Macroeconomics: Economic Growth (Licence 3)
Aamphi IV. PANTHEON
Lesson 12: Revisions

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Acknowledges: some slides and figures are taken or adapted from the supplemental ressources to the textbook "Introduction to Economic Growth" by Charles I. Jones and Dietrich Vollrath, Third Edition, Norton W.W. Company Inc.

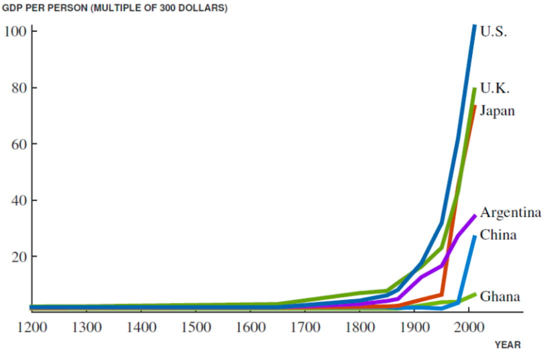
Main reference

- Jones, C. and Vollrath, D. (2013), Introduction to economic growth, 3rd edition (international student edition), WW Norton & Co.

Lesson 1: main stylized facts in empirical works

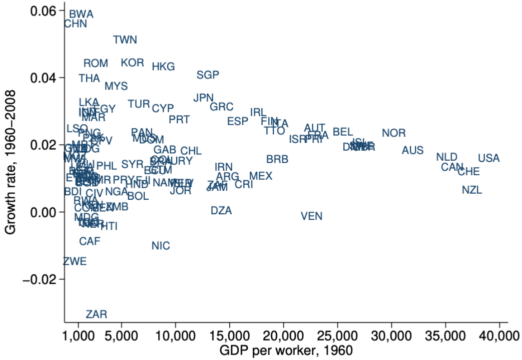
- (1) There are huge **differences** in per capita income across economies
- (2) Countries grow at **different rates**
- (3) Growth rates **vary over time**
- (4) Comparing a country to others reveals that its relative per capita income can **change** over time

Growth rates over time across countries

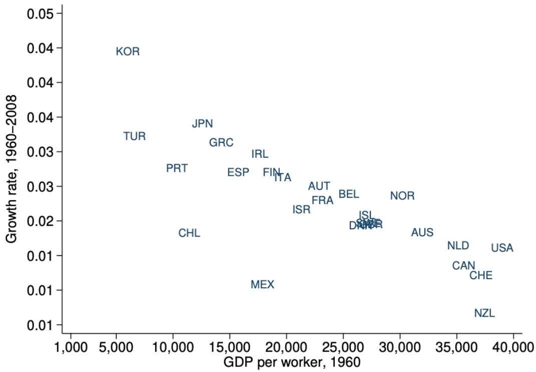


Note: The graph shows GDP per person for various countries. The units are in multiples of 300 dollars and therefore correspond roughly to the ratio between a country's per capita income and the income in the poorest country in the world. Source: The Maddison Project, Bolt and van Zanden (2014).

Lack of evidence of absolute convergence



Evidence of conditional convergence OECD countries (conditional on similar characteristics)



Lesson 2:

- Solow, R., 1956, "A contribution to the theory of economic growth", Quarterly Journal of Economics, vol. 70, pp. 65-94
- Basic version of the Solow growth model without technological progress countries
- Main assumptions:
- Perfect competition
- **A production function:** The Cobb-Douglas with constant returns to scale and no technological progress: $Y = K^{\alpha}(L)^{1-\alpha}$
- **A capital accumulation equation:** $\dot{K} = sY - \delta K$
- The growth rate of the number of workers, $\dot{L}/L$ is constant and equal to n

Lesson 3:

- Extensions of the simple version of the Solow growth model
- A version of the model with **technological progress**
  - incorporate productivity improvements.
  - Exogenous technological progress

Economic Growth

Main assumptions of the Solow Model with exogenous technological progress

- Perfect competition
- (1) **A production function:** The Cobb-Douglas with exogenous technological progress A

$$Y = K^{\alpha}(AL)^{1-\alpha} \quad (1)$$

- Exogenous constant growth of A: $\frac{\dot{A}}{A} = \gamma$
- (2) **A capital accumulation equation:**

$$\dot{K} = sY - \delta K \quad (2)$$

- The growth rate of capital is:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta. \quad (3)$$

- **Other assumptions:** The growth rate of the number of workers, $\dot{L}/L$ is

- Defining $\hat{k} = \frac{K}{AL}$ and taking the log and derivative with respect to time we obtain the accumulation of **capital per unit of effective labor**

$$ln\hat{k} = lnK - lnA - lnL \tag{4}$$

$$\frac{\dot{\hat{k}}}{\hat{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L} \tag{5}$$

$$\frac{\dot{\hat{k}}}{\hat{k}} = s\frac{Y}{K} - \delta - \gamma - n \tag{6}$$

Dividing and multiplying by AL and then replacing $\hat{y} = \hat{k}^{\alpha}$ we get:

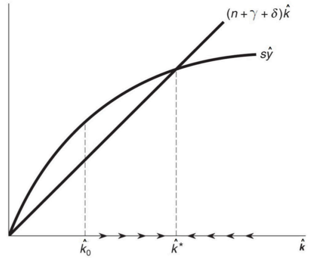
$$\frac{\dot{\hat{k}}}{\hat{k}} = s\frac{\hat{k}^{\alpha}}{\hat{k}} - (\delta + \gamma + n) \tag{7}$$

We get the **dynamic equation of capital (per effective units of labor) accumulation in the Solow model**

$$\dot{\hat{k}} = s\hat{k}^{\alpha} - (\delta + \gamma + n)\hat{k} \tag{8}$$

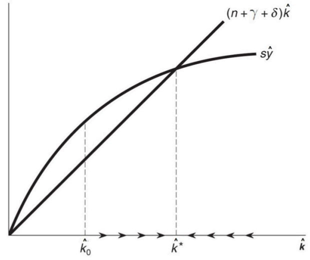
In the long run equilibrium in the steady state, Balance Growth Path (BGP), the $\dot{\hat{k}} = 0$

The Solow Diagram



- The Solow diagram with technological progress: expressed in **per unit of effective labor (in terms of AL)**
- In the long run equilibrium in the steady state, Balance Growth Path (BGP), the $\dot{\hat{k}} = 0 \rightarrow s\hat{k}^{\alpha} = (\delta + \gamma + n)\hat{k}$
- Plotting the two main equations: **production function with A**: $\hat{y} = \hat{k}^{\alpha}$
- **Capital accumulation function:** $\dot{\hat{k}} = s\hat{k}^{\alpha} - (\delta + \gamma + n)\hat{k}$
- $s\hat{k}^{\alpha}$ is the amount of investment per effective labor
- $(\delta + \gamma + n)\hat{k}$ is the amount to keep capital per effective unit of labor constant

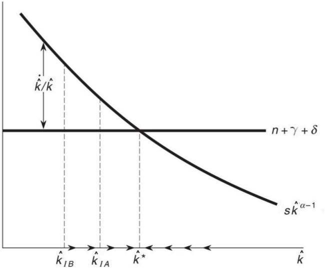
The Solow Diagram



- The diagram shows us that capital per worker will stabilize at some value $\hat{k}^*$ (steady-state),
- where the creation of new capital per worker due to investment ($s\hat{y} = s\hat{k}^{\alpha}$)
- just offsets the capital per worker "lost" due to depreciation and population growth
- Capital deepening capital per effective worker increases over time when **the amount of investment per effective labor exceeds the amount to keep capital-technology ratio constant** when $s\hat{k}^{\alpha} > (\delta + n + \gamma)\hat{k}$
- Capital widening capital per effective worker declines when $s\hat{k}^{\alpha} < (\delta + n + \gamma)\hat{k}$

Transition dynamics

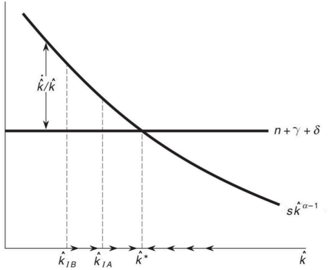
Transition dynamics based on growth rate of capital in effective units of labor



- $\frac{\dot{\hat{k}}}{\hat{k}} = s\hat{k}^{\alpha-1} - (\delta + \gamma + n)$
- The first curve is $s\hat{k}^{\alpha-1}$: the higher the level of capital per worker, the lower the average product of capital (diminishing returns to capital since $\alpha < 1$)
- The second term $(\delta + \gamma + n)$ does not depend on capital so it is a horizontal line.
- The difference between the two lines is the growth rate of capital stock $\frac{\dot{\hat{k}}}{\hat{k}}$.

Transition dynamics

Transition dynamics based on growth rate of capital in effective units of labor



- An economy starting at $k_0 < \hat{k}_*$ will experience a growing capital per effective units of labor (AL)
- **The further to the left of the steady state (the lower the capital per effective units of labor), the higher is the growth rate of the capital per effective units of labor;**
- Growth rates of the capital and income per worker in effective units of labor are proportional to the vertical distance between the $s\hat{k}^{\alpha-1}$ curve and the $(\delta+\gamma+n)$

Long run levels: variables per unit of effective labor and per capita

- Solving the model we find **the steady state equilibrium levels of capital in effective units of labor**
- $\hat{k}^*$ is the value of $\hat{k}$ such that $\dot{\hat{k}} = 0$. So

$$\dot{\hat{k}} = s\hat{k}^{\alpha} - (\delta + \gamma + n)\hat{k} = 0 \tag{9}$$

This solves to

$$\begin{aligned} (\delta + \gamma + n)\hat{k} &= s\hat{k}^{\alpha} && (10)\\ \hat{k}^{1-\alpha} &= \frac{s}{\delta + \gamma + n} && (11)\\ \hat{k} &= \left(\frac{s}{\delta + \gamma + n}\right)^{1/(1-\alpha)} && (12) \end{aligned}$$

which is the capital per worker at steady state, or

$$\hat{k}^* = \left(\frac{s}{\delta + \gamma + n}\right)^{1/(1-\alpha)} \tag{13}$$

Long run levels: variables per unit of effective labor and per capita

- Solving the model we find **the steady state equilibrium levels of capital and income in effective units of labor**
- which depends now also **on the growth rate of the exogenous technological progress (γ)**

$$\hat{k}^* = \frac{k^*}{A} = \left(\frac{s}{\delta + \gamma + n}\right)^{\frac{1}{1-\alpha}} \tag{14}$$

$$\hat{y}^* = \frac{y^*}{A} = \frac{k^{*\alpha}}{A} = \left(\frac{s}{\delta + \gamma + n}\right)^{\frac{\alpha}{1-\alpha}} \tag{15}$$

- We have seen that in the long run $\dot{\hat{k}} = 0$, thus:

$$\frac{\dot{\hat{k}}}{\hat{k}} = 0 \tag{16}$$

- Since $\frac{\dot{\hat{y}}}{\hat{y}} = \alpha \frac{\dot{\hat{k}}}{\hat{k}}$

$$\frac{\dot{\hat{y}}}{\hat{y}} = 0 \tag{17}$$

Long run growth (per worker variables)

- **Main difference with the Solow model without technological progress is in the long run growth of per worker variables:**

- We have defined: $\hat{k} = \frac{k}{A} \rightarrow \frac{\dot{\hat{k}}}{\hat{k}} = \frac{\dot{k}}{k} - \frac{\dot{A}}{A}$

- Since in the log run $\frac{\dot{\hat{k}}}{\hat{k}} = 0$ and $\frac{\dot{A}}{A} = \gamma$

- In the long run (BGP), the growth rate of capital per worker and income per worker are determined by the trend of growth rate of exogenous technological change:

$$\frac{\dot{k}}{k} = \frac{\dot{\hat{k}}}{\hat{k}} + \frac{\dot{A}}{A} = \gamma \tag{18}$$

- $\dot{y}/y = \gamma$

Lesson 4:

- **Comparative statics of Solow model with exogenous technological progress**

- Analyze the response of the model to changes in parameters
  - (1) The effects of a change in the population (n) on economic growth
  - (2) The effects of a change in investment (savings, s) on economic growth

- **Results are similar to the model without technological progress**

- (1) and (2) have **ONLY temporary** effects on the GROWTH RATES during the transition dynamics and **permanent** effects on the long run LEVELS

Lesson 4: Evaluating the Solow model

- How can we measure technological progress?
- **Computing and estimating Multi-factor Productivity**
  - The Solow residual
  - A black box.

Lesson 5:

- **Extensions of the Solow model: Human Capital**
- Mankiw, Romer, and Weil (1992) "A contribution to the Empirics of Economic Growth", Quarterly Journal of Economics
  - Assess how good is the fit with the data of the predictions of the Solow model with technological progress
  - They augment the model with the human capital and show that this version improves the fit
  - By recognizing that labor in different economies may have different levels of education (skills)

Mankiw, Romer, and Weil (1992): a growth regression

- Mankiw, Romer, and Weil (1992) estimate that Solow growth equation with technical progress with a regression using a sample of 98 countries :

$$ln(y_i) = a + \frac{\alpha}{1-\alpha} ln(s_i) - \frac{\alpha}{1-\alpha} ln(n_i + 0.05) + \epsilon_i \qquad (19)$$

- Where i refers to each country
- The dependent variable y_i is the log of GDP per working age person in 1985
- The independent variables are computed as averages over the period 1960-1985 (they repeat the estimation on sub-samples):
- $s_i = I_i / Y_i$ and n_i

Mankiw, Romer, and Weil (1992): a growth regression

- Main conclusion of the previous estimations of the Solow model with technology:
  - gives satisfactory results for what it concerns the signs of the effects of s and n
  - gives quite satisfactory results for what it concerns how well observed outcomes are replicated by the estimated model
  - BUT does not give satisfactory results for what it concerns the estimation of α
- To obtain a better fit with the data using a neoclassical growth model, Mankiw, Romer, and Weil (1992) include in the estimation human capital in a the Solow model
- A better fit since they find that this model specification explains almost the 80% in the variation of incomes ($R2 = 0.78$) and
- the estimated coefficients give $\alpha = 0.31$, which is about the expected value.

- To take into account the empirical findings they modify the Solow model to include *human capital*.

$$Y = K^{\alpha}(AH)^{1-\alpha} \tag{20}$$

where A is the labor-augmenting technology that grows exogenously at rate γ and H is the stock of human capital

$$H = e^{\psi u}L. \tag{21}$$

- u is the amount of time spent acquiring human capital (think of it as years of schooling).
- ψ is the return to education. The increase in H from one more unit of time acquiring human capital.

Extensions of the Solow Model

Solving the model by writing output in per worker terms (dividing by L and denoting in lowercase letters):

$$y = k^{\alpha}(Ah)^{1-\alpha}. \tag{22}$$

Take logs and derivatives,

$$\frac{\dot{y}}{y} = \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A} + (1-\alpha)\frac{\dot{h}}{h}. \tag{23}$$

We assume that

$$\frac{\dot{h}}{h} = 0 \tag{24}$$

$$\frac{\dot{A}}{A} = \gamma \tag{25}$$

or human capital (h) does not have trend growth, but there is trend growth in technology (A). A balanced growth path, as before, is where $\dot{y}/y$ is constant. That required that $\dot{y}/y = \dot{k}/k$. So again we have that

$$\frac{\dot{y}}{y} = \gamma \tag{26}$$

along the balanced growth path. Human capital doesn't change this.

Long run growth (per worker variables)

Proof:

- We can write the production function in terms of efficiency units taking into account human capital and dividing by AH
- Noting $\hat{y} = \frac{Y}{AhL} = \frac{y}{Ah}$ and $\hat{k} = \frac{K}{AhL} = \frac{k}{Ah} \rightarrow \hat{y} = \hat{k}^{\alpha}$
- Since: $\hat{k} = \frac{k}{Ah} \rightarrow \frac{\dot{\hat{k}}}{\hat{k}} = \frac{\dot{k}}{k} - \frac{\dot{A}}{A} - \frac{\dot{h}}{h}$
- Since in the log run steady state equilibrium (BGP), $\frac{\dot{\hat{k}}}{\hat{k}} = 0$, $\frac{\dot{h}}{h} = 0$ and $\frac{\dot{\hat{A}}}{\hat{A}} = \gamma$
- In the long run steady state equilibrium (BGP), the growth rate of capital per worker is

$$\frac{\dot{k}}{k} = \frac{\dot{\hat{k}}}{\hat{k}} + \frac{\dot{A}}{A} + \frac{\dot{h}}{h} = \gamma \qquad (27)$$

So in steady state

$$\dot{k}/k = \gamma = s\frac{y}{k} - (\delta + n). \tag{28}$$

Plug in for y to get

$$\gamma = s\frac{(Ah)^{1-\alpha}}{k^{1-\alpha}} - (\delta + n). \tag{29}$$

Solve for

$$\frac{k}{Ah} = \left(\frac{s}{\delta + n + \gamma}\right)^{1/(1-\alpha)}. \tag{30}$$

we know that

$$y = k^{\alpha}(Ah)^{1-\alpha} = Ah\left(\frac{k}{Ah}\right)^{\alpha} \tag{31}$$

$$y = Ah\left(\frac{s}{\delta + n + \gamma}\right)^{\alpha/(1-\alpha)} \tag{32}$$

$$y(t) = A(t)e^{\psi u}\left(\frac{s}{\delta + n + \gamma}\right)^{\alpha/(1-\alpha)}. \tag{33}$$

- Human capital, as determined by u, influences the level of output per worker, even though it does not change the growth rate of output per worker.

Economic Growth

Lesson 7:

- Endogenous growth
- Economics of ideas
- AK model

- **Basic AK model**

- Assume $\alpha = 1$, so the production function can be

$$Y = AK \tag{34}$$

- Assumptions:

- where A is some positive constant and it is assumed that $\frac{\dot{A}}{A} = 0 \rightarrow$ i.e. no technological progress
- There is no population growth

- Notice the linearity between K and Y: **We have perpetual growth because here we have constant returns to capital accumulation since** $\alpha = 1$

Economic Growth

- **Accumulation of capital is like in the Solow model**

$$\dot{K} = sY - \delta K \tag{35}$$

- **Assumption: total investment is larger than depreciation:** $sY > \delta K$

- Perpetual growth: Capital stock is always growing: Increase in capital accumulation drives economic growth than never stops

- The growth rate of capital is: $\frac{\dot{K}}{K} = sY/K - \delta = sA - \delta$

- The growth rate of the economy's income is equal to the growth rate of capital given by:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\dot{K}}{K} \tag{36}$$

$$\frac{\dot{Y}}{Y} = sA - \delta \tag{37}$$

- **The growth rate of the economy is an increasing function of the investment rate** s $\rightarrow$ policy implication

- **Lesson 8: The Romer Model**
- **Endogenous Growth: technological change is endogenous**
- **Assumptions:**
  - **Technological progress** is the addition of new varieties of intermediate goods to those available in the economy
  - Product innovation is a specific activity motivated by profits
  - **The economy is composed of three sectors:** research, intermediate goods sector, final goods sector
  - Market structure: imperfect competition monopolistic competition in the intermediate good sector

Economic Growth

- **Assumptions: the economy is composed of three sectors:**
    - **A research sector:** creates new ideas (non rivalrous but partially excludable due to patents). Design new variety of machinery and sells the right to produce a capital good to the intermediate good sector;
    - **An intermediate goods sector:** several monopolists (imperfect competition) that gain monopoly power to charge a markup by purchasing the design for specific capital goods and manufacture it and sell it to the final good sector.
    - **A sector producing final goods:** under perfect competition using labor and a number of different capital goods call also intermediate goods

- **Predictions**
    - **The long-run growth rates of the economy and of technological progress are determined by the growth rate of researchers (population n)**
    - Higher population means increasing the share of researchers and more researchers mean more ideas sustaining growth.

Lesson 9: Schumpeterian model

- An alternative way to model endogenous technological progress:
- **Creative destruction**: is to allow innovations to replace existing intermediate good with a more productive (or better quality) goods
- The innovation can be **uncertain** (a probability of being able to find an innovation)
- Improving productivity at each step
- Schumpeterian growth changes the mechanical details of growth, but not the general conclusions
  - The long-run trend growth rate depends on population growth
- One advantage of the Schumpeterian model is that it explicitly allows us to think about **firm dynamics, or the creation and destruction of firms over time.**

- **Malthusian model:**
  - Negative effect of population growth: when there is a fixed resource (e.g. land)
  - Malthusian effect: higher L (higher Y) but lower y: given the fixed factor (land), more people are working, lower is the per capita output
  - Negative relationship between population growth and population size that only explains part of the evidence
- **Kremer (1993) Model**
  - Explain the positive relationship between population growth and population size over the modern history
  - Based on endogenous growth models
- **Other determinants of economic growth:**
  - Institutions: Social infrastructure

Lesson 11

- International Trade and Economic Growth
- The role of MNF
- Does trade liberalization can foster economic growth/ productivity?
- Different impact of import barrier reduction on final goods and on intermediate inputs
- Export activity and productivity: cause or determinant?