Macroeconomics: Economic Growth (Licence 3)
Lesson 6: Overlapping generations model

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Overlapping generations (OLG) model

Lesson 6

- Before starting the presentation of the ‘endogenous growth theories’, we discuss in this course the assumption of the Solow model about **exogenous saving rate**
- This course presents a version of the overlapping generations model (OLG), where saving rate is endogenous
- The OLG model introduces on the demand side an **intertemporal consumption choice**.

Overlapping generations (OLG) model

Demande side

- **The intertemporal consumption choice is influenced by:**
- (1) Interest rate: if today we decide to investment instead of consuming one unit of the good, tomorrow we will receive $1 + r$
- (2) Preferences for today's consumption: one unit of consumption tomorrow is worth $1/(1 + r)$ today.

Overlapping generations (OLG) model

Assumptions

- (1) **The economy is populated by two generations:** young (generation 1) and old (generation 2)
- (2) **There are two periods** individuals of a generation live two periods (period t and period $t+1$)
  - At the beginning of each period, a young generation is born
  - At the end of each period, the generation born in the previous period passes away.

Overlapping generations (OLG) model

Assumptions

- **For this presentation, we will take the point of view of a single generation:**
- (1)x_t is the value of a given variable x for the young generation
- (2) x_{t+1} the value of x for the old generation (i.e., when the generation that was born at time t gets old)

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Assumptions on labor market, savings and consumption

- **Each young individual**
  - (1) **Labor market:** supplies 1 unit of labor when she is young only and receives a wage (w_t)
  - (2) **Consumption and saving decisions:** decides the part of income to be consumed in the present period (c_t) and the part to be saved for the future (s_t)
  - (3) **Budget constraint for young individual at t:** $w_t = c_t + s_t$
- **Each old individual**
  - (4) **Production:** is the owner of capital, which is combined with the labor supplied by the young generation to produce,
  - (5) **Consumption and saving decisions:** consumes all the saved income plus the interest she eventually receives
  - (6) **Budget constraint for old individual at $t + 1$:** $c_{t+1} = s_t(1 + r)$

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Assumptions on labor market, savings and consumption

- Combining both budget constraints we get the intertemporal budget constrain (IBC) for the young individual
- **Budget constraint for young individual at t:** $w_t = c_t + s_t$
- **Budget constraint for old individual at $t+1$:** $c_{t+1} = s_t(1+r)$
- **The intertemporal budget constrain (IBC) for the young individual is**

$$w_t = c_t + \frac{c_{t+1}}{(1+r)} \tag{1}$$

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Assumptions on labor market, savings and consumption

- The **utility function for the representative young** individual is

$$U = u(c_t) + \beta u(c_{t+1}) \tag{2}$$

Where $\beta = 1/(1+\rho)$

- The rate of depreciation of capital $\delta = 1$ (assume it for simplicity)
- The **growth rate of the population** is exogenous and equal to n
- **The production function** is Cobb Douglas: $Y = K_t^{\alpha} L_t^{1-\alpha}$
- **Perfect competition:** Factor markets are competitive (factor prices are equal to their marginal products): $r = \alpha k_t^{\alpha-1}$ and $w = (1-\alpha)k_t^{\alpha}$

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Solving the model: consumers

- The **Lagrangian for the maximisation problem of utility subject to the intertemporal budget constraint for the young generation is**:

$$L = u(c_t) + \beta u(c_{t+1}) + \lambda(w_t - c_t + c_{t+1}/(1+r)) \quad (3)$$

- First order conditions (FOC)
  - $u'(c_t) - \lambda = 0 \rightarrow u'(c_t) = \lambda$
  - $\beta u'(c_{t+1}) - \lambda/(1+r) = 0 \rightarrow \beta u'(c_{t+1})(1+r) = \lambda$
- Then we have:

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r \quad (4)$$

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Solving the model

- **To solve the model we assume that u(c)=ln(c)** :
- The Lagrangian becomes

$$L = ln(c_t) + \beta ln(c_{t+1}) + \lambda(w_t - c_t + c_{t+1}/(1+r)) \quad (5)$$

- First order conditions (FOC)
  - $1/c_t - \lambda = 0 \rightarrow 1/c_t = \lambda$
  - $\beta 1/c_{t+1} - \lambda/(1+r) = 0 \rightarrow \beta(1+r)/c_{t+1} = \lambda$
- Then we have:

$$\frac{c_{t+1}}{c_t} = \beta(1+r) \quad (6)$$

$$c_{t+1} = \beta(1+r)c_t \quad (7)$$

Overlapping generations (OLG) model

Solving the model

- **To solve the model we assume that u(c)=ln(c)** :
- Substitute $c_{t+1} = \beta(1+r)c_t$ into the IBC and solve for c_t

$$w_t = c_t + \frac{c_{t+1}}{(1+r)} \tag{8}$$

$$w_t = c_t + \frac{\beta(1+r)c_t}{(1+r)} = c_t(1+\beta) \tag{9}$$

$$c_t = \frac{w_t}{(1+\beta)} \tag{10}$$

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Solving the model: finding the endogenous saving rate

- Substitute $c_t = \frac{w_t}{(1+\beta)}$ into the first budget constraint and solve for s_t:
- **Budget constraint for young individual at t:** $w_t = c_t + s_t$

$$w_t = \frac{w_t}{(1+\beta)} + s_t \tag{11}$$

$$s_t = \frac{\beta w_t}{(1+\beta)} \tag{12}$$

Overlapping generations (OLG) model

Solving the model:

- Substitute $s_t = \frac{\beta w_t}{(1+\beta)}$ into the second budget constraint and solve for c_{t+1}:
- **Budget constraint for old individual at $t+1$:** $c_{t+1} = s_t(1+r)$

$$c_{t+1} = \frac{\beta w_t (1+r)}{(1+\beta)} \tag{13}$$

- Notice that, when $\beta = 1/(1+r)$, then $c_t = c_{t+1}$

$$c_{t+1} = \frac{w_t}{(1+\beta)} \tag{14}$$

Equilibrium

- In a close economy, in equilibrium, investment is equal to saving:

$$s_t L_t = S = I = K_{t+1} \tag{15}$$

Since $L_t = L_{t+1}/(1+n)$

$$s_t = k_{t+1}(1+n) \tag{16}$$

- Factor markets are competitive (factor prices are equal to their marginal products): $w_t = (1-\alpha)k_t^{\alpha}$ and $s_t = \frac{\beta w_t}{(1+\beta)}$, so:

$$s_t = \frac{\beta(1-\alpha)k_t^{\alpha}}{(1+\beta)} \tag{17}$$

Combining the previous equation, we get k_{t+1}

$$k_{t+1} = \frac{\beta(1-\alpha)k_t^{\alpha}}{(1+\beta)(1+n)} \tag{18}$$

Equilibrium

- At the steady state, $k^* = k_t = k_{t+1}$:

$$k^* = \left(\frac{\beta(1-\alpha)}{(1+\beta)(1+n)}\right)^{\frac{1}{1-\alpha}} \tag{19}$$

Overlapping generations (OLG) model

Equilibrium of golden rule

- Is the previous equilibrium also the one that maximizes the steady-state consumption (i.e., golden rule)?
- At the steady state: $c^* = y^* - sy^* = k^{*\alpha} - (\delta + n)k^*$
- To find the value of k^* that maximizes c^*, we differentiate respect to k^* and set equal to 0:

$$\frac{dc^*}{dk^*} = 0$$

$$\alpha k^{*\alpha-1} - (\delta + n) = 0$$

$$\alpha k^{*\alpha} = (\delta + n)k^*$$

$$k^{*1-\alpha} = \frac{\alpha}{\delta + n}$$

$$k^{gr} = (\frac{\alpha}{\delta + n})^{1/(1-\alpha)} = (\frac{\alpha}{1 + n})^{1/(1-\alpha)}$$

Where *gr* states for the Golden Rule where c is maximized, under assumption that $\delta = 1$

Dynamic inefficiency

- The comparison between k^* and k^{gr} shows that agents' choices do not necessarily lead to the maximum consumption
- If $k^* > k^{gr}$, then we have too much capital accumulation (and lower than maximum consumption)
- In this case, a **public intervention can be optimal**

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Dynamic inefficiency

- In this case, a public intervention can tax the young (thus, they will save less) and redistribute the payments to the old.
- This can lead capital accumulation towards the k^{gr} level and consumption in any period can increase