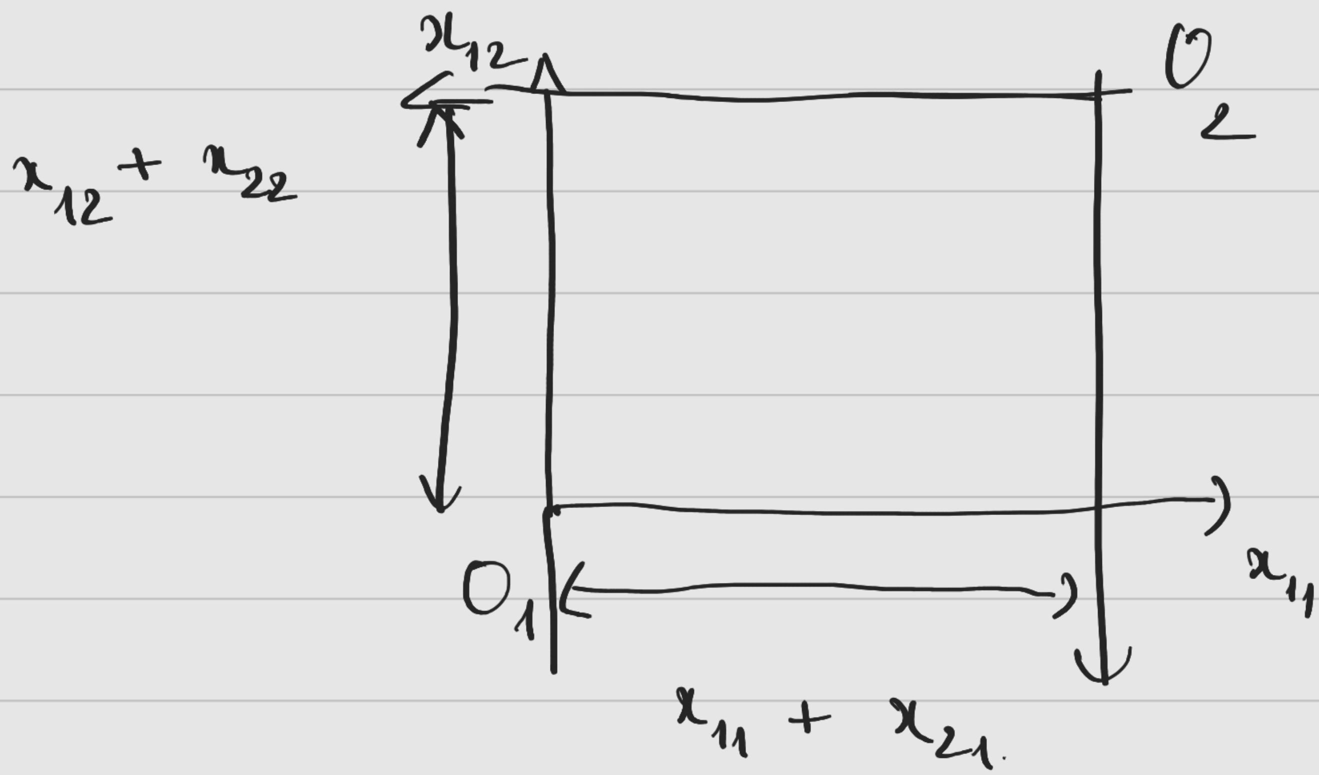
About Ex 17 – 18
MWG – pg 141 – 142
Solution of Ex 18 : Ex 5.C.1 (MWG)

Ex 19 :

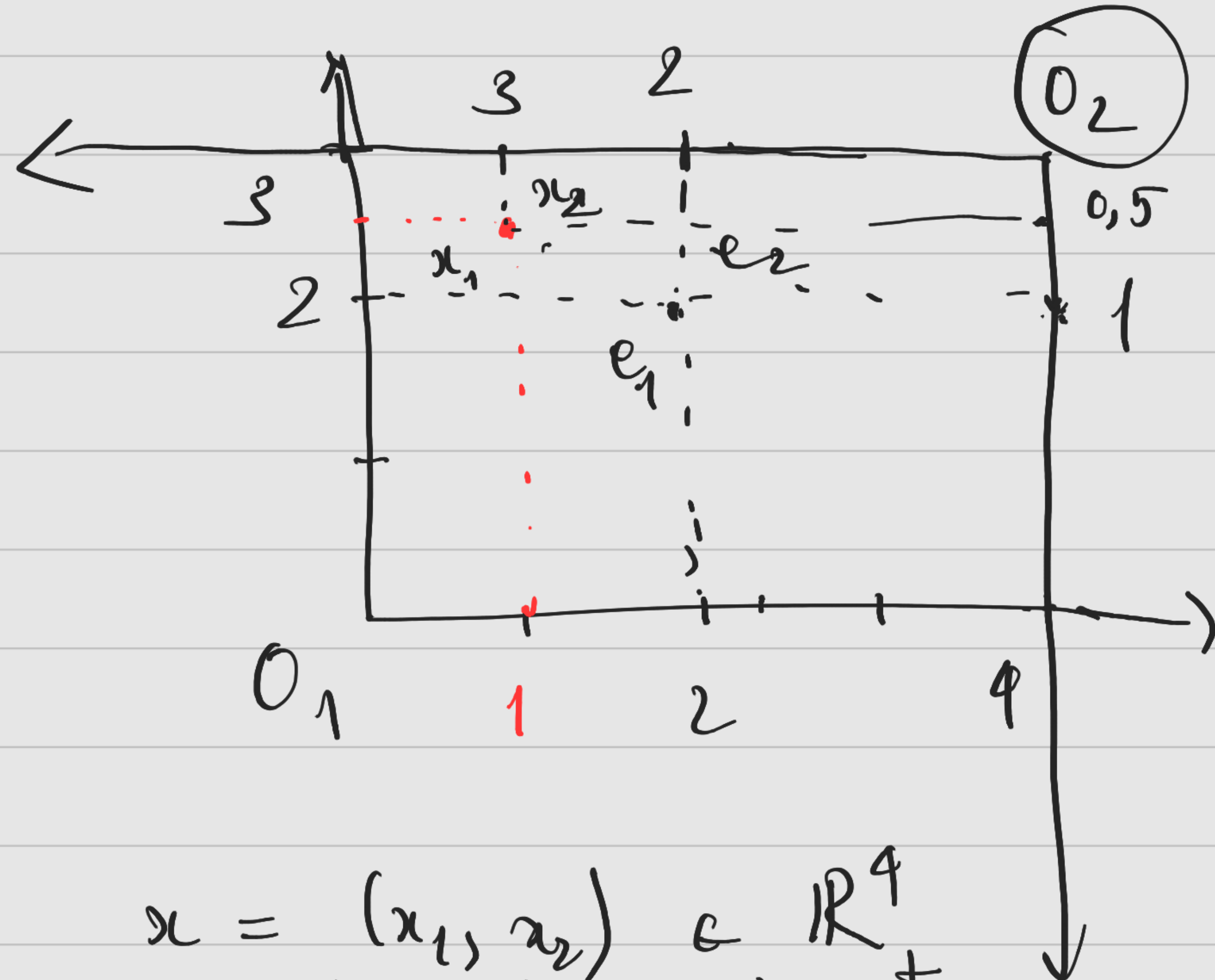
$$u_1(x_{11}, x_{12}) = x_{11} + x_{12}$$
$$u_2(x_{21}, x_{22}) = a x_{21} + b x_{22}$$
$$e_1 = (2,2)$$
$$e_2 = (2,1)$$

1



Total endowment :

$$e = e_1 + e_2 = (4,3)$$



Def: $x = (x_1, x_2) \in \mathbb{R}^4_+$ is a feasible allocation if $x_1 + x_2 = e = (4, 3)$

$\Rightarrow$ It is trivial that (e_1, e_2) is a feasible allocation.

2) At the point $\left(1, \frac{5}{2}\right)$

$x_1 = \left(1, \frac{5}{2}\right)$

$$\begin{cases} x_{11} + x_{21} = 4 \\ x_{12} + x_{22} = 3 \end{cases}$$

$$\Rightarrow \begin{cases} x_{21} = 4 - x_{11} = 4 - 1 = 3 \\ x_{22} = 3 - x_{12} = 3 - \frac{5}{2} = \frac{1}{2} \end{cases}$$

$$\Rightarrow x_2 = \left(3, \frac{1}{2}\right)$$

3) The definition of a competitive eq:

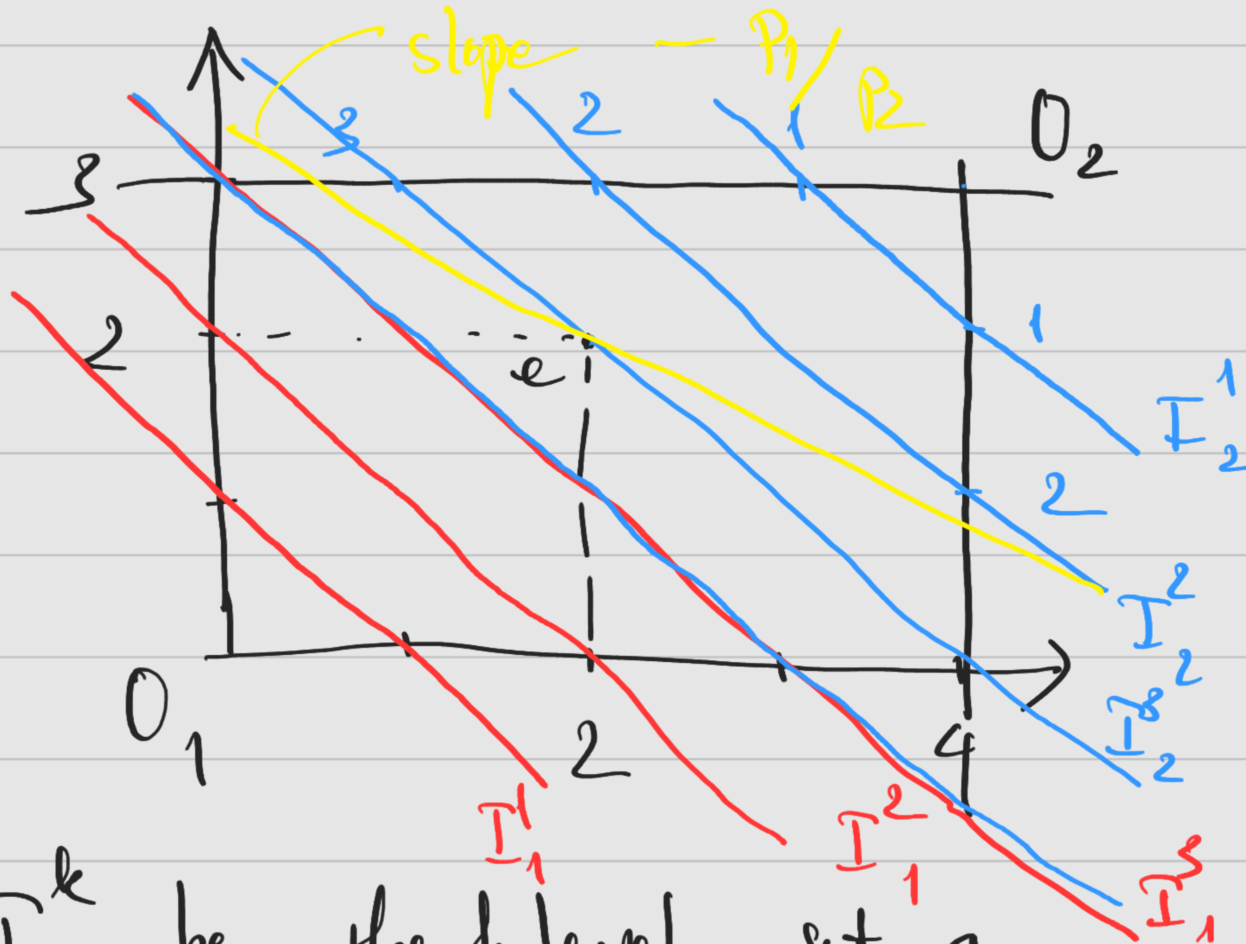
(x_1^*, x_2^*, p^*) is a competitive eq iff:

(i) $\forall i = 1, 2$
$x_i^* \in d_i(p^*, p^* \cdot e_i)$

(ii) Market clearing condition:
$x_1^* + x_2^* = e_1 + e_2$

4) $a = b = 1$

$u_2(x_2) = x_{21} + x_{22}$



Let I_1^k be the k-level set of consumer 1.

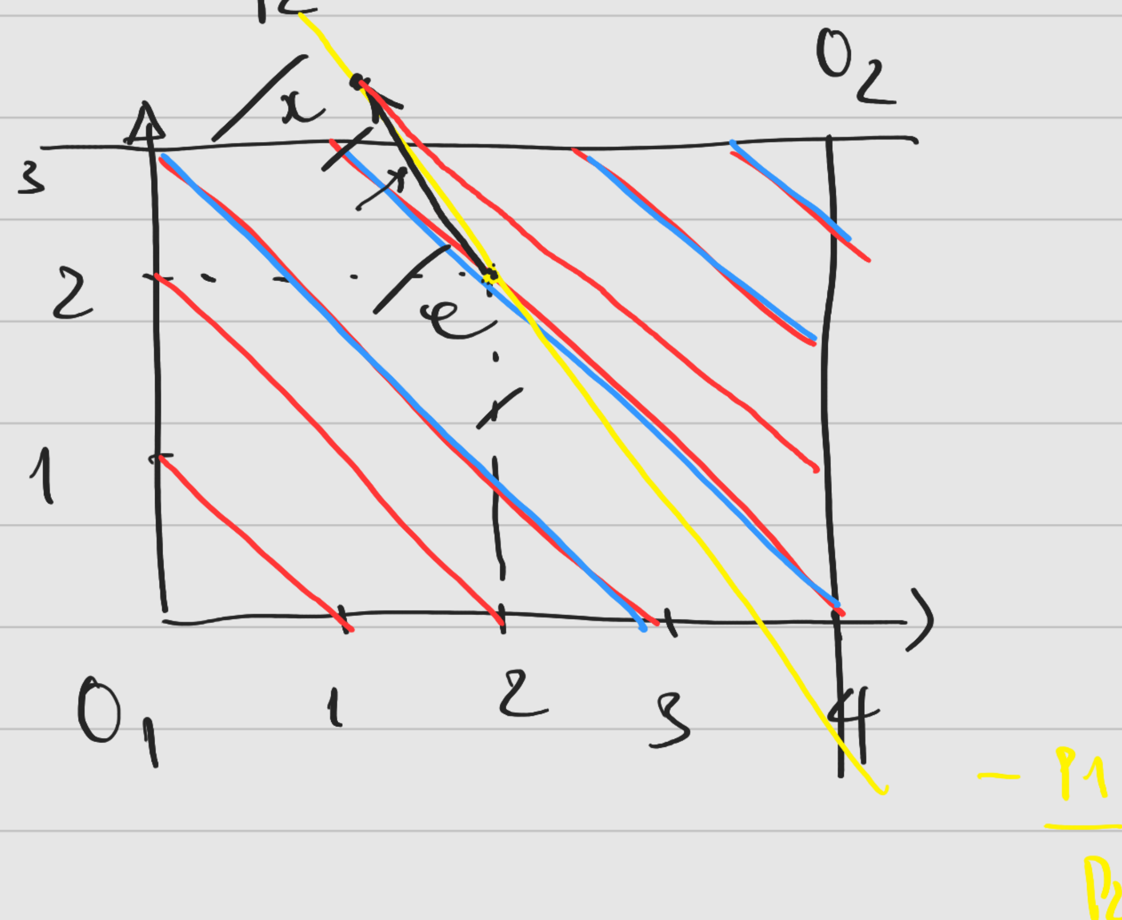
$$I_1^k = \{ (x_{11}, x_{12}) \in \mathbb{R}^2 \mid u_1(x_{11}, x_{12}) = k \}$$

$$\Leftrightarrow I_1^k = \{ (x_{11}, x_{12}) \in \mathbb{R}^2 \mid x_{11} + x_{12} = k \}$$

Similarly, let I_2^k be the k-level set of consumer 2.

$$I_2^k = \{ (x_{21}, x_{22}) \in \mathbb{R}^2 \mid x_{21} + x_{22} = k \}$$

$\circ\ I_f$: $-\dfrac{p_1^*}{p_2^*} < -1$



Budget line for the 1st consumer

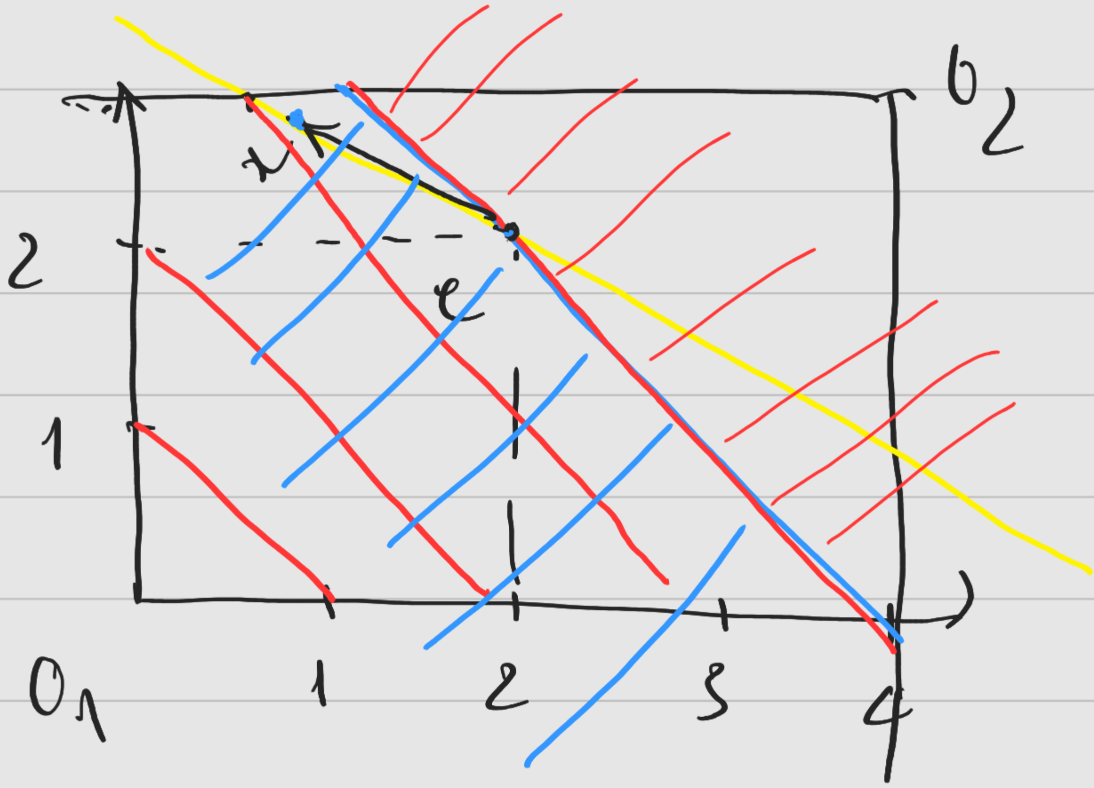
$$\{(x_{11}, x_{12}) \in \mathbb{R}^2_+ : p_1 x_{11} + p_2 x_{12} \leq p_1 e_{11} + p_2 e_{12}\}$$

$\Rightarrow$ The demand of the 1st consumer is

$$\left\{\left(0, \frac{p \cdot e_1}{p_2}\right)\right\}$$

But the point is NOT optimal for consumer 2. $\Rightarrow$ p^* is NOT C.E price

+) I_f : $-\dfrac{p_1^*}{p_2^*} > -1$



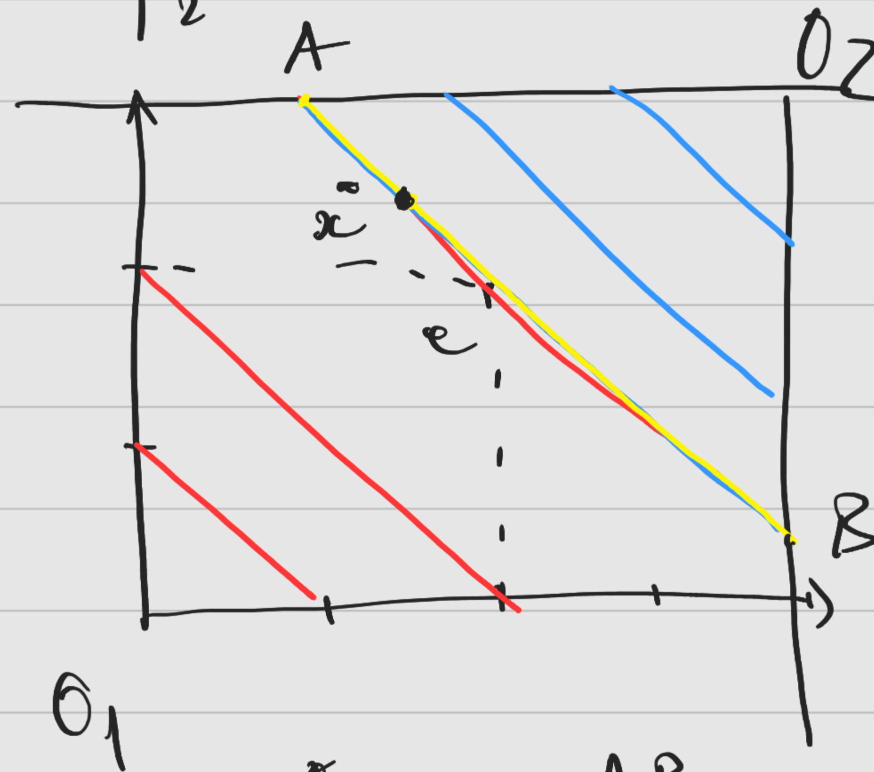
$\Rightarrow$ The demand of the 2^{nd} consumer:

$\left\{ \left(\dfrac{p \cdot e_2}{p_1}, 0 \right) \right\}$.

But the point is not optimal for the 1^{st} consumer.

$\Rightarrow$ (p_1^*, p_2^*) is NOT competitive eq prices.

+) $p_1^* = p_2^*$



At any point $x^* \in AB$

x_1^* is optimal for consumer 1

x_2^* ———————— 2

$x_1^* + x_2^* = e_1 + e_2$

$\Rightarrow (p^*, x_1^*, x_2^*)$ is a CE.