Midterm Exam Micro 1 - 60 minutes

Mobile phones, class notes and problem sets are strictly prohibited

Exercise 1 (20 minutes). There are two commodities. The preference relation $\succsim$ of the consumer is represented by the utility function $u : \mathbb{R}^2_+ \to \mathbb{R}$ defined by

$$u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$$

1. Show that this preference relation is continuous, monotone, and convex.
2. Let $p = (p_1, p_2) \gg 0$ be a price system and $\mathrm{w} > 0$ be the wealth of the consumer.

   Determine the demand of this consumer (carefully justify your answer by stating the properties used for this purpose).

Exercise 2 (30 minutes). There are two commodities. As usual,

$$x(p_1, p_2, \mathrm{w}) = (x_1(p_1, p_2, \mathrm{w}), x_2(p_1, p_2, \mathrm{w}))$$

denotes the demand of the consumer. For every $0 < p_1 < p_2$ and for every $\mathrm{w} > 0$, the demand the consumer is given by

$$x_1(p_1, p_2, \mathrm{w}) = \frac{\mathrm{w}}{p_2} \quad \text{and} \quad x_2(p_1, p_2, \mathrm{w}) = \frac{\mathrm{w}(p_2 - p_1)}{(p_2)^2}$$

1. Show that this demand is homogeneous of degree zero.
2. Show that this demand satisfies Walras's Law.
3. State the Weak Axiom of Revealed Preferences (WARP) in the framework of the demand.
4. Without loss of generality, normalize to 1 the price of commodity 2, and prove that this demand does **not** satisfy WARP.

Exercise 3 (10 minutes). $C = \{c_1, ..., c_n, ..., c_N\}$ is the finite set of outcomes. $\mathcal{L}$ is the set of lotteries over C. Let $\succsim$ be a preference relation over the set $\mathcal{L}$.

1. State the independence axiom.
2. Assume now that $\succsim$ is represented by a function $U : \mathcal{L} \to \mathbb{R}$ that has an expected utility form.
   (a) What does this mean ? (give the formal definition).
   (b) Then show that $\succsim$ satisfies the independence axiom.