Lecture 1 – Optimal Stopping / Card Game

Lecture Goal The aim of this first lecture is to refresh your Python skills through an optimization problem often used in interviews for quantitative finance positions (due to its similarity with American option problems). Specifically, you will practice object-oriented programming and minimize the use of loops for repetitive tasks by leveraging NumPy's vectorized approach.

Problem Description Consider a deck of $2N$ cards, equally split between red and black. The game is played by a single player who draws cards one by one without replacement. Drawing a red card adds +1 point to the player's score, while drawing a black card subtracts 1 point. The player can stop drawing at any point, with the goal of maximizing the expected score.

Simulator Implementation Implement a class `GameSimulator` that includes:

- At least two instance variables: one for N and one for the card deck (you can simplify by focusing on card colors);
- At least two methods: one that simulates the game based on a given strategy and returns the score, and another that runs the game `M` times to estimate the expected score for a strategy (remember to shuffle the deck before each game).

Note: Explain why a reasonable strategy can be described by a threshold vector `v` *of size* $2N$*, where the player stops after step* k *if the score at that point is greater than or equal to* `v[k]`*. We will assume* `v[-1] = 0`*.*

Testing Strategies Implement and test the following strategies using an instance of your simulator:

- A strategy that stops when a certain score is reached (or continues drawing until there is no card left);
- A strategy where the threshold vector is determined by a heuristic with 2 or 3 parameters of your choice – you should use `scipy` and `optuna` to optimize the heuristic parameters;
- The optimal strategy.

Note: Each strategy should be a class that includes a method named `threshold_vector` *that is called when playing.*

Additional Info: For a game with 32 cards, the optimal strategy results in an expected score of approximately 2.05.