Lecture 2 – Optimal Stopping / Asian-like Payoff

Lecture Goal The goal of this lecture is to apply the techniques from Lecture 1 to a financial example. Specifically, we will develop a class to simulate stock price dynamics and use Monte Carlo methods to price an Asian-like payoff and optimize a stopping strategy.

Problem Description We consider a stock whose daily prices follow a geometric Brownian motion under the risk-neutral measure (i.e., $r = 0$):

$$S_{n+1} = S_n \exp\left(-\frac{1}{2}\sigma^2 \Delta t + \sigma\sqrt{\Delta t}\epsilon_{n+1}\right), \quad S_0 \text{ given,} \tag{1}$$

where $\Delta t = \frac{1}{252}$ and (ϵ_{n+1}) are independent, identically distributed standard Gaussian random variables.

We also define the running average:

$$A_n = \frac{S_0 + S_1 + \ldots + S_n}{n+1}.$$

Simulator Implementation Create a class `Simulator` that models the stock price dynamics. The class should include methods to:[1]

- Simulate and plot the trajectory of S and A over a specified time horizon;
- Run M simulations over a given time horizon;
- Use Monte Carlo methods to compute the price of the payoff $\frac{A_N}{S_N}$.

Verify your code using the following parameters: $S_0 = 10$, $\sigma = 0.2$, $\Delta t = \frac{1}{252}$, and $N = 22$ – i.e. compare the result with the theoretical value.

Optimal Stopping Strategies Let $a \geq 1$. Define the stopping time

$$\tau_a = \min\left(\min\{n \in \{0, \ldots, N\}, A_n \geq aS_n\}, N\right).$$

Use the techniques from Lecture 1 to maximize the expected value

$$\mathbb{E}\left[\frac{A_{\tau_a}}{S_{\tau_a}}\right].$$

Additionally, experiment with other heuristic stopping time strategies to approximate the maximization of

$$\mathbb{E}\left[\frac{A_\tau}{S_\tau}\right].$$

Illustrate your results with the following parameters: $S_0 = 10$, $\sigma = 0.2$, $\Delta t = \frac{1}{252}$, and $N = 22$.

[1] **Avoid loops!**