

Midterm Exam (1 hour and 30 minutes)

Read and think before you write, and try to be both concise and precise

**Exercise 1 (30 minutes).** We consider a pure exchange economy with two commodities and two consumers.

- Consumer 1 has initial endowment  $e_1 = (1, 2)$  and his preferences are given by the Cobb-Douglas utility function

$$u_1(x_{11}, x_{12}) = x_{11}^{0.5} x_{1,2}^{0.5}$$

- Consumer 2 has initial endowment  $e_2 = (1, 1)$  and his preferences are given by the Cobb-Douglas utility function

$$u_2(x_{21}, x_{22}) = x_{21}^{0.25} x_{22}^{0.75}$$

1. Can we affirm à priori that there exists an equilibrium in this economy ? Why ?
2. Determine analytically the equilibrium of the economy (the price for good 1 is normalized to 1.)

**Exercise 2 (30 minutes).** We consider a pure exchange economy with two commodities and two consumers.

- The preferences of consumer 1 are given by the utility function  $u_1(x_{11}, x_{12}) = x_{11} + 2x_{1,2}$ .
- The preferences of consumer 2 given by the utility function  $u_2(x_{21}, x_{22}) = x_{21} + x_{22}$
- The total resources of the economy are  $r = (r_1, r_2) = (2, 2)$ .

1. Draw the Edgeworth box and represent the indifference curves of both consumers.
2. Determine (analytically or geometrically in the Edgeworth box) the set of all Pareto optimal allocations.
3. Does the differential characterization of Pareto optima applies in this economy ? Why ?

**Exercise 3 (30 minutes).** We consider an exchange economy with  $n \in \mathbb{N}$  consumers and  $n$  goods.

- All the consumers have the same Leontieff utility function:

$$u(z_1, \dots, z_n) = \min(z_1, \dots, z_n)$$

- Consumer  $i$  has an initial endowment of 1 unit of good  $i$  and zero units of every other good.

1. Determine the demand of consumer  $i$ .
2. Show that every price  $p \in \mathbb{R}_{++}^n$  is an equilibrium price of this economy.