

TD – Wednesday, October 9, 2024

Consumer Theory

The following exercise should be submitted on Wednesday, October 9.

Exercise. Let $X := (-\infty, \infty) \times \mathbb{R}_+^{L-1}$ be the consumption set. The consumer has continuous, strictly convex preferences on X that are represented by a utility function

$$u(x) = x_1 + \varphi(x_2, x_3, \dots, x_L)$$

We assume $p \gg 0$ and we normalize $p_1 = 1$.

1. Show that the demand for commodities $\{2, 3, \dots, L\}$ must be independent of wealth. How does demand for commodity 1 react to changes in wealth w ?
2. Using your previous result, define the **indirect utility function v as usual**, that is, for every $(p, w) \in \mathbb{R}_{++}^{L+1}$,

$$v(p, w) := u(x^*) \text{ with } x^* \in x(p, w)$$

Show that $v(p, w)$ is linear in wealth and $v(p, w) = w + \psi(p)$ for some function $\psi : \mathbb{R}_{++}^L \rightarrow \mathbb{R}$.

3. Now let $L = 2$ and $\varphi(x_2) = \alpha \ln(x_2)$. Solve the UMP as a function of (p, w) (*Recall that in this exercise we allow demand of commodity 1 to be negative*).