), SIG (X), or S_x', is the equivalent of the l_x column in a life table. It shows at each age the proportion single or the number "surviving" the previous year's risk of nuptiality. Its value at the beginning of the nuptiality span, S_0', is the radix of the table and is usually equal to 100,000.

$$S_{x+1}' = S_x'(1 - n_x) . \qquad (18)$$

Column (4), M (X), gives the number of people marrying during each year of age. It is equal to the difference between the successive values of the previous column. In a life table, this column would represent the deaths during each age interval.

$$H_x = S_x' - S_{x+1}' . \qquad (19)$$

Column (5), E (X), gives the number of people who will ever marry during the particular age interval under consideration and all further intervals. Using ω to indicate the closing age of the tables and a for the particular age under consideration, its value is given by

$$E_x = \sum_{x=a}^{\omega} (S_x' - S_{x+1}') = \sum_{x=a}^{\omega} H_x. \quad (20)$$

Column (6), SL (X), is equivalent to the L_x column of a life table. It gives the number of single-person years lived during each age interval. Assuming marriages to be equally distributed over the year of age, its value is given by

$$C_x = \frac{S_x' + S_{x+1}'}{2}. \qquad (21)$$

Column (7), STA2 (X), gives the total number of years lived as single after each age. Its value for age x is simply the summation of the values of column (6) for the particular age and all further ages. It is equivalent to the stationary population in the life table.

$$T_x = \sum_{x=a}^{\omega} C_x . \qquad (22)$$

Column (8), PRE (X), gives the probability at age x of ever marrying.

$$h_x = \frac{E_x}{S_x'}. \qquad (23)$$

Column (9), ADS (X), gives the mean expectation of remaining single before first marriage. It is the average number of years to be lived as single for each member of the cohort at a particular age. It is obtained by dividing T_x, the total number of years lived as single, by its respective S_x' value:

$$s_x = \frac{T_x}{S_x'}. \qquad (24)$$

The mean expectation of remaining single at the beginning of the nuptiality span will be given by

$$\mathring{s} = \frac{H_0}{S_x'}. \qquad (25)$$

The net nuptiality table as contrasted to the gross nuptiality table takes account of two factors of decrement for the population single—nuptiality and mortality. By nature it is, thus, a multiple decrement table. There are thus two attrition rates in such a table, which are expressed by the dependent rates of mortality and nuptiality; added together they will form the total rate of decrement.

The proportion single and alive at exact age x will decrease by these two factors. The S_{x+1}' values can then be written as

$$S_{x+1}' = S_x'\{1 - [q_x'^{(s)} + n_x']\}\ . \quad (26)$$

Assuming the relations between dependent and independent rates to exist as given by formulae (11) and (12), it can also be written as

$$S_{x+1}' = S_x'[1 - q_x^{(s)}](1 - n_x)\ . \quad (27)$$

Equation (26) seems to be most useful in constructing net nuptiality tables, as the number dying and marrying can be shown separately. If, however, one has to start from the independent rate of nuptiality, as will be the case for the nuptiality rate derived from census data, equation (27) may be more practical if one does not want to use the conversion formulae to get dependent rates. Using the last equation, it is sufficient to take a series of survival rates from an appropriate life table and to multiply it by the complement of the independent nuptiality rate, $(1 - n_x)$, to get the series of those surviving as single persons. From this series the other functions of the multiple decrement table can be derived. They are not materially different from those of a single decrement table. The final column will give the average expectancy of remaining in life as a single person.

3. *The nuptiality rate inferred from an enumeration procedure.*—The nuptiality rates used in this study are not based on data obtained through a registration system; they are completely derived from census data. To gain more clarity in the method used to derive these nuptiality rates, a simplified model will be given first. Progressively, this model will be adapted for ready use of the method.

Suppose a cohort is followed in which all the members were born at the beginning of a year; suppose that there is no migration; it is a closed cohort. Suppose, further, that this cohort as it proceeds through time would be completely and correctly enumerated at the beginning of each year, according to sex and marital status. For the purpose of this study, only two categories of marital status will be considered—persons single and persons ever married. Suppose, also, that the mortality conditions of the cohort are known specific for sex, age, and marital status as defined previously; they are given by the independent rates of mortality as shown above. They are $q_x^{(t)}$ for the total population, $q_x^{(s)}$ for the single population, and $q_x^{(m)}$ for the ever-married population. Their complementary values, the survival rates, are given respectively by $p_x^{(t)}$, $p_x^{(s)}$, and $p_x^{(m)}$.

Let n_x be the primary nuptiality rate, the value of which is to be derived from the enumerated data on marital status and the above-mentioned survival rates.

The task set out for us in this model is to compute nuptiality rates from the data obtained by successive enumerations of this cohort. Without using symbols we can say that:

The number living at exact age x + 1 = *the number living at exact age* x minus number dying during year of age x.

The number single and living at exact age x + 1 = *the number single and living at exact age* x minus the number single and dying during year of age x and the number marrying for the first time during the year of age x.

The number ever married and living at exact age x + 1 = *the number ever married and living at exact age* x plus the number single at exact age x and marrying during year of age x, minus the number ever married at exact age x and dying during year of age x and the number single at exact age x, marrying and subsequently dying during year of age x.

The italicized items would be obtained from the successive annual enumerations of the cohort.

Using enumeration data for the cohort under consideration together with the appropriate survival rates, the above model can be translated into three algebraic equations. The first one concerns the whole population and can be written as

$$P_{x+1} = P_x[1 - q_x^{(t)}] = P_x p_x^{(t)}\ . \quad (28)$$

Equation (28), if expressed on the base of 100,000, would give the values of the l_x column in a life table for the total population. This population consists of single and ever-married persons, the proportions of which are changing with age. The mortality function for this total population is the result of mortality for the single and ever-married populations. According to research findings, mortality has a different impact on these two categories—the global mortality rate is the result of these two separate mortality functions, each for one of these two categories. The combination of mortality for these two categories does not constitute a "multiple decrement table" but a "multiple category decrement table." The two mortality functions do not constitute risks for the whole population but for separate categories. In a multiple decrement table the risks concern the whole population or at least are assumed to be of such nature. There is, however, a complication, because during each age interval there is a transfer from one category to the other.

The second equation stating that the number single at the end of the year of age is equal to the number single at the beginning of the year of age multiplied by the survival rate for persons single and the complement of the independent rate of nuptiality is given as follows:

$$\begin{aligned} S_{x+1} &= S_x[1 - q_x{}^{(s)}](1 - n_x) \\ &= S_x p_x{}^{(s)}(1 - n_x) \, . \end{aligned} \tag{29}$$

This equation can be seen as related to a net nuptiality table. This may be easily seen when compared with equations (26) and (27).

The third equation states that the population ever married at the end of the year of age is equal to the sum of the survivors of the ever-married population at the beginning of the year of age and of those single marrying during that year of age and surviving death. This equation assumes that marriage takes place at the mid-point of the age interval so that for the first half of the year of age those newly marrying are subject to the mortality conditions for persons single and during the second half to the mortality conditions of the ever married.

$$\begin{aligned} M_{x+1} &= M_x[1 - q_x{}^{(m)}] + S_x n_x \\ &\quad [1 - \tfrac{1}{2}q_x{}^{(s)}][1 - \tfrac{1}{2}q_x{}^{(m)}] = M_x p_x{}^{(m)} \\ &\quad + S_x n_x [1 - \tfrac{1}{2}q_x{}^{(s)}][1 - \tfrac{1}{2}q_x{}^{(m)}] \, . \end{aligned} \tag{30}$$

Equation (30) can be seen as related to what in actuarial practice is called a combined table. In the multiple decrement table, upon which equation (29) may be founded, no information is given concerning the influence of mortality on those individuals who leave the group single because of nuptiality. If this is taken into account, one has a combined table. The category ever married is related to a decrement factor—mortality—and an increment factor—nuptiality.

The nuptiality rate indicating the probability of first marriage between ages x and $x + 1$ for single people at exact age x can be computed from equation (29) or (30).

From Equation (29),

$$n_x = \frac{S_x p_x{}^{(s)} - S_{x+1}}{S_x p_x{}^{(s)}} = 1 - \frac{S_{x+1}}{S_x p_x{}^{(s)}} \, . \tag{31}$$

From Equation (30),

$$n_x = \frac{M_{x+1} - M_x p_x{}^{(m)}}{S_x[1 - \frac{1}{2}q_x{}^{(s)}][1 - \frac{1}{2}q_x{}^{(m)}]} \, . \tag{32}$$

For the first form of the formula, only one survival rate is needed. In addition to the survival rate for persons single, a survival rate (or its complementary value, the mortality rate) for the ever married is needed for the formula obtained from equation (30).

4. *The nuptiality rate obtained from the enumeration of a hypothetical cohort.*—The nuptiality data used in this study, however, are not based on regular successive enumerations of the same cohort; they are based on one single enumeration of a whole set of cohorts, each one at a different age. The marital status of this popula-

tion at each single year of age could then be presumed to represent the nuptiality experience of a hypothetical cohort. Assuming still a closed population, the differences in number between different age groups may be partially due to previous fluctuations of fertility rates. In such a case taking only absolute figures would give rise to awkward results. No individual enumerated in S_x will be found back in the group S_{x+1}; the same is true for P_x and M_x. "Negative nuptiality rates" would be obtained in some cases. Therefore, proportions single and ever married are used. The absolute numbers of single and ever married are weighted with the reciprocals of the figures for the total population. This procedure does not make all "negative nuptiality rates" disappear but only eliminates those based on past differences in fluctuations of fertility. In several cases these "negative nuptiality rates" still remain a function of nuptiality, differential mortality, and (for an open population) differential migration. In case of an incorrect or incomplete enumeration, they may also be the consequence of differential quality of enumeration. (The word "differential" is here understood as differential according to marital status.)

In going from successive enumeration of the same real cohort to the enumeration of one artificial cohort, we come closer to the method used in this study. Essentially it is a method of computing completely from census material the measures which in classic demography are partially based on vital statistics. This procedure has become more frequent in recent times, as it is a valuable tool for computing rates where no vital statistics are available. However, the intrinsic worth of several of the measures computed from census data is increasingly recognized. Very often they make possible more rapid calculations. In some cases they even have the additional property of shedding new light. As yet, no general methodological theory exists concerning the inference of demographic measures usually based on vital statistics from census data.[8] They may be described as the derivation of flow concepts from stock concepts.

When using proportions, equations (29) and (30), after having been divided by equation (28), can be written as

$$S_{x+1}' = S_x' \frac{p_x^{(s)}}{p_x^{(t)}}(1 - n_x)\,; \tag{33}$$

$$M_{x+1}' = M_x' \frac{p_x^{(m)}}{p_x^{(t)}} + S_x' \frac{[1 - \frac{1}{2} q_x^{(s)}][1 - \frac{1}{2} q_x^{(m)}] n_x}{p_x^{(t)}}. \tag{34}$$

The nuptiality rates are again obtained from these equations. From equation (33),

$$n_x = \frac{S_x' [p_x^{(s)}/p_x^{(t)}] - S_{x+1}'}{S_x' [p_x^{(s)}/p_x^{(t)}]} = 1 - \frac{S_{x+1}'}{S_x'} \frac{p_x^{(t)}}{p_x^{(s)}}. \tag{35}$$

From equation (34),

$$n_x = \frac{M_{x+1}' - M_x' [p_x^{(m)}/p_x^{(t)}]}{S_x' [1 - \frac{1}{2} q_x^{(s)}][1 - \frac{1}{2} q_x^{(m)}]/p_x^{(t)}}. \tag{36}$$

As was the case for formula (32), formula (36) requires one more survival or mortality rate than the previous formula based on the differences between proportions single. With proportions, however, both formulae require already one more survival rate than formulae (31) and (32) to take account of the total population.

5. *The assumption of no differential mortality.*—For the actual computations of the nuptiality rates, it will be assumed that there is no differential mortality between the single and ever-married population. This assumption implies that at all ages the survival rates are the same and consequently that both categories have the same survival functions.

This assumption not only simplifies the

[8] Attempts have been made, Cf. *inter alia:* Ivo Lah, *Analytische Ausgleichung der aus den Ergebnissen der Volkszaehlung berechneten demographischen Tafeln* (Internationaler Bevoelkerungskongress Wien, 1959), pp. 192–202; Ajit Das Gupta, *Estimation of Vital Rates for Developing Countries* (Ottawa, 1963); and Giorgio Mortara, *Sur l'utilisation des recensements pour la reconstruction du mouvement de la population* (Ottawa, 1963).

computations for inferring nuptiality rates from census data, but also it makes it possible to consider these rates as seizing the phenomenon of nuptiality in its pure state. The observation of many demographic phenomena consists of a mixture of two or more phenomena. Mortality has a parasitic influence on nuptiality; this makes difficult the comparison of nuptiality patterns between countries with different mortality patterns. Only with "true proportions single," that is, the proportion single in the absence of all mortality, can a valid comparison be made. This may be seen more clearly by showing the relation between the "true proportion single," that is, the proportion single as computed from census data.[9]

Let

$S(x)$ = Function giving number single at beginning of year of age in the population under study.

$\nu(x)$ = Force of nuptiality for single persons.

$\mu(x)$ = Force of mortality for single persons.

$\gamma(x)$ = Proportion of people who would not have experienced marriage if mortality did not exist. This is the "true proportion single."

$\sigma(x)$ = Proportion of people who would not have experienced mortality if marriage did not exist. This is equal to the survival function for a population which has mortality characteristics of a single population.

$S'(x)$ = Proportion single as given by the census.

From

$$d\gamma(x) = -\nu(x)\gamma(x)dx\,, \qquad \gamma(x) = e^{-N(x)}\,, \tag{37}$$

with

$$N(x) = \int_0^x \nu(x)\,dx\,,$$

$$d\sigma(x) = -\mu(x)\sigma(x)dx\,, \qquad \sigma(x) = e^{-Q(x)}\,, \tag{38}$$

with

$$Q(x) = \int_0^x \mu(x)\,dx\,.$$

In both instances the integration constants have the values of the radix, which are considered to be one here.

We have also

$$dS(x) = -[\nu(x) + \mu(x)]S(x)dx\,;$$

$$S(x) = Ce\,\exp - \int_0^x [\nu(x) + \mu(x)]\,dx\,.$$

For

$$x = 0 \qquad C = S(0) = S_0\,;$$

$$S(x) = S_0 e\,\exp - \int_0^x [\nu(x) + \mu(x)]\,dx\,;$$

$$S(x) = S_0 e\,\exp - [N(x) + Q(x)]\,;$$

$$S(x) = S_0\gamma(x)\sigma(x)\,. \tag{39}$$

We have further

$$S'(x) = \frac{S(x)}{S_0\,s(x)}\,, \tag{40}$$

where $s(x)$ is the survival function for the total population. From (39) and (40) we have

$$S'(x) = \frac{\gamma(x)\sigma(x)}{s(x)}\,. \tag{41}$$

By assuming that the survival functions are identical for both categories, single and ever married, it is automatically assumed that the proportions single based on census data are "true proportions single." The nuptiality rate obtained in this way is thus assumed to be a measure of pure nuptiality.

This can be easily seen by replacing S_x' in formula (35) for the nuptiality rate by its equivalent in (41). Formula (19) becomes

$$n_x = \frac{[\gamma(x)\sigma(x)/s(x)][p_x^{(s)}/p_x^{(t)}] - [\gamma(x+1)\sigma(x+1)/s(x+1)]}{[\gamma(x)\sigma(x)p_x^{(s)}/s(x)p_x^{(t)}]}\,. \tag{42}$$

[9] Louis Henry, "D'un probleme fondamental de l'analyse démographique," *Population*, XIV, No. 1 (January–March, 1959), 10–32, and "Approximations et erreurs dans les tables de nuptialité," *Population*, XVIII, No. 4 (October-December, 1963).

The assumption of $p_x^{(s)}/p_x^{(t)}$ equals 1 for all ages means also that $\sigma(x)/s(x) = 1$. Formula (42) thus becomes

$$n_x = \frac{\gamma(x) - \gamma(x+1)}{\gamma(x)}. \quad (43)$$

This will give the independent rate of primary nuptiality or the "true nuptiality rate."

The value of

$$n_x = \frac{S_x' - S_{x+1}'}{S_x'}, \quad (44)$$

which is the one used for the construction of the nuptiality tables of this project is, by the above assumptions, equal to (43) and is consequently an independent rate of nuptiality. It should be stressed that these assumptions involve certain simplifications, since it is known from facts that there exists differential mortality according to marital status.

6. *Migration introduced.*—The whole impact of migration as intertwined with mortality and nuptiality can be represented in terms of decrement factors for emigration and in terms of increment factors for immigration. All the equations of the fundamental system would be related at the same time to a multiple decrement table and a combined table.

The accompanying tabulation shows succinctly how many and to which decrement and increment factors the equations would be related if full account would be taken of mortality, nuptiality, and migration.

	Decrement Factors	Increment Factors
Equation (1)........	Mortality, emigration	Immigration
Equation (2)........	Mortality, nuptiality, emigration	Immigration
Equation (3)........	Mortality, emigration	Nuptiality, immigration

To write this system in equations would become rather elaborate, especially if one would like to take account of the possibility of differential mortality and nuptiality by emigration and immigration, and before and after migration, as well as of the influence of marital status on migration behavior. Exactly the same point can be developed for migration as has been done for mortality in item 5 above. By assuming that migration behavior does not differ according to marital status and that mortality and nuptiality are no different according to migration behavior, it automatically follows that the parasitic influences of migration on the phenomenon of nuptiality have been removed. Pure nuptiality rates are thus again obtained, however, with the same reservations as indicated before.

If mortality and migration are considered together, it is sufficient to assume that their combined effect is not different according to marital status.

7. *The problem of differential enumeration.*—Thus far, it has been assumed that the total population as well as the categories single and ever married has been correctly enumerated. Real facts of life, however, are never correctly enumerated. There will be overcounts and undercounts.

To have correct figures, the data from the census should be multiplied by correction factors. The balance of these correction factors to be applied on our data will very often be for underenumeration. They should thus usually be greater than one would expect for cases of age-heaping. The following symbols are used to represent the correction factors:

$u_x^{(t)}$ = Correction factor for the total population.

$u_x^{(s)}$ = Correction factor for the single population.

$u_x^{(m)}$ = Correction factor for the ever-married population.

Supposing mortality and migration

have been taken account of, the basic formulae can be written as:

$$n_x = \frac{[S_x u_x^{(s)}/P_x u_x^{(s)}] - [s_{x+1}/P_{x+1}][u_{x+1}^{(s)}/u_{x+1}^{(t)}]}{(S_x/P_x)[u_x^{(s)}/u_x^{(t)}} = \frac{S_x'[u_x^{(s)}/u_x^{(t)}] - S_{x+1}'[u_{x+1}^{(s)}/u_{x+1}^{(t)}]}{S_x'[u_x^{(s)}/u_x^{(t)}]}. \quad (45)$$

The formula based on the successive differences between ever married becomes

$$n_x = \frac{M_{x+1}'[u_{x+1}^{(m)}/u_{x+1}^{(t)}] - M_x'[u_x^{(s)}/u_x^{(t)}]}{S_x'[u_x^{(s)}/u_x^{(t)}]}. \quad (46)$$

From these formulae, it becomes clear that so stringent a condition as that of complete correct enumeration is not necessary. Even differential enumeration according to marital status can be assumed without the need of having to adjust the figures in the nuptiality rate. As long as it can be assumed that this differential enumeration is in the same ratio for all ages, there is no need of adjustment for computing nuptiality rates from census material. This assumption will be made for the construction of the nuptiality tables.

II. COMPUTATIONAL ASPECTS

A letter asking for reliable census or survey data on marital status for the twentieth century was sent to most of the government statistical institutes in the world. An effort was made to obtain data by single years of age, especially for the age groups 15–19 and 20–24. The use of several graduation processes to obtain proportions ever married or single by single year age from five-year age groupings gave unsatisfactory results for the first ages of the nuptiality span.[10] Starting from the proportions ever married, negative rates will be obtained for the first ages of the nuptiality span (usually for ages 15, 16, and 17). Starting from proportions, single figures larger than 100,000 will be obtained. This and other difficulties were resolved by use of the computer to adjust the data as well as to construct the nuptiality tables.

To get a better insight into the actual organization of the data for the computer, a distinction is made between a Card Data Unit and a Program Data Unit. The data were punched on IBM cards. Every such card is considered a Card Data Unit. Every card gives information on marital status for one particular age group, sex, country, and census year. Depending on the available information, these age groups were by single year of age, by five years of age, and in several cases by irregular intervals. Every set of such cards for one country, one census year, and one sex is called a Program Data Unit because the whole of the program applies to every such Program Data Unit. The first card of a Program Data Unit is a title card indicating country, census year, sex, number of cards in the particular Program Data Unit, age at which marriage starts, and other similar information necessary for the computer to handle the data. All the data cards are preceded by a master title card indicating the number of Program Data Units. The Sprague multipliers were also read in as data.

To obtain the nuptiality tables, a FORTRAN program was written for the University of Chicago's IBM 7094 computer. It has been adapted for the University of Montreal's CDC 3400. The actual construction of a nuptiality table, once

[10] Nuptiality span is defined as the interval containing those ages during which first marriages take place, that is, the ages during which proportions single are decreasing. This interval differs from country to country and from census to census. It is also different for males and females. It begins somewhere between ages ten and twenty. Very often it is assumed to close at age 50. This convention is followed in the tables. First marriages at later ages are so few that they usually have no sensible influence on the proportions single after age 50.

relatively accurate nuptiality rates have been obtained, should not give rise to many difficulties. However, two secondary aspects gave rise to some rather intricate programming. One was the fact that the data obtained were in different form—some by single year of age, others by five years of age, and a substantial part of mixed nature. The other difficulty, and a more important one, concerns the negative figures obtained as nuptiality rates. The process of successively taking differences between proportions ever married or single (and the last was done in this study) gave for almost every country and census for some ages "negative nuptiality rates." In some instances graduation processes created additional such negative rates. Therefore, after every step which could give rise to such negative rates, the results had to be checked for increasing proportions single. Wherever this occurred a correction had to be made and in some cases the data even had to be discarded for constructing nuptiality tables.

The whole project comprehends three groups of programs:

1. *Programs to construct gross nuptiality tables.* In this group a program exists for data by single year of age, by five years of age, and for data partially by single year of age and partially by five years of age.
2. *Program to construct net nuptiality tables.* Values for mortality rates from life tables for the corresponding countries and census years whenever available have been punched on IBM cards. The nuptiality rates obtained in the previous programs in conjunction with the mortality rates will be used in formula (27), from which it is possible to construct net nuptiality tables.
3. *Program to construct nuptiality tables based on a longitudinal approach.* Once there are two or more censuses containing the necessary information on marital status for a country, a longitudinal approach may be used. The construction and testing of this program are still in their initial phase.

In addition to these programs especially written for this project, a program for regression analysis from the Social Science Library of the University of Chicago has been used for inferring values by single age for the interval 15–19 from values by five years of age. The regression equations based on these computations are used in the program to help in the interpolation of data published in five-year age groupings as single years of age.

As an example, the procedure for computing the gross nuptiality tables from data by single year of age will be given together with the program and its output. To follow the treatment, no knowledge of FORTRAN or even of the general principles of programming is necessary. The only thing required to know is that for matters of clarity and simplification any kind of computational procedure can be divided in subgroups which in programming language are called subroutines and in FORTRAN must have as title a word of no more than six letters. Anyone not having a computer at his disposal can do the same calculations on a desk calculator following the same steps as will be given here. This is the reason why the program itself is given in Appendix I, where it can be consulted by those familiar with FORTRAN.

After having given the general computational outline, all the computational steps together with their output will summarily be treated. The general computational outline is given in a flow diagram presented in Table 2. The words in capital letters are the names of the subroutines as used in the program.

In giving the outline of the computational strategy, it should be taken into account that although this program has been mainly constructed to compute nuptiality rates and hence gross nuptiality tables from census material some additional features have been computed too. The other proportions of marital status have been computed although only proportions single are needed to calculate the nuptiality rates. Besides the nuptiality tables singulate mean and median ages at marriage have been computed. With the almost unlimited possibilities of the computer it was also possible to construct several sets of nuptiality tables according to different graduation methods.

The fundamental relation used for the nuptiality rate is the one given by formula (44) in the first part:

$$n_x = \frac{S_x' - S_{x+1}'}{S_x'}.$$

The computation of this formula implies the following important steps:

1. Computation of proportions single from census data (Step 1: Subroutine RAWPER).
2. Computations of proportions single at exact age x (Step 5: Subroutines GREVIL and VIKRO).
3. Computation of the nuptiality rate n_x (Step 6: Subroutine NUPTIL).

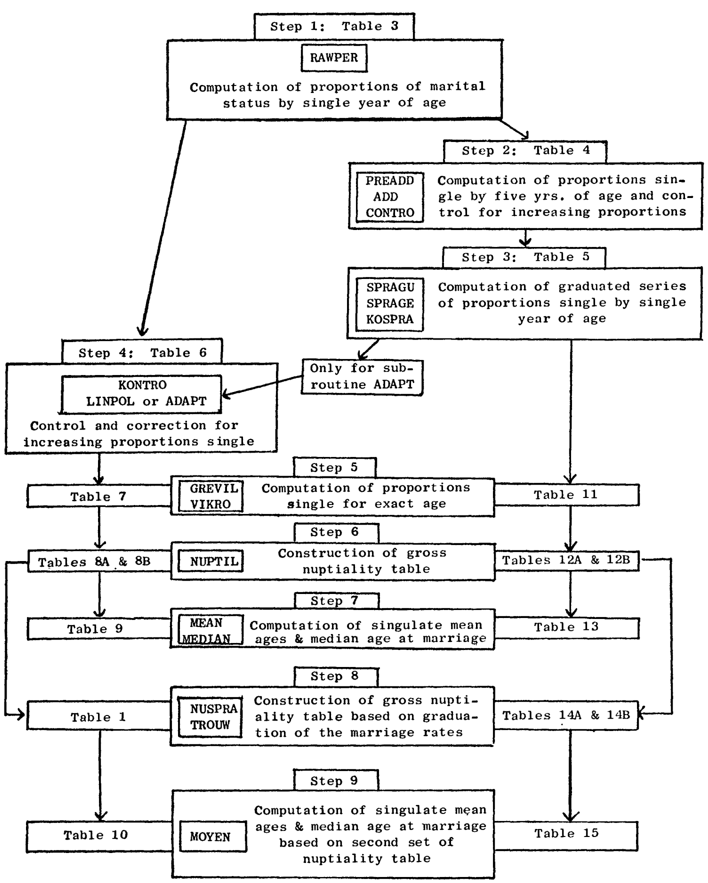
It is from this probability rate that gross nuptiality tables can be constructed. From the data by single year of age, it was possible to construct two series of proportions single by single year of age:

1. One series directly derived from the raw data (Step 1: Subroutine RAWPER).
2. Another series obtained through graduation from the proportions single by five years of age (Step 3: Subroutines SPRAGU, SPRAGE, and KOSPRA).

For each of these two series of proportions single by single year of age, two sets of nuptiality tables have been computed:

1. The first set is directly obtained by using the proportions single as given by one of the previous series of proportions single in

Table 2.—Flow Diagram of Computational Strategy to Construct Gross Nuptiality Tables from Census



formula (44) for the nuptiality rate (Step 5: Subroutine NUPTIL).

2. For the second set the marriage rates obtained in the previous set with subroutine NUPTIL were added in groups by five years of age. From these five-year-age totals a new series of marriage rates by single year of age has been computed with Sprague graduation (Step 8: Subroutine NUSPRA). Once this new series of marriage rates obtained, the way was open to build another nuptiality table (Step 9: Subroutine TROUW).

Because of these two series of proportions single by single year of age and because of these two ways of constructing the table, four kinds of nuptiality tables can be made from the census data on marital status by single year of age.

All the steps summarily outlined in Table 2, the flow diagram, will be briefly analyzed together with their respective output as given by the computer.

Step 1: Computation of raw proportions of marital status (Table 3, subroutine RAWPER).—Columns (1) to (6) are straightforward and give the proportions by marital status on a base of 100,000. They have been calculated after the cate-

Table 3.—PROPORTIONS OF MARITAL STATUS FOR THE NUPTIALITY TABLE FOR NORWAY, 1900—FEMALES

Age	Single (1)	Married (2)	Consensually married (3)	Widowed (4)	Divorced (5)	Separated (6)	Difference (7)	Unknown (8)
15	99943.	53.	0.	4.	0.	0.	0.	0.0
16	99781.	214.	0.	4.	0.	0.	0.	0.0
17	99247.	711.	0.	42.	0.	0.	0.	0.0
18	97713.	2273.	0.	14.	0.	0.	0.	0.1
19	94427.	5474.	0.	94.	0.	5.	0.	0.1
20	90499.	9397.	0.	89.	15.	0.	0.	0.1
21	83839.	15965.	0.	191.	0.	5.	0.	0.1
22	77157.	22532.	0.	261.	15.	35.	0.	0.1
23	70283.	29303.	0.	388.	5.	21.	0.	0.1
24	63739.	35700.	0.	492.	21.	48.	0.	0.1
25	57495.	41681.	0.	771.	16.	37.	0.	0.1
26	51749.	47335.	0.	875.	18.	24.	0.	0.1
27	46868.	51962.	0.	1107.	25.	37.	0.	0.1
28	44627.	53826.	0.	1430.	41.	76.	0.	0.1
29	38594.	59663.	0.	1631.	42.	70.	0.	0.1
30	38067.	59782.	0.	2054.	32.	65.	0.	0.1
31	32829.	64663.	0.	2327.	79.	101.	0.	0.1
32	32050.	65083.	0.	2629.	94.	144.	0.	0.1
33	29964.	66785.	0.	3059.	78.	114.	0.	0.1
34	28152.	68498.	0.	3126.	84.	140.	0.	0.1
35	27493.	68807.	0.	3482.	70.	148.	0.	0.1
36	25935.	69891.	0.	3898.	104.	173.	0.	0.1
37	24269.	71308.	0.	4144.	172.	108.	0.	0.1
38	24529.	70765.	0.	4462.	111.	134.	0.	0.1
39	23127.	71391.	0.	5262.	102.	119.	0.	0.1
40	25185.	68709.	0.	5776.	125.	205.	0.	0.1
41	20659.	72847.	0.	6267.	101.	126.	0.	0.1
42	21283.	71311.	0.	7121.	122.	163.	0.	0.1
43	19689.	72481.	0.	7568.	114.	149.	0.	0.1
44	20425.	71071.	0.	8216.	136.	153.	0.	0.1
45	19306.	71408.	0.	9077.	81.	129.	0.	0.1
46	17895.	72140.	0.	9756.	117.	92.	0.	0.1
47	17946.	71620.	0.	10163.	118.	154.	0.	0.1
48	18752.	69882.	0.	11166.	136.	63.	0.	0.1
49	17724.	69905.	0.	12038.	176.	157.	0.	0.1
50	19995.	66500.	0.	13251.	105.	149.	0.	0.1
51	16660.	69359.	0.	13642.	144.	195.	0.	0.1
52	16728.	67918.	0.	15127.	99.	128.	0.	0.1
53	16362.	67821.	0.	15575.	136.	105.	0.	0.1
54	15749.	67334.	0.	16670.	146.	101.	0.	0.1
55	16757.	64672.	0.	18327.	138.	106.	0.	0.1
56	16324.	63953.	0.	19443.	146.	135.	0.	0.1
57	15754.	64264.	0.	19804.	89.	89.	0.	0.1
58	15784.	62520.	0.	21526.	61.	109.	0.	0.2
59	15649.	61659.	0.	22450.	142.	100.	0.	0.2
60	18325.	57136.	0.	24327.	94.	117.	0.	0.3
61	14712.	58944.	0.	26230.	49.	65.	0.	0.2
62	16012.	56702.	0.	27132.	77.	77.	0.	0.2
63	15327.	55350.	0.	29131.	74.	118.	0.	0.2
64	15259.	53428.	0.	31157.	85.	71.	0.	0.1

gory unknown had been subtracted from the value for the total population. Column (8) expresses the unknown as a percentage of the original total. Column (7) does not have any analytical purposes. Although the cards had been checked for punching errors with a preliminary program for the IBM 1620, it was easy to insert an additional check, as the proportions by marital status for every age should add up to 100,000. Column (7), DIF, is the difference between 100,000 and the sum of the values for the marital status categories. This column should always give zero if no errors are involved.

Table 3 is a crucial table, as column (1) gives the proportions single from which ultimately the nuptiality rate will be calculated.

Step 2: Computation of proportions single by five-year-of-age groups (Table 4, subroutines PREADD, ADD, and CONTRO).—Even for the data by single age, it was thought useful for matters of comparison and correction to have a graduate series by single year of age. To obtain this graduated series, five-year-of-age proportions single were necessary. They are obtained in subroutines PREADD and ADD. The subroutine CONTRO checks for increases in proportions single by five years of age. It does not only compare successive proportions; it compares every proportion with all previous ones. From the moment a proportion larger than any previous one is discovered, an asterisk is placed after the larger proportion. The results are found in Table 4. As in this particular example, there were no increasing proportions, no asterisks were necessary.

Table 4.—FIVE-YEAR RAW PROPORTIONS SINGLE FOR NORWAY, 1900—FEMALES

Age	Single
15	98335.
20	77316.
25	48364.
30	32308.
35	25151.
40	21569.
45	18350.
50	17218.
55	16096.
60	16063.

Step 3: Computation of a graduated series of proportions single (Table 5, subroutines SPRAGU, SPRAGE, and KOSPRA).—The first two subroutines give the graduated figures based on the five year totals obtained in Step 2 with subroutines ADD and PREADD and shown in Table 4. Graduated figures are computed for ages 20–51. Whenever possible mid-panel Sprague multipliers were used. If the proportions by five years of age were increas-

Table 5.—GRADUATED PROPORTIONS SINGLE FOR NUPTIALITY TABLE FOR NORWAY, 1900—FEMALES

Age	AERCT(IN) (1)	HERCT(IN) (2)
15	99943.	99943.
16	99781.	99781.
17	99247.	99247.
18	97713.	97713.
19	94427.	94427.
20	87890.	87890.
21	83229.	83229.
22	77934.	77934.
23	71945.	71945.
24	65580.	65580.
25	59183.	59183.
26	52646.	52646.
27	47054.	47054.
28	42971.	42971.
29	39968.	39968.
30	37008.	37008.
31	34249.	34249.
32	31910.	31910.
33	29979.	29979.
34	28394.	28394.
35	27076.	27076.
36	26008.	26008.
37	25076.	25076.
38	24202.	24202.
39	23394.	23394.
40	22752.	22752.
41	22271.	22271.
42	21712.	21712.
43	20971.	20971.
44	20139.	20139.
45	19398.	19398.
46	18691.	18691.
47	18145.	18145.
48	17835.	17835.
49	17683.	17683.
50	17545.	17545.
51	17461.	17461.

(1) Proportions single obtained through graduation with Sprague multipliers from proportions single by five years of age

(2) Graduated series of column (1) after correction for increasing proportions single if necessary

ing after age group 45–49, graduated figures were obtained by using the Sprague end-panel or next-to-the-end-panel multipliers. If the proportions were increasing from age 45 or earlier, no graduation was made. In such a case the computer was programmed to report *increases in proportions single by five-year age group from group 45–49*. For the age group 15–19 the original figures were used. All these operations are done in subroutines SPRAGU and SPRAGE. The result is shown in column (1).

Even by avoiding the use of increasing proportions by five years of age increasing proportions by single year of age showed up in the graduated series. Therefore, a similar control for increasing proportions, as mentioned previously, was made. Similarly an asterisk was placed after an increasing proportion. A correction was made with linear interpolation. Checking and correcting are done in subroutine KOSPRA; column (2) is the result of this subroutine.

In this particular example, the graduated series does not contain any increasing proportion. The values of column (2) are, thus, completely identical to column (1).

Step 4: Computation of proportions single corrected for increasing proportions (Table 5, subroutines KONTRO, LINPOL, and ADAPT).—KONTRO is the subroutine checking for increasing proportions single obtained directly from the data in subroutine RAWPER and shown in column (1) of Table 3. An effort was made to distinguish between those resulting from inaccurate enumeration and those resulting from changing nuptiality patterns. It was arbitrarily assumed that, if more than three immediately succeeding increasing proportions were found, this was due to changing nuptiality patterns. Three categories could thus be distinguished with respect to the problem of increasing proportions single: (1) no proportions single increasing; (2) not more than three immediately succeeding proportions increasing; and (3) more than three successive proportions increasing.

Increasing proportions single are marked with an asterisk. Column (1) of Table 6 shows the results of this operation. This column is simply a repetition of column (1) of Table 2. It shows again the same proportions single computed from the census with the only element added, an asterisk, for increasing proportions if this was necessary.

For the first category no corrections are necessary. For the second category the values of the particular five-year age group in which such increasing proportions occurred were replaced with a graduated series by means of subroutine ADAPT. This is the case for the particular example of Norway given here. The values for ages 40–44 and 45–49 in column (1) of Table 6 have been replaced by the graduated values for these ages in column (2) of Table 5. Column (2) of Table 6 is the result of this operation.

An additional control was made if no new increasing proportions showed up at the beginning and immediately after the end of the replacing series. These operations are also performed in subroutine ADAPT and shown in column (3) of Table 6. In this case no new increasing proportions showed up, and column (3) is thus identical to column (2) of the same table. If there had been increasing proportions, the correction would have been made with linear interpolation.

For the third category, simple linear interpolation was used. At first sight this may seem rather crude. However, the apparition of more than three immediately succeeding increasing proportions was usually limited to the last part and very often to the last ages of the conventional nuptiality span. If the increasing proportions were found in the very last ages of the nuptiality span, the last nonincreasing proportion single was held constant for these ages. This is equivalent to closing the tables at an earlier age than 50, thereby reducing the nuptiality span. This linear interpolation, if necessary, is performed in subroutine LINPOL. If this was the case, Table 6 would have only two columns, the first identical in scope with column (1) of Table 6. The second column would give the values corrected with linear interpolation.

Step 5: Computation of proportions single at exact age x *(Table 7, subroutines GREVIL and VIKRO)*.—The census data refer on the average to exact age $x + \frac{1}{2}$. To obtain values for exact age x, interpolation is necessary. Two options were possible: A linear interpolation between the successive proportions single given in the census or a more refined method of interpolation could be used. For both options the necessary programming has been done. Linear interpolation has the advantage of not creating additional increasing proportions single. As marriage rates are changing rather rapidly during a short interval, the use of a more refined method of interpolation or graduation is theoretically preferred. Several methods have been tried. One possibility was to divide the one year interval with Sprague graduation in five parts and to make a linear interpolation between the values for ages $x - \frac{1}{2} + 0.4$ and $x - \frac{1}{2} + 0.6$. Another possibility was a set of multipliers developed by Grabill to divide the values for one year in ten parts. The method, however, which after many experiments seemed to give the fewest increasing proportions, was the one developed by Gre-

Table 6.—Proportions Single for Nuptiality Table for Norway, 1900—Females

Age	SINGLE(RAW) (1)	SINGLE(VERCT) (2)	SINGLE(WERCT) (3)
15	99943.	99943.	99943.
16	99781.	99781.	99781.
17	99247.	99247.	99247.
18	97713.	97713.	97713.
19	94427.	94427.	94427.
20	90499.	90499.	90499.
21	83839.	83839.	83839.
22	77157.	77157.	77157.
23	70283.	70283.	70283.
24	63739.	63739.	63739.
25	57495.	57495.	57495.
26	51749.	51749.	51749.
27	46868.	46868.	46868.
28	44627.	44627.	44627.
29	38594.	38594.	38594.
30	38067.	38067.	38067.
31	32829.	32829.	32829.
32	32050.	32050.	32050.
33	29964.	29964.	29964.
34	28152.	28152.	28152.
35	27493.	27076.	27076.
36	25935.	26008.	26008.
37	24269.	25076.	25076.
38	24529.*	24202.	24202.
39	23127.	23394.	23394.
40	25185.*	22752.	22752.
41	20659.	22271.	22271.
42	21283.*	21712.	21712.
43	19689.	20971.	20971.
44	20425.*	20139.	20139.
45	19306.	19398.	19398.
46	17895.	18691.	18691.
47	17946.*	18145.	18145.
48	18752.*	17835.	17835.
49	17724.	17683.	17683.
50	19995.*	17545.	17545.
51	16660.	16660.	16660.

(1) Proportions single as computed from census data with asterisk for increasing proportions

(2) Proportions single corrected for increasing proportions with graduated figures

(3) Proportions single after final correction has been made if still necessary

ville.[11] These multipliers have been primarily developed for obtaining values for single age from five year totals. The particular coefficients used to compute the values for the first ten ages from the first five year totals can be applied to the values by single age for obtaining values for tenths of one year interval. Only one column of multipliers had to be used as only one value from the ten was needed. Even the use of Greville multipliers does not prevent occasional increasing proportions single.

The figures from which the values for exact age have to be obtained are first shown in column (1) of Table 7. This column is identical to column (3) of Table 6. Column (2) of Table 7 gives the values for exact age, computed in subroutine GREVIL. The ones shown here have been obtained with linear interpolation. Column (3) of Table 6 gives the values of column (2) after a final check and correction, if necessary, have been made for increasing proportions. This column is the result of subroutine VIKRO.

Step 6: Computation of gross nuptiality table (Tables 8A and 8B, subroutine NUPTIL).—Once the previous preparatory calculations have been completed, the nuptiality rate as given by formula (44), and hence gross nuptiality tables, can

[11] H. H. Wolfenden, *Population Statistics and Their Compilation* (Chicago, 1954), p. 151, Table 2, A.

Table 7.—Proportions Single at Exact Age X for Norway, 1900—Females

Age	WERCT(I) (1)	CPERCT(I) (2)	ANUP(I,3) (3)
15	99943.	100000.	100000.
16	99781.	99862.	99862.
17	99247.	99514.	99514.
18	97713.	98480.	98480.
19	94427.	96070.	96070.
20	90499.	92463.	92463.
21	83839.	87169.	87169.
22	77157.	80498.	80498.
23	70283.	73720.	73720.
24	63739.	67011.	67011.
25	57495.	60617.	60617.
26	51749.	54622.	54622.
27	46868.	49309.	49309.
28	44627.	45748.	45748.
29	38594.	41611.	41611.
30	38067.	38331.	38331.
31	32829.	35448.	35448.
32	32050.	32440.	32440.
33	29964.	31007.	31007.
34	28152.	29058.	29058.
35	27076.	27614.	27614.
36	26008.	26542.	26542.
37	25076.	25542.	25542.
38	24202.	24639.	24639.
39	23394.	23798.	23798.
40	22752.	23073.	23073.
41	22271.	22512.	22512.
42	21712.	21992.	21992.
43	20971.	21341.	21341.
44	20139.	20555.	20555.
45	19398.	19768.	19768.
46	18691.	19044.	19044.
47	18145.	18418.	18418.
48	17835.	17990.	17990.
49	17683.	17759.	17759.
50	17545.	17614.	17614.

(1) Proportions single as computed from census and corrected for increasing proportions (= Column 3 of Table 6)

(2) Proportions single at exact age before final correction for increasing proportions if necessary

(3) Proportions single at exact age after final correction for increasing proportions if necessary

be computed. This is done in subroutine NUPTIL.

Table 8A is the main part of the nuptiality table. Table 8B consists of additional columns useful but not necessary for the final presentation of the nuptiality table. The interpretation of a gross nuptiality table has already been clarified in Section I, 2, and an example was given in Table 1. Table 1 is nothing else but the combination of all the columns of a table such as Table 8A with column (2) of Table 8B. The interpretation of columns (1) to (6) of Table 8A is thus the same as for columns (1) to (6) of Table 1. Columns (1) and (2) give, respectively, the marriage and the nuptiality rates. The marriage rate is inferred from the nuptiality rate according to the inverse of formula (17):

$$m_x = \frac{2n_x}{2 - n_x}.$$

Column (3) of Table 8A, identical to column (3) of Table 7, gives the proportions single at exact age x. As indicated in the above treatment of the nuptiality table, this column is analogous to the l_x column of an ordinary life table. Since the nuptiality rate according to formula (44) is inferred from successive differences between proportions single, column (3) gives the values of the fundamental function

Table 8A.—NUPTIALITY TABLE FOR NORWAY, 1900—FEMALES

Age	MAR(X) (1)	NUP(X) (2)	SIG(X) (3)	M(X) (4)	E(X) (5)	SL(X) (6)	PRE(X) (7)	ADS(X) (8)
15	1.38	0.0014	100000.	138.	82386.	99931.	0.8239	16.0
16	3.49	0.0035	99862.	348.	82248.	99688.	0.8236	15.0
17	10.44	0.0104	99514.	1034.	81900.	98997.	0.8230	14.1
18	24.78	0.0245	98480.	2410.	80866.	97275.	0.8211	13.2
19	38.26	0.0375	96070.	3607.	78456.	94267.	0.8167	12.6
20	58.94	0.0573	92463.	5294.	74849.	89816.	0.8095	12.0
21	79.57	0.0765	87169.	6671.	69555.	83834.	0.7979	11.7
22	87.90	0.0842	80498.	6778.	62884.	77109.	0.7812	11.7
23	95.35	0.0910	73720.	6709.	56106.	70366.	0.7611	11.7
24	100.20	0.0954	67011.	6394.	49397.	63814.	0.7371	11.8
25	104.04	0.0989	60617.	5995.	43003.	57619.	0.7094	12.0
26	102.25	0.0973	54622.	5313.	37008.	51965.	0.6775	12.3
27	74.92	0.0722	49309.	3561.	31695.	47528.	0.6428	12.5
28	94.72	0.0904	45748.	4137.	28134.	43679.	0.6150	12.5
29	82.06	0.0788	41611.	3280.	23997.	39971.	0.5767	12.7
30	78.14	0.0752	38331.	2883.	20717.	36889.	0.5405	12.7
31	88.63	0.0849	35448.	3008.	17834.	33944.	0.5031	12.7
32	45.16	0.0442	32440.	1433.	14826.	31723.	0.4570	12.8
33	64.90	0.0629	31007.	1949.	13393.	30032.	0.4319	12.4
34	50.96	0.0497	29058.	1444.	11444.	28336.	0.3938	12.2
35	39.59	0.0388	27614.	1072.	10000.	27078.	0.3621	11.8
36	38.40	0.0377	26542.	1000.	8928.	26042.	0.3364	11.2
37	36.00	0.0354	25542.	903.	7928.	25090.	0.3104	10.7
38*	34.71	0.0341	24639.	841.	7025.	24218.	0.2851	10.0
39	30.93	0.0305	23798.	725.	6184.	23436.	0.2599	9.4
40*	24.64	0.0243	23073.	562.	5459.	22792.	0.2366	8.7
41	23.36	0.0231	22512.	520.	4898.	22252.	0.2176	7.9
42*	30.01	0.0296	21992.	650.	4378.	21667.	0.1991	7.0
43	37.56	0.0369	21341.	787.	3728.	20948.	0.1747	6.2
44*	39.01	0.0383	20555.	786.	2941.	20162.	0.1431	5.5
45	37.30	0.0366	19768.	724.	2154.	19406.	0.1090	4.6
46	33.43	0.0329	19044.	626.	1431.	18731.	0.0751	3.8
47*	23.51	0.0232	18418.	428.	805.	18204.	0.0437	2.9
48*	12.95	0.0129	17990.	231.	376.	17875.	0.0209	2.0
49	8.20	0.0082	17759.	145.	145.	17686.	0.0082	1.0
50*	0.	0.	17614.	0.	0.	0.	0.	0.

(1) Marriage rate

(2) Nuptiality rate

(3) Proportion single at exact age X

(4) Persons marrying for the first time during year of age X

(5) Persons marrying during year of age X and all further years

(6) Number of single-persons lived during each age interim

(7) Probability of ever marrying at age X

(8) Average expectancy of remaining single

from which the other functions of the nuptiality table have to be derived when using census data on marital status. Column (4) gives these successive differences indicating the number of people marrying during each age interval. These differences divided by the proportions single at exact age x will give the nuptiality rates of column (2). Column (8) of Table 8A is the average duration of bachelorhood or spinsterhood. It has the same interpretation as column (9) of Table 1. In order to interpret correctly this average duration of remaining single, it should be stressed that these nuptiality tables close at age 50, assuming no further first marriages after exact age x.

Column (1) of Table 8B expresses the average expectancy of remaining single given in column (8) of Table 8A as a percentage of the maximum possible duration of remaining single until the closing age of the table. This last value is given in column (3) of Table 8B. Column (2) of the same table gives the total number of single-person years lived after age x. It is equivalent to the values of column (7) in Table 1. Column (4) gives the number of persons ever married at age x.

Table 8B.—NUPTIALITY TABLE FOR NORWAY, 1900—FEMALES

Age	PRADS(X) (1)	STA2(X) (2)	DIF (3)	EMA(X) (4)
15	45.78	1602371.	35.	0.
16	44.25	1502440.	34.	138.
17	42.72	1402752.	33.	486.
18	41.37	1303754.	32.	1520.
19	40.51	1206479.	31.	3930.
20	40.10	1112213.	30.	7537.
21	40.44	1022397.	29.	12831.
22	41.64	938563.	28.	19502.
23	43.28	861454.	27.	26280.
24	45.41	791088.	26.	32989.
25	47.99	727274.	25.	39383.
26	51.08	669655.	24.	45378.
27	54.47	617690.	23.	50691.
28	56.65	570161.	22.	54252.
29	60.25	526482.	21.	58389.
30	63.46	486512.	20.	61669.
31	66.76	449622.	19.	64552.
32	71.19	415679.	18.	67560.
33	72.84	383955.	17.	68993.
34	76.12	353923.	16.	70942.
35	78.61	325587.	15.	72386.
36	80.33	298509.	14.	73458.
37	82.06	272467.	13.	74458.
38*	83.67	247377.	12.	75361.
39	85.25	223159.	11.	76202.
40*	86.56	199723.	10.	76927.
41	87.33	176931.	9.	77488.
42*	87.92	154679.	8.	78008.
43	89.04	133012.	7.	78659.
44*	90.87	112064.	6.	79445.
45	92.98	91903.	5.	80232.
46	95.17	72496.	4.	80956.
47*	97.30	53765.	3.	81582.
48*	98.83	35561.	2.	82010.
49	99.59	17686.	1.	82241.
50*	0.	0.	0.	82386.

(1) Average expectancy of remaining single expressed as a percentage of difference between age X and age 50

(2) Total number of single-person years lived after age X

(3) Difference between age X and age 50

(4) Persons ever married at exact age X

Step 7: Computation of singulate mean age and median age at marriage (Table 9, subroutines MEAN and MEDIAN).—Four slightly different singulate mean ages at marriage have been computed in this step, according to a slightly different formula.[12] The first two are based on the proportions single by single year of age; the next two are derived from the porportions single by five years of age.

Step 8: Computation of a gross nuptiality table based on the graduation of the marriage rates (Table 1, subroutines NUSPRA and TROUW).—From the marriage rates by single year of age obtained in subroutine NUPTIL and presented in column (1) of Table 8A, totals by five years of age were obtained. Sprague graduation applied on these totals gave a new series of marriage rates by single year of age. This is done in subroutine NUSPRA. From this new series of marriage rates by single year of age, all the other functions of the gross nuptiality table have been computed in subroutine TROUW. The output is in exactly the same form as given for Tables 8A and 8B. The tables are not presented

[12] For a treatment of the singulate mean age at marriage, see J. Jajnal, "Age at Marriage and Proportions Marrying," *Population Studies*, VIII, 1953–54, pp. 111–36.

Table 9.—SINGULATE MEAN AND MEDIAN AGE AT MARRIAGE FOR NORWAY, 1900—FEMALES

Singulate mean age at marriage (1) . . .	27.47
Singulate mean age at marriage (2) . . .	26.97
Singulate mean age at marriage (3) . . .	26.35
Singulate mean age at marriage (4) . . .	26.35
Median age at marriage	25.30

here, as Table 1 is derived from this output.

Step 9: Computation of singulate mean age and median age at marriage based on the proportions single obtained in Step 8 (Table 10, subroutine MOYEN).—As indicated before, two series of proportions single can be obtained from the census data on marital status by single year of age. The first series is directly obtained from the raw data and was the base of the tables shown thus far. The second series is computed from the proportions single by five years of age. These values have been computed in Step 3 and are shown in Table 5. The right-hand side of the flow diagram presented in Table 2 gives the computational strategy for the construction of the nuptiality tables based on the graduated series of the proportions single. Steps 5 to 9 are identical with the ones described above. The tables are presented in Appendix 2.

5. Computation of proportions single for exact age x (Table 14, subroutines GREVIL and VIKRO).
6. Computation of nuptiality rate and the gross nuptiality table (Tables 15A and B, subroutine NUPTIL).
7. Computation of singulate mean age and median age at marriage (Table 16, subroutines MEAN and MEDIAN).
8. Computation of gross nuptiality tables based on the graduation of five-year totals of marriage rates obtained in Step 6 (Tables 17A and 17B, subroutines NUSPRA and TROUW).
9. Computation of singulate mean ages and median age at marriage based on the results obtained in Step 8 (Table 18, subroutine MOYEN).

From the data by five years of age, only one series of proportions single by single year of age can be obtained. This is done with Sprague multipliers, as described in Step 3 for the data by single year of age. For the ages 15–16–17–18–19, linear regression equations were used. The flow diagram of the computational strategy used for census data on marital status is presented in Table 11. With the exception of the use of regression equations to obtain proportions single for ages 15–16–17–18–19, this flow diagram is not substantially different from the one presented in Table 2. The flow diagram in Table 11 refers to the tables obtained from the data by five years of age. The tables have been reproduced in Appendix 3[13]. The computational strategy presented in Table 11 comprehends nine steps:

1. Computation of proportions of marital status by five-year-of-age groups (Table 19, subroutine RAWPER).
2. Control for increasing proportions by five years of age (Table 20, subroutine CONTRO).
3. Computations of proportions single for ages 15–16–17–18–19 with linear regression (Table 21, subroutine GODIVA).
4. Computation of graduated series of proportions single by single year of age with Sprague multipliers (Table 22, subroutines SPRAGU, SPRAGE, and KOSPRA).
5. Computations of proportions single for exact age x (Table 23, subroutines GREVIL and VIKRO).
6. Computation of nuptiality rate and the gross nuptiality table (Tables 24A and 24B, subroutine NUPTIL).
7. Computation of singulate mean age and median age at marriage (Table 25, subroutines MEAN and MEDIAN).
8. Computation of gross nuptiality tables based on the graduation of the marriage rates obtained in Step 5 and added up by five years of age (Table 26A and 26B, subroutines NUSPRA and TROUW).

Table 10.—Singulate Mean and Median Age at Marriage for Norway, 1900—Females

Singulate mean age at marriage (5) . . .	27.05
Singulate mean age at marriage (6) . . .	26.55
Median age at marriage	23.10

[13] Illustrations of the procedure for calculating nuptiality tables from data by single year of age based on a graduated series derived from their five-year-of-age totals are reproduced in Appendix 2. Illustrations of the procedure for calculating from data for five-year-age groups are presented in Appendix 3.

Copies of these two appendixes may be obtained from the Community and Family Study Center, University of Chicago, for the cost of reproducing. The textual material presented here describes contents of these appendixes briefly.

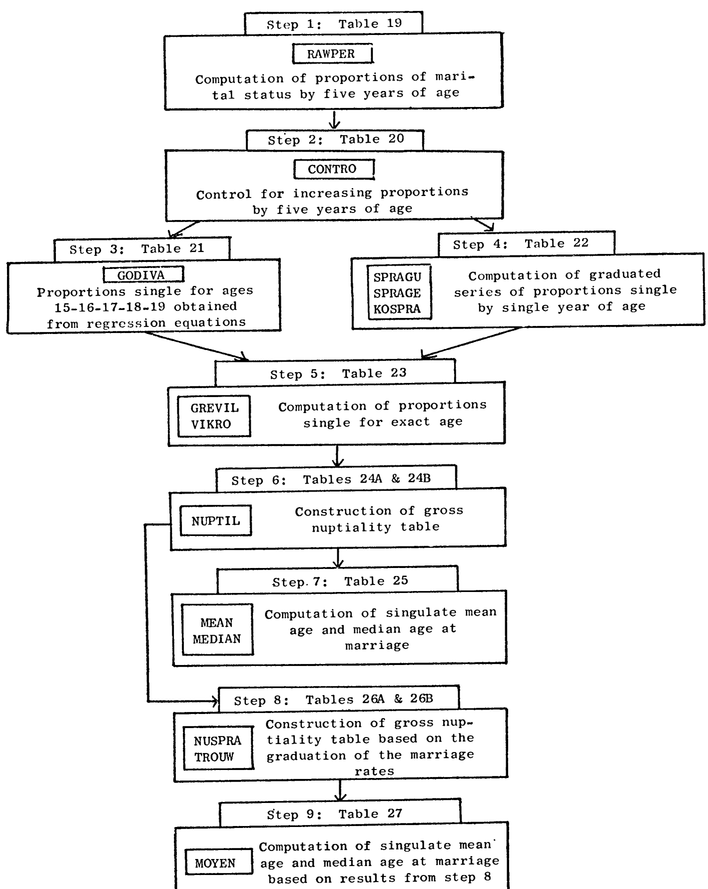
9. Computation of singulate mean age and median age at marriage based on the results obtained in Step 8 (Table 27, subroutine MOYEN).

With the exception of the use of regression equations in Step 3 to calculate values of proportions single for ages 15–16–17–18–19, these steps do not need further clarification, as they are on the whole similar to the ones used in the treatment of the data on marital status by single year of age.

Two sets of regression equations have been computed—one for males, the other for females. They are presented in Table 12. They are based on 79 cases for the males and upon 78 for the females. The amount of variation explained by the regression equation is quite substantial, as can be seen from Table 13, showing the square of the multiple correlation coefficient between the dependent variable and the independent variables which are the values for the five-year age groups between 15 and 64. The same table also shows that the presence of correlation is significant at a high level.

These regression equations seem to give far better results for ages 15–16–17–18–19 than the use of graduation procedures. Al-

Table 11.—Flow Diagram of Computational Strategy to Construct Gross Nuptiality Tables from Census Data by Five Years of Age



though proportions greater than 100,000 show up occasionally, their occurrence is far less frequent. On a total of 294 cases,44 cases came up with proportions greater than 100,000. Of these 44 cases the excess over 100,000 was mostly limited to a very small amount, so that only in a few cases the data had to be discarded for the construction of nuptiality rables. The regression equation does not give good results either for countries, such as the case for several censuses of India with a substantial amount of married people in the 10–14 group shows. Very small values or even negative values for ages 18 and 19 will show up. This is because these regression equations involve a bias. They are based on information by single year of age. This is on the whole only available for Western countries. The nuptiality patterns of those countries for ages 15–16–17–18–19 which happen to be mostly countries of Western

Table 12.—REGRESSION EQUATIONS TO INFER PROPORTIONS SINGLE FOR AGES 15–16–17–18–19 FROM PROPORTIONS BY FIVE YEARS OF AGE

A. Regression Equations for Males

for age 15

BERCT(1) = 93269.515 + 0.072254* FERCT(1) - 0.007097* FERCT(2) - 0.014785* FERCT(10) + 0.020449* FERCT(9) - 0.018300* FERCT(6) - 0.018227* FERCT(8) + 0.017246* FERCT(7) + 0.011471* FERCT(5) + 0.003891* FERCT(3) - 0.002904* FERCT(4)

for age 16

BERCT(2) = 85692.157 + 0.142704* FERCT(1) + 0.025832* FERCT(4) - 0.012898* FERCT(3) - 0.033173* FERCT(5) - 0.018505* FERCT(10) + 0.023889* FERCT(9) + 0.024204* FERCT(6) - 0.020911* FERCT(8) + 0.004330* FERCT(2) + 0.005200* FERCT(7)

for age 17

BERCT(3) = 53884.849 + 0.461554* FERCT(1) - 0.099155* FERCT(6) + 0.071046* FERCT(7) - 0.015888* FERCT(3) + 0.038174* FERCT(5) - 0.026438* FERCT(10) + 0.013654* FERCT(4) + 0.022067* FERCT(9) + 0.005935* FERCT(2) - 0.014875* FERCT(8)

for age 18

BERCT(4) = -41765.558 + 1.420524* FERCT(1) - 0.110339* FERCT(6) + 0.104520* FERCT(7) + 0.049811* FERCT(5) - 0.038974* FERCT(10) + 0.048774* FERCT(9) - 0.043859* FERCT(8) - 0.010256* FERCT(4) - 0.0028505* FERCT(3) - 0.000135* FERCT(2)

for age 19

BERCT(5) = -193472.019 + 2.909600* FERCT(1) + 0.104382* FERCT(10) - 0.109239 FERCT(9) + 0.106229* FERCT(6) - 0.076129* FERCT(5) + 0.020117* FERCT(2) + 0.018367* FERCT(3) - 0.041102* FERCT(7) + 0.020035* FERCT(8) - 0.006563* FERCT(4)

B. Regression Equation for Females

for age 15

BERCT(1) = 79259.362 + 0.249757* FERCT(1) - 0.061636* FERCT(2) - 0.157709* FERCT(10) + 0.227754* FERCT(9) - 0.215434* FERCT(8) + 0.153719* FERCT(7) + 0.105389* FERCT(5) - 0.107586* FERCT(6) + 0.007418* FERCT(3) + 0.000204* FERCT(4)

for age 16

BERCT(2) = 45268.838 + 0.630286* FERCT(1) - 0.118218* FERCT(2) - 0.204354* FERCT(10) + 0.238162* FERCT(5) + 0.217695* FERCT(7) - 0.272702* FERCT(9) - 0.239204* FERCT(6) - 0.236575* FERCT(8) + 0.027740* FERCT(3) - 0.045761* FERCT(4)

for age 17

BERCT(3) = -3789.620 + 1.129758* FERCT(1) - 0.124618* FERCT(2) - 0.345252* FERCT(6) + 0.300259* FERCT(5) + 0.183819* FERCT(7) - 0.103905* FERCT(4) + 0.044672* FERCT(3) - 0.054085* FERCT(10) + 0.011926* FERCT(8) -0.006713* FERCT(9)

for age 18

BERCT(4) = -41089.083 + 1.370458* FERCT(1) + 0.092354* FERCT(2) -0.325678* FERCT(9) + 0.256625* FERCT(5) + 0.178362* FERCT(10) - 0.229278* FERCT(6) - 0.133230* FERCT(4)+0.187099* FERCT(8) - 0.058672* FERCT(3) + 0.084287* FERCT(7)

for age 19

BERCT(5) = -59486.532 + 1.312561* FERCT(1) + 0.428181* FERCT(2) - 0.145256*FERCT(3) + 0.197381* FERCT(10) - 0.326364* FERCT(9) + 0.221899* FERCT(8) - 0.133641* FERCT(4) + 0.081841* FERCT(5) - 0.022645* FERCT(7) - 0.009782* FERCT(6)

* BERCT (I), the independent variable, indicates the proportions single for ages 15-16-17-18-19. FERCT (I), the dependent variable, indicates the proportions single by five year age groups going from 15-19 till 60-64 included. FERCT (1) indicates thus the proportion single for age group 15-19 and FERCT(10) the proportion single for age group 60-64. The order in which the independent variables are given in the regression equation is according to the F-value of the partial correlation coefficients.

civilization are thus reflected in these regression equations.[14]

[14] For this project, nuptiality tables for different countries have been computed. With the same program, nuptiality tables for territorial subdivisions or social groups can be computed if the necessary data are available. This program on IBM cards can be obtained for a nominal cost from the author, Centre d'Etudes Démographiques et Département de Sociologie, C. P. 6128, Université de Montréal, Montréal, Canada.

Table 13.—Square of Multiple Correlation Coefficients with Their F-Values between Proportions Single for Ages 15–16–17–18–19 and the Proportions Single by Five Years of Age between Ages 15 and 64

Dependent Variable	Males		Females	
	R^2	$F_{(10,68)}$	R^2	$F_{(10,67)}$
15........	0.57523	9.21	0.89144	55.02
16........	.69472	15.47	.95496	142.07
7.........	.88402	51.83	.97966	322.71
18........	.97183	234.59	.99167	797.55
19........	0.98619	485.48	0.99065	709.96

APPENDIX 1

Program to Construct Nuptiality Tables from Census Data on Marital Status by Single Year of Age

Preliminary Remark: As the DIMENSION and COMMON statements are identical for all subroutines they have been only given in the main program and have not been repeated in the subroutines.

```
      MAIN PROGRAM
      PROGRAM TO COMPILE NUPTIALITY TABLES
      DIMENSION           TITLE(10),IAGE(100),DATA(100,8),TOTA(100),CHE
     1CK(100),FERCT(100),TATB(100,8),LAGE(20),           TATA(100,8),GER
     2CT(80),AERCT(100),JAGE(100),SIG2N(90),HERCT(100),SIGIN(100),NCONT(
     390),WERCT(100),NCOMT(100),NDIS(50),VERCT(100),AMULT(5),STAR(50),CP
     4ERCT(50),ANUP(50,12),AIAGE(50),JIAGE(50),PERCT(100,7),SIG5N(100),
     5SPRAG(15,5),SIG4N(100),SAGIN(100),LUTCH(50),LETCH(50),C(50),
     6KATCH(50),KITCH(50),B(50),LATCH(50),LITCH(50),A(50),ENUP(10),UNUP(
     750,12),SPRIG(10,4),ETOL(50)
      COMMON SPRAG, TITLE,IAGE,DATA,TOTA,CHECK,AERCT,JAGE,SIG2N,HERCT,SI
     1GIN, NCONT,PERCT,TATB,LAGE,FERCT,TATA,GERCT,WERCT,NCOMT,NDIS,VERCT
     2,AMULT,STAR,CPERCT,ANUP,AIAGE,JIAGE,KALFA,KAR,KOR,KENA,NENA,NTEL,N
     3SIM,NENO1,NTEL1,NSIM1,IT,NGROUP,NUM,NALFA,IMEGA,KEND,NEND,      KE
     4N4,NENC,KENB,NENB,SIG5N,NENE,NUMI,KAN,SIG4N,SAGIN,KEND1,LAK,LUTCH,
     5C,KATCH,KITCH,B,LATCH,LITCH,A,LETCH,LF,NAGES,NFGES,NBETA,
     6ENUP,UNUP,SPRIG,NCOD ,ETOL
      READ INPUT TAPE 5,1,((SPRAG(II,JJ),JJ=1,5),II=1,15)
    1 FORMAT(5F8.4)
      READ INPUT TAPE 5,3,((SPRIG(II,JJ),JJ=1,4),II=1,10)
    3 FORMAT(4F8.4)
      READ INPUT TAPE 5,2,NCOUNT
    2 FORMAT(I3)
      WRITE OUTPUT TAPE 6,6,NCOUNT
    6 FORMAT(12H1 NCOUNT IS I3)
      DO 1000 K=1,NCOUNT
      READ INPUT TAPE 5,4,TITLE,NGROUP,NUM,NAGES,NFGES,NALFA,NBETA,NCOD
    4 FORMAT(10A6,I1,5I2,4X,I5)
      WRITE OUTPUT TAPE 6,9,TITLE,NGROUP,NUM,NAGES,NFGES,NALFA,NBETA
    9 FORMAT(10H1           10A6,I2,5I2)
      READ INPUT TAPE 5,5,(IAGE(I),(DATA(I,J),J=1,8),I=1,NUM)
    5 FORMAT(I2,3X,7F9.0,F7.0,5X)
      IF(IGROUP-2)10,30,30
   10 IF(IAGE(NUM)-64)40,41,41
   40 NUMI=NUM
      KAN=(IAGE(NUM)-14)/5
      IMEGA=IAGE(NUM)-3-IAGE(1)
      GO TO 42
   41 NUMI=65-IAGE(1)
      KAN=10
      IMEGA=61-IAGE(1)
   42 CALL RAWPER
      CALL PREADO
      CALL ACD
      DO 11 IR=1,10
   11 GERCT(IR+1)=FERCT(IR)
      DO 600 I=1,10
      IF(IAGE(I)-15)600,650,600
  600 CONTINUE
  650 IAG=I
      DO 700 M=1,5
      AERCT(M)=PERCT(IAG,1)
  700 IAG=IAG+1
      CALL SPRAGU
      KEND=NALFA-IAGE(1)+1
      NEND=51-IAGE(1)+1
      KEND1=KEND+1
      NEND1=NEND-1
      KENA=KEND
      NENA=NENC
      NENC=NENA-KENA
      NENE=NENC-1
      NENB=NENA-1
      KENB=KENA-1
      CALL KONTRC
      IF(LAK-2)16,50,55
   50 IF(KAR-2)55,60,60
   60 CALL ADAPT
      GO TO 16
   55 CALL LINPCL
   16 CALL GREVIL
      CALL NUPTIL
      CALL MEAN
      CALL MEDIAN
      CALL NUSPRA
      CALL TROUW
      CALL MOYEN
      CALL PUNCH
      GO TO 1000
   30 WRITE OUTPUT TAPE 6,31,(TITLE(N),N=1,10)
   31 FORMAT(9H1 ABSURD,   10A6)
 1000 CONTINUE
 1001 CALL EXIT
      END
      RAW PERCENTAGES
      SUBROUTINE RAWPER
      DO 50 I=1,NUMI
      TOTA(I)=DATA(I,1)-DATA(I,8)
      CHECK(I)=100000.
      PERCT(I,7)=100.*DATA(I,8)/DATA(I,1)
      DO 50 LL=1,6
      PERCT(I,LL)=100000.*DATA(I,LL+1)/TOTA(I)
   50 CHECK(I)=CHECK(I)-PERCT(I,LL)
      DO 80 J=1,7
```

```
      DO 80 I=1,NUMI
      IF(PERCT(I,J))81,81,80
   81 PERCT(I,J)=ABSF(PERCT(I,J))
   80 CONTINUE
      DO 90 I=1,NUMI
      IF(CHECK(I))91,91,90
   91 CHECK(I)=ABSF(CHECK(I))
   90 CONTINUE
      WRITE OUTPUT TAPE 6,51,(TITLE(N),N=1,10)
   51 FORMAT(26H1 RAW PERCENTAGES FOR THE 10A6 ////)
      WRITE OUTPUT TAPE 6,52
   52 FORMAT(87H     AGE     SINGLE    MARRIED CONS. MR.   WIDOWED   DIVORCE
     1D     SEPAR.        DIF.    UNKNOWN    ///)
      WRITE OUTPUT TAPE 6,53,(IAGE(I),(PERCT(I,J),J=1,6),CHECK(I),PERCT(
     1I,7),I=1,NUMI)
   53 FORMAT(5X,I2,7F10.0,F10.1)
      RETURN
      END
      PREPARATION FOR THE ADDITION
      SUBROUTINE PREADD
      DIMENSION             TITLE(10),IAGE(100),DATA(100,8),TOTA(100),CHE
     1CK(100),FERCT(100),TATB(100,8),LAOE(20),          TATA(100,8),GER
     2CT(80),AERCT(100),JAGE(100),SIG2N(90),HERCT(100),SIGIN(100),NCONT(
     390),WERCT(100),NCOMT(100),NDIS(50),VERCT(100),AMULT(5),STAR(50),CP
     4ERCT(50),ANUP(50,12),AIAGE(50),JIAGE(50),PERCT(100,7),SIG5N(100),
     5SPRAG(15,5),SIG4N(100),SAGIN(100),LUTCH(50),LETCH(50),C(50),
     6KATCH(50),KITCH(50),B(50),LATCH(50),LITCH(50),A(50),ENUP(10),UNUP(
     750,12),SPRIG(10,4),ETOL(50)
      COMMON SPRAG, TITLE,IAGE,DATA,TOTA,CHECK,AERCT,JAGE,SIG2N,HERCT,SI
     1GIN, NCONT,PERCT,TATB,LAGE,FERCT,TATA,GERCT,WERCT,NCOMT,NDIS,VERCT
     2,AMULT,STAR,CPERCT,ANUP,AIAGE,JIAGE,KALFA,KAR,KOR,KENA,NENA,NTEL,N
     3SIM,NEND1,NTEL1,NSIM1,IT,NGROUP,NUM,NALFA,IMEGA,KEND,NEND,       KE
     4N4,NENC,KENB,NENB,SIG5N,NENE,NUMI,KAN,SIG4N,SAGIN,KEND1,LAK,LUTCH,
     5C,KATCH,KITCH,B,LATCH,LITCH,A,LETCH,LF,NAGES,NFGES,NBETA,
     6ENUP,UNUP,SPRIG,NCOD ,ETOL
      IF(15-IAGE(1))100,105,110
  100 KALFA=21-IAGE(1)
      KKLFA=20-IAGE(1)
      DO 101 J=1,8
      TATB(1,J)=DATA(1,J)
      DO 101 KR=2,KKLFA
  101 TATB(1,J)=TATB(1,J)+DATA(KR,J)
      FERCT(1)=100000.*TATB(1,2)/(TATB(1,1)-TATB(1,8))
      LAGE(1)=IAGE(1)
      LAGE(2)=20
      DO 102 IT=2,9
  102 LAGE(IT+1)=LAGE(IT)+5
      IT=2
      GO TO 116
  105 KALFA=1
      GO TO 111
  110 KALFA=16-IAGE(1)
  111 LAGE(1)=15
      DO 112 IT=1,9
  112 LAGE(IT+1)=LAGE(IT)+5
      IT=1
  116 RETURN
      END
      ADDITION
      SUBROUTINE ADD
      DO 201 I=KALFA,IMEGA,5
      DO 200 J=1,8
      TATA(I,J)=DATA(I,J)
      DO 200 KN =1,4
      KIN=I+KN
  200 TATA(I,J)=TATA(I,J)+DATA(KIN,J)
      FERCT(IT)=100000.*TATA(I,2)/(TATA(I,1)-TATA(I,8))
  201 IT=IT+1
      CALL CONTRC
      WRITE OUTPUT TAPE 6,205,(TITLE(N),N=1,10)
  205 FORMAT(36H1 FIVE YEAR RAW PERCENTAGES FOR THE 10A6////)
      WRITE OUTPUT TAPE 6,206
  206 FORMAT(18H     AGE     SINGLE ///)
      WRITE OUTPUT TAPE 6,207,(LAGE(IT),FERCT(IT),SIG4N(IT),IT=1,KAN)
  207 FORMAT(5X,I2,F9.0,A1)
      RETURN
      END
      CONTROL FOR INCREASES IN PROPORTIONS SINGLE BY FIVE YEAR OF AGE
      SUBROUTINE CONTRL
      KAN1=KAN-1
      DO 121 IT=1,KAN
  121 SIG4N(IT)=(+6H
      DO 150 IT=1,KAN1
      KIT=KAN-IT
      DO 150 KT=1,KIT
      KTI=IT+KT
      IF(FERCT(IT)-FERCT(KTI))122,125,150
  122 SIG4N(KTI)=(+6H*      )
      GO TO 150
  125 IF(IT+1-KTI)122,150,150
  150 CONTINUE
      RETURN
      END
      USE OF SPRAGUE MULTIPLIERS
      SUBROUTINE SPRAGU
      GERCT(1)=100000.
      DO 205 IG=1,7
      IF(GERCT(IG+1)-GERCT(IG))205,205,210
  205 CONTINUE
      IF (GERCT(9)-GERCT(8))206,206,215
  206 IF(GERCT(10)-GERCT(9))207,207,220
  207 IF(GERCT(11)-GERCT(10))208,208,225
  210 WRITE OUTPUT TAPE 6,211,(TITLE(N),N=1,10)
  211 FORMAT(76H0 INCREASES IN PROPORTIONS SINGLE BY FIVE YEAR AGE GROUP
     1S   FROM   AGE 45 FOR            )
      KAR=1
      GO TO 155
  215 LF=6
      CALL SPRAGE
      IK=26
      DO 216 II=6,15
      AERCT(IK)=5.*(SPRAG(II,1 )*GERCT(5)+SPRAG(II,2)*GERCT(6)+SPRAG(II,
     1 3)*GERCT(7)+SPRAG(II,4)*GERCT(8))
```

```
216 IK=IK+1
    AERCT(36)=AERCT(35)
    AERCT(37)=AERCT(35)
    GO TO 250
220 LF=7
    CALL SPRAGE
    IK=31
    DO 221 II=6,15
    AERCT(IK)=5.*(SPRAG(II,1 )*GERCT(6)+SPRAG(II,2)*GERCT(7)+SPRAG(II,
   1 3)*GERCT(8)+SPRAG(II,4)*GERCT(9))
221 IK=IK+1
    GO TO 250
225 LF=8
    CALL SPRAGE
    AERCT(36)=5.*(SPRAG(6 ,1 )*GERCT(7)+SPRAG(6 ,2)*GERCT(8)+SPRAG(6 ,
   1 3)*GERCT(9)+SPRAG(6 ,4)*GERCT(9))
    AERCT(37)=5.*(SPRAG(7 ,1 )*GERCT(7)+SPRAG(7 ,2)*GERCT(8)+SPRAG(7 ,
   1 3)*GERCT(9)+SPRAG(7 ,4)*GERCT(9))
    GO TO  250
208 LF=9
    CALL SPRAGE
250 DO 251 IN=1,37
251 JAGE(IN)=14+IN
    DO 255 IN=1,37
    SIG2N(IN)=(+6H       )
255 HERCT(IN)=AERCT(IN)
    KNAL=0
    DO 150 IN=1,36
    KOTA=37-IN
     DC 150 NT=1,KOTA
    INT=IN+NT
    IF(AERCT(IN)-AERCT(INT))122,125,150
122 IF(SIG2N(INT)-(+6H*      ))2,150,2
  2 SIG2N(INT)=(+6H*      )
    KNAL=KNAL+1
    GC TO 150
125 IF(IN+1-INT)122,150,150
150 CCNTINUE
    IF(KNAL)700,700,710
700 KAR=3
    GO TO 155
710 KAR=2
155 IF(KAR-2)161,720,156
720 CALL KOSPRA
156 WRITE OUTPUT TAPE 6,157
157 FORMAT(10H1          ///////)
    WRITE OUTPUT TAPE 6,158,(TITLE(N),N=1,10)
158 FORMAT(35HO GRADUATED PROPORTIONS SINGLE FOR 10A6 /////)
    WRITE OUTPUT TAPE 6,159
159 FORMAT(36H     AGE      AERCT(IN)      HERCT(IN)     ///)
    WRITE OUTPUT TAPE 6,160,(JAGE(IN),AERCT(IN),SIG2N(IN),HERCT(IN),IN
   1=1,37)
160 FORMAT(5X,I2,F14.0,A1,F15.0)
161 RETURN
    END
    MIDPANEL SPRAGUE MULTIPLIERS
    SUBROUTINE SPRAGE
    IK=6
    DO 250 I =3,LF
    DO 250 II=1,5
    AERCT(IK)=5.*(SPRAG(II,1)*GERCT(I-2)+SPRAG(II,2)*GERCT(I-1)+SPRAG(
   1II,3)*GERCT(I)+SPRAG(II,4)*GERCT(I+1)+SPRAG(II,5)*GERCT(I+2))
250 IK=IK+1
    RETURN
    END
    CONTROL FOR SPRAGUE GRADUATION
    SUBROUTINE KOSPRA
    LUNI=1
    NTAL=0
    DO 201  IN =2,37
    IF(SIG2N( IN )  -(+6H*      ))201,202,201
202 KCNU= IN -1
    IF(SIG2N(KQNU)-(+6H*      ))203,204,203
203 LUTCH(LUNI)=KCNU
    NTAL=NTAL+1
204 LQNU= IN +1
    IF(LQNU-37  )210,215,215
210 IF(SIG2N(LQNU)  -(+6H*      ))205,201,205
205 KRU=2
    LETCH(LUNI)=LQNU
    LUNI=LUNI+1
    GO TO 201
215 IF(SIG2N(37)   -(+6H*      ))205,207,205
207 LETCH(LUNI)=37
    KRU=1
201 CONTINUE
    IF(KRU-1)220,220,225
220 NNUTCH=LUTCH(NTAL)
    NNOTCH=  37-NNUTCH
    DO 221 KUL=1,NNOTCH
    IKLN=NNUTCH+KUL
221 HERCT(IKLN)=AERCT(NNUTCH)
    NTALI=NTAL-1
    IF(NTALI)700,700,226
225 NTALI=NTAL
226 DO 230 LUNI=1,NTALI
    NNATCH=LUTCH(LUNI)
    NNITCH=LETCH(LUNI)
    ULIKLA=NNITCH-NNATCH
    ULAKLI=ULIKLA-1.
    NLUKLI=ULAKLI
    DO 230 KUL=1,NLUKLI
    IKLE=NNATCH+KUL
    C(KUL)=KUL
230 HERCT(IKLE)=AERCT(NNATCH)-C(KUL)*(AERCT(NNATCH)-AERCT(NNITCH))/ULI
   1KLA
700 RETURN
    END
    CONTROL FOR DECREASING PERCENTAGES
    SUBROUTINE KONTRU
    DO 121 I=KEND,NEND
121 SIGIN(I)=(+6H       )
    NTEL=0
    KL=1
```

```
      DO 150 I=KEND,NEND1
      NONA=NEND-I
      DO 150 K=1,NONA
      KI=I+K
      IF(PERCT(I,1)-PERCT(KI,1))122,125,150
  122 IF(SIGIN(KI)-(+6H*      ))2,150,2
    2 SIGIN(KI)=(+6H*      )
      NTEL=NTEL+1
      NCOMT(KL)=KI
      KL=KI+1
      GO TO 150
  125 IF(I+1-KI)122,150,150
  150 CONTINUE
      IF(NTEL)130,130,185
  130 DO 142 I=KEND,NEND
  142 WERCT(I)=PERCT(I,1)
      LAK=1
      GO TO 172
  185 IF(NTEL-4)170,195,195
  195 NEND3=NEND-3
      DO 160 I =KEND,NEND3
      IF(SIGIN(I   ) - (+6H*      ))160,151,160
  151 IF(SIGIN(I+1) - (+6H*      ))160,152,160
  152 IF(SIGIN(I+2) - (+6H*      ))160,153,160
  153 IF(SIGIN(I+3) - (+6H*      ))160,154,160
  160 CONTINUE
  170 LAK=2
      GO TO 172
  154 LAK=3
  172 RETURN
      END
      LINEAR INTERPOLATICN
      SUBROUTINE LINPOL
      NTCL=0
      DO 165 I=KEND,NEND
  165 WERCT(I )=PERCT(I,1)
      KANI =1
      DO 201  I=KEND1,NEND
      IF(SIGIN( I)-(+6H*      ))201,202,201
  202 KQI= I-1
      IF(SIGIN(KQI)-(+6H*      ))203,204,203
  203 KATCH(KANI)=KQI
      NTOL=NTCL+1
  204 LQI= I+1
      IF(LQI-NEND)210,215,215
  210 IF(SIGIN(LQI)-(+6H*      ))205,201,205
  205 KRI=2
      KITCH(KANI)=LQI
      KANI=KANI+1
      GO TO 201
  215 IF(SIGIN(NEND)-(+6H*      ))205,207,205
  207 KITCH(KANI)=NEND
      KRI=1
  201 CONTINUE
      IF(KRI-1)220,220,225
  220 KLUTCH=KATCH(NTOL)
      KLCTCH=NEND-KLUTCH
      DO 221 KIL=1,KLOTCH
      IKLO=KLUTCH+KIL
      WERCT(IKLO)=PERCT(KLUTCH,1)
  221 SIGIN(IKLO)=(+6H**     )
      NTIL=NTOL-1
      IF(NTIL)700,700,226
  225 NTIL=NTCL
  226 DO 230 KANI=1,NTIL
      KLATCH=KATCH(KANI)
      KLITCH=KITCH(KANI)
      BLIKLA=KLITCH-KLATCH
      BLAKLI=BLIKLA-1.
      KLAKLI=BLAKLI
      DO 230 KOL=1,KLAKLI
      IKLA=KLATCH+KOL
      B(KOL)=KOL
      WERCT(IKLA)=PERCT(KLATCH,1)-B(KOL)*(PERCT(KLATCH,1)-PERCT(KLITCH,1
     1))/BLIKLA
  230 SIGIN(IKLA)=(+6H**     )
  700 WRITE OUTPUT TAPE 6,180,(TITLE(N),N=1,10)
  180 FORMAT( 31H1 LINEAR INTERPOLATION FOR THE 10A6 ////)
      WRITE OUTPUT TAPE 6,156
  156 FORMAT(20H  PROPORTIONS SINGLE////)
      WRITE OUTPUT TAPE 6,157
  157 FORMAT(41H    AGE       SINGLE(RAW)       SINGLE(LIN)///)
      WRITE OUTPUT TAPE 6,159,(IAGE(I),PERCT(I,1),SIGIN(I),WERCT(I),I=KE
     1ND,NEND)
  159 FORMAT(5X,I2,F17.0,A2,F15.0)
      RETURN
      END
      ADJUSTMENTS FOR DECREASING PERCENTAGES
      SUBROUTINE ADAPT
      DO 406 I= 1,NUMI
  406 VERCT(I)=PERCT(I,1)
      DO 407 LK=1,NTEL
      IERP=NCOMT(LK)
      IF(IAGE(IERP)-15)410,411,411
  410 IERP1=IERP-1
      IERP2=IERP+1
      VERCT(IERP)=0.5*(VERCT(IERP1)+VERCT(IERP2))
      GO TO 407
  411 IF(IAGE(IERP)-50)412,440,441
  440 VERCT(IERP)=HERCT(36)
      GO TO 407
  441 VERCT(IERP)=HERCT(37)
      GO TO 407
  412 DO 416 IN=1,31,5
      DO 416 KA=1,5
      IF(IAGE(IERP)-JAGE(IN)+1-KA)416,417,416
  416 CONTINUE
      GO TO 407
  417 IMP=IERP-KA
      INR=IN-1
      DO 418 LA=1,5
      IMPLA=LA+IMP
      INRLA=LA+INR
  418 VERCT(IMPLA)=HERCT(INRLA)
```

```
  407 CONTINUE
      DO 420 I=KEND,NEND
      WERCT(I)=VERCT(I)
  420 SIG5N(I)=(+6H      )
      DO 422 I=KEND1,NEND1
      IF(VERCT(I)-VERCT(I-1))422,422,421
  421 SIG5N(I)=(+6H*     )
      WERCT(I)=0.5*(VERCT(I-1)+VERCT(I+1))
  422 CONTINUE
      IF(VERCT(NEND)-VERCT(NEND1))426,426,425
  425 SIG5N(NEND)=(+6H*     )
      WERCT(NEND)=VERCT(NEND1)
  426 WRITE OUTPUT TAPE 6,427,(TITLE(N),N=1,10)
  427 FORMAT(25H1 PROPORTIONS SINGLE FOR 10A6////)
      WRITE OUTPUT TAPE 6,428
  428 FORMAT(54H    AGE      SINGLE(RAW)  SINGLE(VERCT)  SINGLE(WERCT)
     1      ///)
      WRITE OUTPUT TAPE 6,429,(IAGE(I),PERCT(I,1),SIG1N(I),VERCT(I),SIG5
     1N(I),WERCT(I),I=KEND,NEND)
  429 FORMAT(5X,I2,F16.0,A1,F14.0,A1,F15.0)
      RETURN
      END
      INTERPOLATION WITH GREVILLE MULTIPLIERS
      SUBROUTINE GREVIL
       AMULT(1)=0.0420
       AMULT(2)=0.1936
       AMULT(3)=-0.0248
       AMULT(4)=-0.0192
       AMULT(5)=0.0084
       CPERCT(1)=100000.
       CPERCT(2)= 5.*((AMULT(1)+AMULT(2))*100000.+WERCT(KENA)*AMULT(3)+WE
     1RCT(KENA+1)*AMULT(4)+WERCT(KENA+2)*AMULT(5))
       CPERCT(3)=5.*(AMULT(1)*100000.+WERCT(KENA)*AMULT(2)+WERCT(KENA+1)*
     1AMULT(3)+WERCT(KENA+2)*AMULT(4)+WERCT(KENA+3)*AMULT(5))
       IV=4
       KEN4=KENA+3
       DO 500 I=KEN4,NENB
       CPERCT(IV)= 5.*(WERCT(I-3)*AMULT(1)+WERCT(I-2)*AMULT(2)+WERCT(I-1)
     1*AMULT(3)+WERCT(I)*AMULT(4)+WERCT(I+1)*AMULT(5))
  500 IV=IV+1
       DO 505 I=1,NENC
       STAR(I)=(+6H      )
  505 ANUP(I,3)=CPERCT(I)
       CALL VIKRO
       WRITE OUTPUT TAPE 6,520,(TITLE(N),N=1,10)
  520 FORMAT(28H1 GRADUATED FIGURES FOR THE 10A6////)
       WRITE OUTPUT TAPE 6,521
  521 FORMAT(50H    AGE      WERCT(I)     CPERCT(I)       ANUP(I,3)///)
       IF(KENA-1)523,523,524
  523 WRITE OUTPUT TAPE 6,526,(JAGE(I),WERCT(I),CPERCT(I),STAR(I),ANUP(I
     1,3),I=KENA,NENC)
  526 FORMAT(5X,I2,F13.0,F14.0,A1,F15.0)
       GO TO 525
  524 DO 530 I=1,NENC
       IKAB=I+KENB
  530 WRITE OUTPUT TAPE 6,526,(IAGE(IKAB),WERCT(IKAB),CPERCT(I),STAR(I),
     1ANUP(I,3))
  525 RETURN
      END
      CONTROL FOR GREVILLE GRADUATUON
      SUBROUTINE VIKRO
      KNUL=0
      DO 150 I =1,NENE
      NINA=NENC-I
      DO 150 KK=1,NINA
      KINA=I +KK
      IF(CPERCT(I  )-CPERCT(KINA))122,125,150
  122 IF(STAR(KINA)-(+6H*     ))2,150,2
    2 STAR(KINA)=(+6H*     )
      KNUL=KNUL+1
      GO TO 150
  125 IF(I +1-KINA)122,150,150
  150 CONTINUE
      LANI=1
      NTUL=0
      DO 201 I=2,NENC
      IF(   STAR( I  )-(+6H*     ))201,202,201
  202 KQNA= I  -1
      IF(   STAR(KQNA)-(+6H*     ))203,204,203
  203 LATCH(LANI)=KQNA
      NTUL=NTUL+1
  204 LQNA= I  +1
      IF(LQNA-NENC)210,215,215
  210 IF(   STAR(LQNA)-(+6H*     ))205,201,205
  205 KRA=2
      LITCH(LANI)=LQNA
      LANI=LANI+1
      GO TO 201
  215 IF(  STAR(NENC)-(+6H*     ))205,207,205
  207 LITCH(KANI)=NENC
      KRA=1
  201 CONTINUE
      IF(KNUL)800,800,810
  800 KRA=0
  810 IF(KRA-1)330,220,225
  220 LLUTCH=LATCH(NTUL)
      LLOTCH=NENC-LLUTCH
      DO 221 KAL=1,LLOTCH
      IKLU=LLUTCH+KAL
  221 ANUP( IKLU,3)=CPERCT(LLUTCH)
      NTULI=NTUL-1
      IF(NTULI)330,330,226
  225 NTULI=NTUL
  226 DO 230 LANI=1,NTULI
      LLATCH=LATCH(LANI)
      LLITCH=LITCH(LANI)
      ALIKLA=LLITCH-LLATCH
      ALAKLI=ALIKLA-1.
      NLAKLI=ALAKLI
      DO 230 KAL=1,NLAKLI
      IKLI=LLATCH+KAL
      A(KAL)=KAL
  230 ANUP(IKLI,3)=CPERCT(LLATCH)-A(KAL)*(CPERCT(LLATCH)-CPERCT(LLITCH))
```

```
   1/ALIKLA
330 RETURN
    ENC
    NUPTIALITY TABLES
    SUBROUTINE NUPTIL
    ANUP(1,5)=0.0
    ANUP(1,10)=0.0
    DO 613 I=1,NENC
    IKENB=I+KENB
    ANUP(I,12)=100000.-ANUP(I,3)
    IF(KENA-1)610,610,611
610  AIAGE(I)=JAGE(IKENB)
    GO TO 680
611 AIAGE(I)=IAGE(IKENB)
680 SAGIN(I)=SIGIN(IKENB)
613 ANUP(I,11)=50.-AIAGE(I)
612 DO 620 I=1,NENE
    ANUP(I,4)=ANUP(I,3)-ANUP(I+1,3)
    ANUP(I,2)=ANUP(I,4)/ANUP(I,3)
    ANUP(I,6)=0.5*(ANUP(I,3)+ANUP(I+1,3))
620 ANUP(I,1)=1000.*ANUP(I,4)/ANUP(I,6)
    DO 622 I=1,NENE
622 ANUP(1,5)=ANUP(1,5)+ANUP(I,4)
    DO 624 I=2,NENE
624 ANUP(I,5)=ANUP(1,5)-ANUP(I   ,12)
    DO 626 I=1,NENE
    ANUP(I,7)=ANUP(I,5)/ANUP(I,3)
626 ANUP(1,10)=ANUP(1,10)+ANUP(I,6)
    DO 625 I=2,NENE
625 ANUP(I,10)=ANUP(I-1,10)-ANUP(I-1,6)
    DO 627 I=1,NENE
    ANUP(I,8)=ANUP(I,10)/ANUP(I,3)
627 ANUP(I,9)=100.*ANUP(I,8)/ANUP(I,11)
    DO 630 I=1,NENC
630 JIAGE(I)=AIAGE(I)
    ANUP(NENC,1)=0.0
    ANUP(NENC,2)=0.0
    ANUP(NENC,4)=0.0
    ANUP(NENC,5)=0.0
    ANUP(NENC,6)=0.0
    ANUP(NENC,7)=0.0
    ANUP(NENC,8)=0.0
    ANUP(NENC,9)=0.0
    ANUP(NENC,10)=0.0
    DO 80 J=1,12
    DO 80 I=1,NENC
    IF(ANUP(I,J))81,81,80
 81 ANUP(I,J)=ABSF(ANUP(I,J))
 80 CONTINUE
    WRITE OUTPUT TAPE 6,631,(TITLE(N),N=1,10)
631 FORMAT(10H1              10A6  ////)
    WRITE OUTPUT TAPE 6,634
634 FORMAT(119H  AGE  MAR(X)     NUP(X)      SIG(X)       M(X)       E(X)
   1   SL(X)     PRE(X)    ADS(X)
   2////)
    WRITE CUTPUT TAPE 6,632,(JIAGE(I),SAGIN(I),(ANUP(I,J),J=1,8 ),I=1,
   1NENC)
632 FORMAT(1X,I2,A2,1X,
   1             F7.2,F10.4,4F10.0,F10.4,F10.1)
    WRITE OUTPUT TAPE 6,631,(TITLE(N),N=1,10)
    WRITE OUTPUT TAPE 6,650
650 FORMAT (41H  AGE  PRADS(X)   STA2(X)   DIF    EMA(X)         ////)
    WRITE OUTPUT TAPE 6,652,(JIAGE(I),SAGIN(I),(ANUP(I,J),J=9,12),I=1,
   1NENC)
652 FORMAT(1X,I2,A2,1X,
   1                                          F10.2,F10.0,F6.0,F10.0)
    RETURN
    ENC
    SINGULATE MEAN AGE AT MARRIAGE
    SUBROUTINE MEAN
    BNUP1=0.0
    BNUP2=0.0
    CNUP=0.0
    ALFA=NALFA
    AENE=NENE
    DO 900 I=1,NENE
    BNUP1=BNUP1+ANUP(I,3)
900 BNUP2=BNUP2+ANUP(I,6)
    SMAM1=ALFA+(BNUP1-AENE*ANUP(NENC,3))/(100000.-ANUP(NENC,3))
    SMAM2=ALFA+(BNUP2-AENE*ANUP(NENC,3))/(100000.-ANUP(NENC,3))
    HALF=0.5*ANUP(NENC,12)
    DO 910 I=1,NENE
    IF (ANUP(I,12)-HALF)910,915,920
910 CONTINUE
915 AMAM=JIAGE(I)
    GO TO 922
920 AGE=JIAGE(I-1)
  . AMAM=AGE+(HALF-ANUP(I-1,12))/(ANUP(I,12)-ANUP(I-1,12))
922 WRITE CUTPUT TAPE 6,921,AMAM
921 FORMAT(27HO MEDIAN AGE AT MARRIAGE = F7.2)
    RETURN
    ENC
    PREPARATION FOR NUPTIALTY TABLES BASED ON GRADUATION OF MAR. RATES
    SUBROUTINE NUSPRA
    DO 950 IT=1,7
950 CNUP=CNUP+FERCT(IT)
    FARCT=0.5*(FERCT(6)+FERCT(7))
    SMAM3 = ALFA + (5.* CNUP - 35.*FARCT)/(100000. - FARCT)
    SMAM4=(ALFA*100000.+(5.*CNUP-50.*FARCT))/(100000.-FARCT)
    WRITE CUTPUT TAPE 6,631,(TITLE(N),N=1,10)
631 FORMAT(10H1              10A6  ////)
    WRITE OUTPUT TAPE 6,901,SMAM1
    WRITE OUTPUT TAPE 6,902,SMAM2
    WRITE OUTPUT TAPE 6,903,SMAM3
    WRITE OUTPUT TAPE 6,904,SMAM4
901 FORMAT(37HO SINGULATE MEAN AGE AT MARRIAGE(1)= F7.2)
902 FORMAT(37HO SINGULATE MEAN AGE AT MARRIAGE(2)= F7.2)
903 FORMAT(37HO SINGULATE MEAN AGE AT MARRIAGE(3)= F7.2)
904 FORMAT(37HO SINGULATE MEAN AGE AT MARRIAGF(4)= F7.2)
    RETURN
    ENC
    MEDIAN AGE AT MARRIAGE
    SUBROUTINE MEDIAN
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```
      IF(JIAGE(1)-15)300,400,401
  300 N15 = 15-JIAGE(1)
      DO 203 I=1,N15
  203 UNUP(I,1)=ANUP(I,1)
  400 KOND=15-JIAGE(1)+1
      KGEN=5
      IA=4
      N25=25-JIAGE(1)+1
      GO TO 410
  401 N20=20-JIAGE(1)
      KGEN=4
      IA=3
      DO 402 I=1,N20
  402 UNUP(I,1)=ANUP(I,1)
      KOND=20-JIAGE(1)+1
      N25=30-JIAGE(1)     +1
  410 NFIF=45-JIAGE(1)+1
      IS= 1
      DO 209 IL=KOND,NFIF,5
      ENUP(IS)=ANUP(IL,1)
      DO 200 KS=1,4
      KIS=IL+KS
  200 ENUP(IS)=ENUP(IS)+ANUP(KIS,1)
  209 IS=IS+1
      DO 206 JJ=1,10
      UNUP(KOND,1)=SPRIG(JJ,1)*ENUP(1)+SPRIG(JJ,2)*ENUP(2)+SPRIG(JJ,3)*E
     1NUP(3)+SPRIG(JJ,4)*ENUP(4)
  206 KOND=KOND+1
      DO 225 IS=3,KGEN
      DO 225 II=1,5
      UNUP(N25,1)=SPRAG(II,1)*ENUP(IS-2)+SPRAG(II,2)*ENUP(IS-1)+SPRAG(II
     1,3)*ENUP(IS)+SPRAG(II,4)*ENUP(IS+1)+SPRAG(II,5)*ENUP(IS+2)
  225 N25 = N25+1
      N40 = 40-JIAGE(1)+1
      DO 227 II=6,15
      UNUP(N40,1)=SPRAG(II,1)*ENUP(IA)+SPRAG(II,2)*ENUP(IA+1)+SPRAG(II,3
     1)*ENUP(IA+2)+SPRAG(II,4)*ENUP(IA+3)
  227   N40=N40+1
      UNUP( NENC,1)=ANUP(NENC,1)
      DO 503 I=1,NENC
  503 ETCL(I)=(+6H       )
      DO 500 I=1,NENE
      IF(ANUP(I,1))500, 502,500
  502 UNUP(I,1)=ANUP(I,1)
      ETCL(I)=(+6HG      )
  500 CONTINUE
      DO 552 I=1,NENE
      IF(UNUP(I))551,552,552
  551 ETCL(I)=(+6HV      )
      IF(ANUP(I,1) - 5.)560,560,561
  560 UNUP(I,1) = ANUP(I,1)
      GO TO 552
  561 UNUP(I,1) = 0.50
  552 CONTINUE
      IF (UNUP(1,1)-ANUP(1,1))630,630,600
  600 UNUP(1,1) = ANUP(1,1)
  630 RETURN
      END
      NUPTIALTY TABLES BASED ON THE GRADUATICN OF THE MARRIAGE RATES
      SUBROUTINE TROUW
      UNUP(1,10)=0.0
      UNUP(1,5 )=0.0
      UNUP(1,3)=100000.
      DO 100 I=1,NENE
  100 UNUP(I,2)=UNUP(I,1)/(1000.+0.5*UNUP(I,1))
      DO 203 I=2,NENC
  203 UNUP(I,3)=(1.-UNUP(I-1,2))*UNUP(I-1,3)
      DO 613 I=1,NENC
      UNUP(I,11)=ANUP(I,11)
  613 UNUP(I,12)=100000.-UNUP(I,3)
      DO 620 I=1,NENE
      UNUP(I,4)=UNUP(I,3)-UNUP(I+1,3)
  620 UNUP(I,6)=0.5*(UNUP(I,3)+UNUP(I+1,3))
      DO 622 I=1,NENE
  622 UNUP(1,5)=UNUP(1,5)+UNUP(I,4)
      DO 624 I=2,NENE
  624 UNUP(I,5)=UNUP(1,5)-UNUP(1,12)
      DO 626 I=1,NENE
      UNUP(I,7)=UNUP(I,5)/UNUP(I,3)
  626 UNUP(1,10)=UNUP(1,10)+UNUP(I,6)
      DO 625 I=2,NENE
  625 UNUP(I,10)=UNUP(I-1,10)-UNUP(I-1,6)
      DO 627 I=1,NENE
      UNUP(I,8)=UNUP(I,10)/UNUP(I,3)
  627 UNUP(I,9)=100.*UNUP(I,8)/UNUP(I,11)
      UNUP(NENC,1)=0.0
      UNUP(NENC,2)=0.0
      UNUP(NENC,4)=0.0
      UNUP(NENC,5)=0.0
      UNUP(NENC,6)=0.0
      UNUP(NENC,7)=0.0
      UNUP(NENC,8)=0.0
      UNUP(NENC,9)=0.0
      UNUP(NENC,10)=0.0
      DO 80 J=1,12
      DO 80 I=1,NENC
      IF(UNUP(I,J))81,81,80
   81 UNUP(I,J)=ABSF(UNUP(I,J))
   80 CONTINUE
      WRITE OUTPUT TAPE 6,631,(TITLE(N),N=1,10)
  631 FORMAT(10H1               10A6    //)
      WRITE OUTPUT TAPE 6,670
  670 FORMAT(65H                (BASED ON THE GRADUATION OF THE MARRIAGE
     1 RATES)                                                      ////)
      WRITE OUTPUT TAPE 6,634
  634 FORMAT(119H  AGE  MAR(X)    NUP(X)    SIG(X)      M(X)      E(X)
     1    SL(X)     PRE(X)     ADS(X)
     2////)
      WRITE OUTPUT TAPE 6,632,(JIAGE(I),SAGIN(I), ETCL(I),
     1(UNUP(I,J),J=1,8 ) ,I=1,NENC)
  632 FORMAT(1X,I2,A2,A1,
     1              F7.2,F10.4,4F10.0,F10.4,F10.1)
      WRITE OUTPUT TAPE 6,631,(TITLE(N),N=1,10)
      WRITE OUTPUT TAPE 6,670
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      WRITE OUTPUT TAPE 6,650
  650 FORMAT (41H  AGE  PRADS(X)   STA2(X)   DIF    EMA(X)          /////)
      WRITE OUTPUT TAPE 6,652,(JIAGE(I),SAGIN(I), ETOL(I),
     1(UNUP(I,J),J=9,12) ,I=1,NENC)
  652 FORMAT(1X,I2,A2,A1,
     1                                         F10.2,F10.0,F6.0,F10.0)
      RETURN
      END
      SINGULATE MEAN AND MEDIAN AGE AT MARRIAGE
      SUBROUTINE  MOYEN
      BNIP1=0.C
      BNIP2=0.0
      ALFA=NALFA
      AENE=NENE
      DO 900 I=1,NENE
      BNIP1=BNIP1+UNUP(I,3)
  900 BNIP2=BNIP2+UNUP(I,6)
      SMAM5=ALFA+(BNIP1-AENE*UNUP(NENC,3))/(1CC000.-UNUP(NENC,3))
      SMAM6=ALFA+(BNIP2-AENE*UNUP(NENC,3))/(1COC00.-UNUP(NENC,3))
      BALF=0.5*UNUP(NENC,12)
      DO 910 I=1,NENE
      IF (UNUP(I,12)-BALF)910,915,920
  910 CONTINUE
  915 UMAM=JIAGE(I)
      GO TO 922
  920 UGE=JIAGE(I-1)
      UMAM=UGE+(BALF-UNUP(I-1,12))/(UNUP(9,12)-UNUP(I-1,12))
  922 WRITE CUTPUT TAPE 6,631,(TITLE(N),N=1,1C)
  631 FORMAT(1CH1              10A6  /////)
      WRITE OUTPUT TAPE 6,901,SMAM5
      WRITE OUTPUT TAPE 6,902,SMAM6
  901 FORMAT(37HO SINGULATE MEAN AGE AT MARRIAGE(5)= F7.2)
  902 FORMAT(37HO SINGULATE MEAN AGE AT MARRIAGE(6)= F7.2)
      WRITE OUTPUT TAPE 6,921,UMAM
  921 FORMAT(27HO MEDIAN AGE AT MARRIAGE = F7.2)
      RETURN
      ENC
      PUNCH 90,TITLE,NGROUP,NUM,KAR,NFGES,NALFA,NBETA,NCOD
   90 FORMAT(3H A 10A6,I1, 5I2,1X,I5 )
      IF(KAR-2)91,92,92
   91 PUNCH 95      ,NCOD
   95 FORMAT(46H A PROPORTIONS SINGLE INCREASING BEFORE AGE 45  29X,I5)
      GO TO 205
   92 PUNCH 100, (JAGE(I),AERCT(I),FERCT(I ),SIG2N(I),NCOD,I=1,37)
  100 FORMAT(3H A  I2,3X,2F10.0,A1,46X,I5)
  205 PUNCH 190, TITLE,NGROUP,NUM,NAGES,NENC ,NALFA,NBETA,NCOD
  190 FORMAT(3H B  10A6,I1,5I2,1X,I5)
      PUNCH 200, (JIAGE(I),ANUP(I,2),ANUP(I,3),UNUP(I,2),UNUP(I,3),SAGIN
     1(I),ETCL(I),NCOD,I=1,NENC)
  200 FORMAT(3H B I2,3X,  F12.9,F10.0,F12.9, F10.0,A2,A1,20X,I5)
      PUNCH 206, TITLE,NGROUP,NUM,NAGES,NUMI ,NALFA,NBETA,NCOD
  206 FORMAT(3H C  10A6,I1,5I2,1X,I5)
      PUNCH 207, (IAGE(I),PERCT(I,1),NCOD,I=1,NUMI)
  207 FORMAT(3H C I2,3X,F10.0,57X,I5)
      PUNCH 210,TITLE,NGROUP,KAN,KAR,NFGES,NALFA,NBETA,NCOD
  210 FORMAT(3H D 10A6,I1, 5I2,1X,I5 )
      PUNCH 212 , (LAGE(IT),FERCT(IT),SIG4N(IT),NCOD,IT=1,KAN)
  212 FORMAT(3H D  I2,3X,F10.0,A1,56X,I5)
      RETURN
      END
```