Microeconomics 1: Equilibria and Optimality
Master M1 IMMAEF

TD – Monday, November 25, 2024

The following exercises should be submitted on Monday, November 25.

Competitive equilibrium in a private ownership economy

Exercise 1. Consider a private ownership economy $\mathcal{E}$ with L commodities, m consumers and n firms. Let $(p^*, x^*, y^*) \in \mathbb{R}_+^L \times \mathbb{R}_+^{Lm} \times \prod_{j=1}^n Y_j$ a competitive equilibrium of the economy $\mathcal{E}$. Using the definition of a competitive equilibrium, prove the following basic properties.

1. At equilibrium, Walras's Law holds true for all the consumers.

2. Price normalization: for all $t > 0$, (tp^*, x^*, y^*) is a competitive equilibrium of the same economy $\mathcal{E}$.

3. If there exists at least one consumer h such that u_h is monotone increasing, then the equilibrium price p^* is different from zero, i.e., $p^* > 0$.

4. If there exists at least one consumer h such that u_h is **strictly** monotone increasing, then the equilibrium price of all commodities must be strictly positive, i.e., $p^* \gg 0$.

5. If $p^* \gg 0$ and u_i is monotone increasing for all $i = 1, ..., m$, then in the definition of a competitive equilibrium, requiring Market Clearing Conditions for all commodities is equivalent to require Market Clearing Conditions for $L - 1$ commodities.

Exercise 2. Consider a private ownership economy with two commodities, two consumers and one firm. The firm produces commodity 2 by using commodity 1 as an input. The production set of the firm is
$$Y = \{y = (y^1, y^2) \in \mathbb{R}^2 : y^1 \leq 0 \text{ and } \alpha y^1 + y^2 \leq 0\}$$
with $\alpha > 0$. The two consumers have the same preferences represented by the utility function

$$u_i(x_i^1, x_i^2) = x_i^1 x_i^2$$

for every $i = 1, 2$.

The initial endowments are $e_1 = (1, 2)$ and $e_2 = (4, 1)$.

The price of commodity 1 is normalized to 1, i.e., $p_1 = 1$.

1. Compute the demands of the consumers with respect to the price p_2 and the individual wealth.

2. Compute the supply and the profit function of the producer with respect to the price p_2 and the marginal productivity α.

3. The shares of the consumers on the profit of the firm have no influence on the competitive equilibria of this economy. Why so?

4. Compute the unique competitive equilibrium of this economy with respect to α.