

TD – Monday, November 18, 2024

Producer Theory

The following exercises should be submitted on Monday, November 18.

**Exercise 1.**  $L = 2$  is the number of commodities. The firm produces commodity 2 by using commodity 1 as an input. The production function is  $f(z) = \alpha\sqrt{z}$  with  $\alpha > 0$  and  $z \geq 0$ .

1. For every level of output  $\bar{y}_2 \geq 0$ , determine and draw the following set:

$$Y(\bar{y}_2) := \{z \in \mathbb{R} : z \geq 0 \text{ and } f(z) \geq \bar{y}_2\}$$

2. Write the cost minimization problem of this firm.
3. Determine the demand of inputs and the cost function of this firm.

**Exercise 2.**  $L = 3$  is the number of commodities. The firm produces commodity 3 by using commodities 1 and 2 as inputs. The production function is:

$$f(z_1, z_2) = (z_1)^\alpha (z_2)^\beta \text{ for all } z_1 \geq 0 \text{ and } z_2 \geq 0,$$

with  $\alpha > 0$ ,  $\beta > 0$ , and  $\alpha + \beta \leq 1$ .

1. Determine the demand of inputs and the cost function of this firm.
2. Using the demand of inputs and the cost function, determine the supply and the profit function of this firm [*Suggestion*: Distinguish the two cases:  $\alpha + \beta < 1$  and  $\alpha + \beta = 1$ ].

**Exercise 3.** Let  $L$  be the number of commodities. A firm produces commodity  $L$  using the other  $L - 1$  commodities as inputs.  $z := (z_1, \dots, z_l, \dots, z_{L-1}) \in \mathbb{R}_+^{L-1}$  denotes a generic bundle of inputs. Show that if the production function  $f : \mathbb{R}_+^{L-1} \rightarrow \mathbb{R}_+$  is **concave**, then the cost function  $C$  is a **convex** function of the output level.