

TD – Wednesday, November 6, 2024

Producer Theory

The following exercises should be submitted on Wednesday, November 6.

**Exercise 1.** Let  $L$  be the finite number of commodities. A firm produces commodity  $L$  using the other  $L - 1$  commodities as inputs.  $z := (z_1, \dots, z_l, \dots, z_{L-1}) \in \mathbb{R}_+^{L-1}$  denotes a generic bundle of inputs. Show that if the production function  $f : \mathbb{R}_+^{L-1} \rightarrow \mathbb{R}_+$  is **concave**, then the transformation function defined by

$$t_f(y) = y_L - f(z)$$

is **quasi-convex** on the convex set  $A = \{y = (-z, y_L) \in \mathbb{R}^L : z \geq 0 \text{ and } y_L \geq 0\}$ .

**Exercise 2.**  $L = 2$  is the number of commodities. The firm produces commodity 2 by using commodity 1 as an input. The production function is

$$f(z) = \alpha\sqrt{z}$$

with  $\alpha > 0$  and  $z \geq 0$ .

1. Show that if  $\bar{y} = (\bar{y}_1, \bar{y}_2)$  belongs to the supply of the firm, then  $\bar{y}_1 < 0$  and  $\bar{y}_2 > 0$ .
2. Write the first order conditions associated with (PMP), and determine if these conditions are necessary and/or sufficient to solve (PMP).
3. Compute the supply and the profit function of the firm.