(C4). $\succsim$ rational. MMG 1.B.4.

Show that if $u(x) = u(y) \Rightarrow x \sim y$
and $u(x) > u(y) \Rightarrow x \succ y$,
then $u(\cdot)$ is a utility fct representing $\succsim$.

Reminder.
Def: $u(\cdot)$ is said to represent $\succsim$
iff. $\forall x, y$. $x \succsim y \Rightarrow u(x) \geq u(y)$.

Def: given a fct $u : X \to \mathbb{R}$,
we can define a preference relation $\succsim_u$
by. $\forall x, y$. $u(x) \geq u(y)$
$\Rightarrow$ ~~x~~ $x \succsim_u y$.

($x \succsim y$ iff. $u(x) \geq u(y)$.).

Microeconomics 1A

W2

September 25
and 27, 2023.

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i.e. we want to show that

$$\left.\begin{aligned} &u(x) = u(y) \implies x \sim y. \\ \text{and} \quad &u(x) > u(y) \implies x \succ y. \end{aligned}\right\}.$$

$(\Leftarrow) \implies. \quad u(x) \geq u(y).$

$\implies x \succsim y.$

$$u(x) = u(y) \implies x \sim y.$$

$$\Updownarrow \qquad\qquad\qquad\qquad \Updownarrow$$

$$\begin{pmatrix} u(x) \geq u(y) \\ \text{and } u(x) \leq u(y) \end{pmatrix} \implies \begin{pmatrix} x \succsim y. \\ \text{and. } x \precsim y \end{pmatrix}.$$

and. (separately ...

$$u(x) > u(y) \implies x \succ y.$$

$$\Updownarrow$$

$$\begin{pmatrix} u(x) \geq u(y) \\ \text{and } u(x) \leq u(y) \end{pmatrix} \implies \begin{pmatrix} x \succsim y. \\ \text{and} \\ y \not\succsim x \end{pmatrix}.$$

Let

$$u(x) \geq u(y).$$

$$\Updownarrow$$

$$u(x) > u(y) \xrightarrow{\text{by hyp.}} x \succ y$$

or

$$u(x) = u(y). \xrightarrow{\text{by hyp.}} x \sim y$$

$$\Updownarrow$$

$$x \succsim y.$$

this equivalence of $\succsim$ and. ($\succ$ or $\sim$). can be deduced from completeness.

$\square$.

CS · Suppose $\succsim$ is rational.
Show that if X is finite, then there exists an utility fct u representing $\succsim$.

Intuitively, $\succsim$ rational. $\Longrightarrow$ $\succsim$ transitive and complete.

so all couples of two elements can be compared.

there is no contradictory loops...

same as for $\geq$ on $\mathbb{R}$.

X finite. $\Longrightarrow$ there is also a finite number of pairings to be compared.

We can try to construct such a utility fct representing $\succsim$.

let $n = \dim X$.
— start with $n = 2$.
$X = \{x_1, x_2\}$.
$\succsim_X$ is complete, so either $x_1 \succsim x_2$
or $x_1 \precsim x_2$
(or both).

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~~[illegible]~~

.... $\Longleftrightarrow$ either $u_1 \succ x_2$.

or $x_1 \sim x_2$.

or $x_1 < x_2$.

~~assume that~~

the [illegible] was

with . ...

* Assume the prop was true for n,
and prove it on $n+1$.) $\Leftarrow$

i.e.
proof by
INDUCTION.

... still the
INITIALIZATION.

we will thus prove
that the property
is true $\forall n$.

then.
if. $x_1 < x_2$, define u on X.
by. $u(x_1) = u_1$.
and $u(x_2) = u_2$.
with. $u_1 < u_2$.

if. $x_1 \sim x_2$. with. $u_1 = u_2$. ...

Assume the prop was true for n, i.e. $\exists u$ s.th.

$$\forall x,y \in X \quad x \succsim y \implies u(x) \geq u(y)$$

$$\Updownarrow \text{ result of C4.}$$

$$\Big((x \succ y \implies u(x) > u(y)) \quad \underline{\text{and}} \quad (x \sim y \implies u(x) = u(y)) \Big).$$

so, it means that ~~[illegible]~~, in X of dim $n+1$, we can order (and index) any subset of dim $= n$ (removing an element, say we eliminate the maximal element (one of), and we call it x_{n+1}.

$x_{n+1} =$ one of the x satisfying

$$x_{n+1} \succsim x \quad \forall x \in X.$$

I say this because you could have more than one element satisfying this.

$X \setminus \{x_{n+1}\}$ is of dim n, so, by assumption, $\exists$ indexation and $\exists u$, with.

$x_1 \lesssim x_2 \lesssim \dots \lesssim x_{n-1} \lesssim x_n$

$\Updownarrow$

$$u(x_1) \leq u(x_2) \leq \dots \leq \underbrace{u(x_{n-1})}_{u_{n-1}} \leq \underbrace{u(x_n)}_{\substack{\| \\ u_n}}$$

$\| \qquad \|$

$u_1 \qquad u_2$

$<$

or

$=$.

depending on whether

$x_1 < x_2$

or $x_1 \sim x_2$.

Now. if $x_{n+1} \succ x_n$ $\Longrightarrow$, assign to it.

$$\underset{\substack{\| \\ u_{n+1}}}{u(x_{n+1})} > u(x_n).$$

if $x_{n+1} = x_n$ $\Longrightarrow$, assign to it.

$$\underset{\substack{\| \\ u_{n+1}}}{u(x_{n+1})} = u(x_n)$$

$\square$. 7/

C16 MNG 366.

$X = \mathbb{R}^2_+$. $u(x) = \left[\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{1/\rho}$

CES utility fct.

a) Show that: $\rho = 1 \Rightarrow$ indifference curves are linear.

b) $\rho \longrightarrow 0$, CES $\longrightarrow$ Cobb-Douglas.

$$u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$$

c) $\rho \longrightarrow \infty$; indifference curve become right angles.

ie CES $\longrightarrow$ Léontief

$$u(x) = \min\{x_1, x_2\}$$

a) for $\rho = 1$, $u(x) = \alpha_1 x_1 + \alpha_2 x_2$.

that's the linear pref that we played with, a bit.

~~Ind~~ Indifference curves are defined as

$\forall y$: $I(y) = \{x, u(x) = u(y)\}$.

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h). $\rho \longrightarrow 0$.

$$\lim_{\rho \to 0} \left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{1/\rho}.$$

what is the definition of a limit ??

$$\cancel{\lim_{\rho\to 0} f(x)}$$

$$\left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{1/\rho} = \exp\left(\frac{1}{\rho} \ln\left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)\right).$$

this is continuous, so it will preserve limits!

$$\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho} \longrightarrow \alpha_1 + \alpha_2$$

if $\alpha_1 + \alpha_2$. , we thus have.

$$\frac{1}{\rho} \ln\left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right) \longrightarrow \frac{0}{0}$$

INDETERMINATE. !!

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Do we have some theorems that could help us solve indeterminate forms?

YES — L'Hôpital's rule.

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f'(c)}{g'(c)}$$

??

let f, g s.th. $f(c) = 0 = g(c)$.
and $g'(c) \neq 0$.

then.

Proof:

$$\lim_{x \to c} \frac{f(x)}{g(x)}.$$

what is the definition of a derivative?

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}.$$

So...

$\Longrightarrow$ from $f(c) = 0 = g(c)$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \left(\frac{\frac{f(x) - f(c)}{x - c}}{\frac{g(x) - g(c)}{x - c}} \right).$$

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using $g'(c) \neq 0$.

$$\Rightarrow \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} \frac{f(x)-f(c)}{x-c}}{\lim_{x \to c} \frac{g(x)-g(c)}{x-c}}.$$

$$\boxed{= \frac{f'(c)}{g'(c)}.}$$

$\square$.

$\longrightarrow$ Apply it to our problem!

$$\frac{1}{\ell} \ln \underbrace{\left(\alpha_1 x_1^{\ell} + \alpha_2 x_2^{\ell}\right)}_{= f(\ell)} \xrightarrow[\ell \to 0]{} ?$$

$= g(\ell)$.

we have. $f(\ell) \xrightarrow[\ell \to 0]{} 0$

$g(\ell) \xrightarrow[\ell \to 0]{} 0$

and. $g'(\ell) = 1 \not\xrightarrow[\ell \to 0]{} 0$

$f'(\ell) = ??$ ...

$$f(\ell) = \ln\left(\alpha_1 \underbrace{x_1^{\ell}}_{e^{\ell \ln x_1}} + \alpha_2 x_2^{\ell}\right)$$

$$\ln' = \frac{1}{\cdot}$$

$$f'(\ell) = \frac{\left(\alpha_1 x_1^{\ell} + \alpha_2 x_2^{\ell}\right)'}{\alpha_1 x_1^{\ell} + \alpha_2 x_2^{\ell}}$$

$$\frac{d}{d\ell}\left(\alpha_1 x_1^{\ell}\right) = \frac{d}{d\ell} e^{\ell \ln x_1} = \ln x_1 \, e^{\ell \ln x_1}$$

$$\Longrightarrow f'(\ell) = \frac{\alpha_1 x_1^{\ell} \ln x_1 + \alpha_2 x_2^{\ell} \ln x_2}{\alpha_1 x_1^{\ell} + \alpha_2 x_2^{\ell}}$$

$$\xrightarrow[\ell \to 0]{} \frac{\alpha_1 \ln x_1 + \alpha_2 \ln x_2}{\alpha_1 + \alpha_2} \quad (= 1)$$

$$\parallel$$

$$\ln\left(x_1^{\alpha_1} x_2^{\alpha_2}\right).$$

$$\Longrightarrow \lim_{\ell \to 0} \left(\alpha_1 x_1^{\ell} + \alpha_2 x_2^{\ell}\right)^{1/\ell} = \exp\left(\frac{f'(0)}{g'(0)}\right) = x_1^{\alpha_1} x_2^{\alpha_2}$$

$\square$

c) Show that $\rho \longrightarrow \infty$?? indifference curve become right angles.

$$u(x) \longrightarrow \min\{x_1, x_2\}.$$

Leontief.

$$u(x) = \underbrace{\left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)}^{1/\rho}.$$

$$= \left(\frac{x_1}{x_2}\right)^{-\rho}$$

$0 < . < 1$

if $x_1 < x_2$.

$$\underset{\rho \to -\infty}{\sim} \alpha_1 x_1^{\rho} \left(1 + \frac{\alpha_2}{\alpha_1} \left(\frac{x_2}{x_1}\right)^{\rho}\right)$$

$$\left(\ \right)^{1/\rho}$$

$\rho \to -\infty$.

0

So, $u(x) \underset{\rho \to -\infty}{\sim} \alpha_1^{1/\rho} x_1$

$$\xrightarrow[\rho \to -\infty]{} x_1.$$

if $x_2 > x_1$.

$$\Longrightarrow u(x) \longrightarrow x_2$$

if $x_1 = x_2$.

$$\Longrightarrow . \longrightarrow x_1 = x_2 \ \ldots !!$$

So $u(x) = \min\{x_1, x_2\}$. □

Summing up.

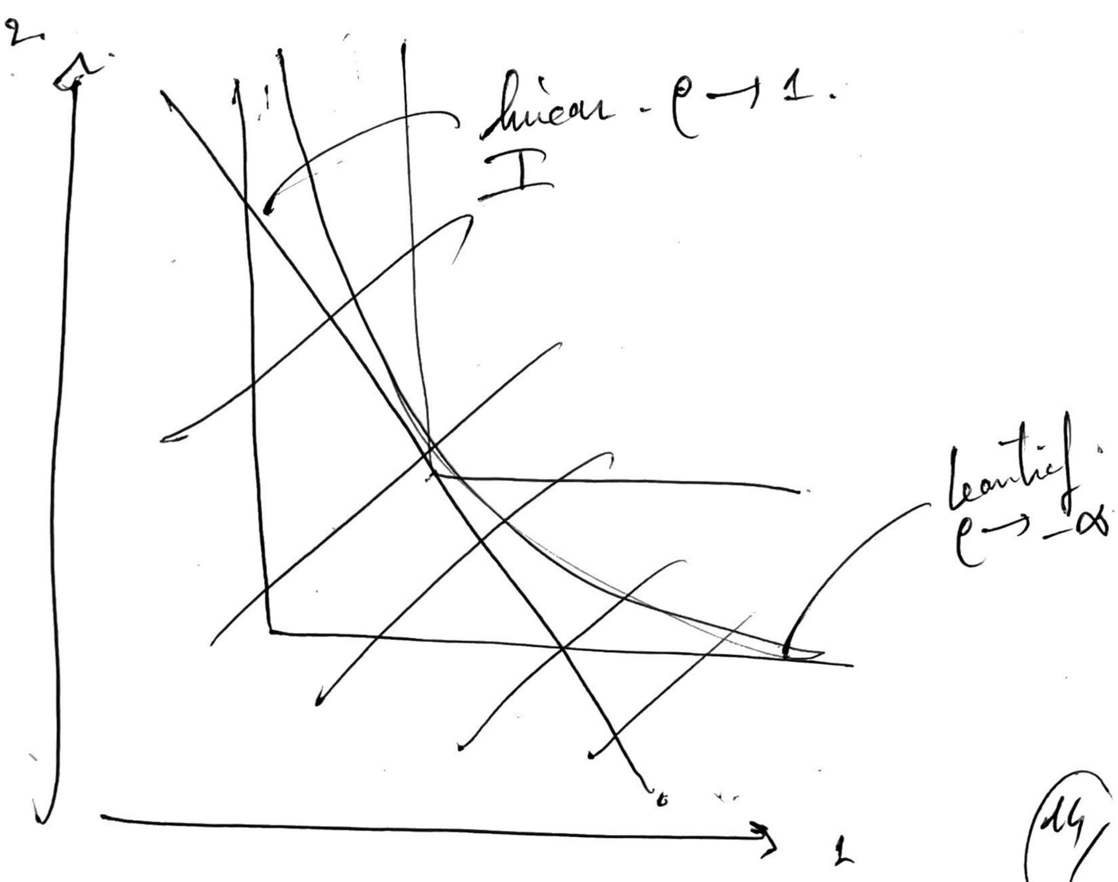
$$u(x) = \left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{1/\rho}.$$

$\rho \to 1$: $\alpha_1 x_1 + \alpha_2 x_2$.

$\rho \to 0$: $x_1^{\alpha_1} x_2^{\alpha_2}$. ← plot that ...

$\rho \to -\infty$: $\min\{\alpha_1, \alpha_2\}$.

DRAW THE INDIFFERENCE CURVES.



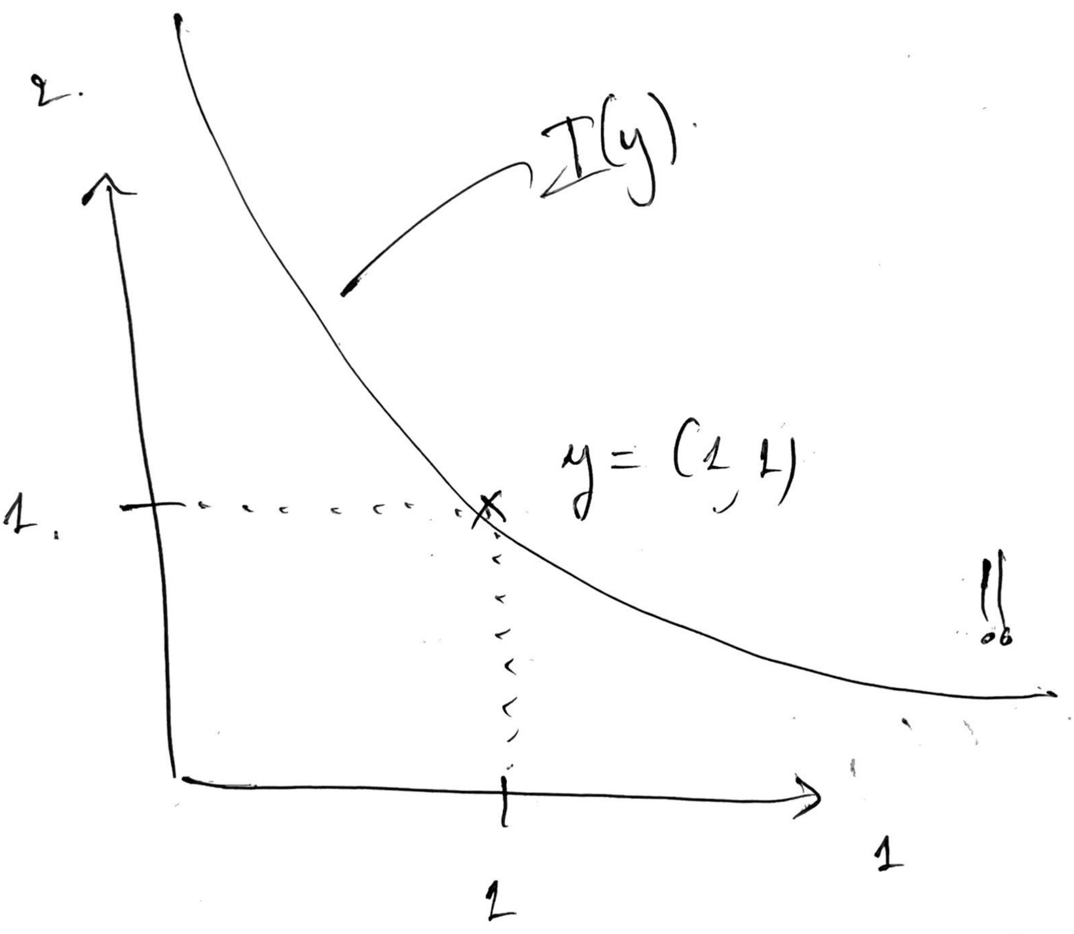
(14)

Cobb-Douglas.

$u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$ $\qquad \alpha_1 + \alpha_2 = 1.$

Idee. $\alpha_1 = \alpha_2 = \frac{1}{2}.$

Draw $I(y)$.
for, e.g. $y = (1,1)$.



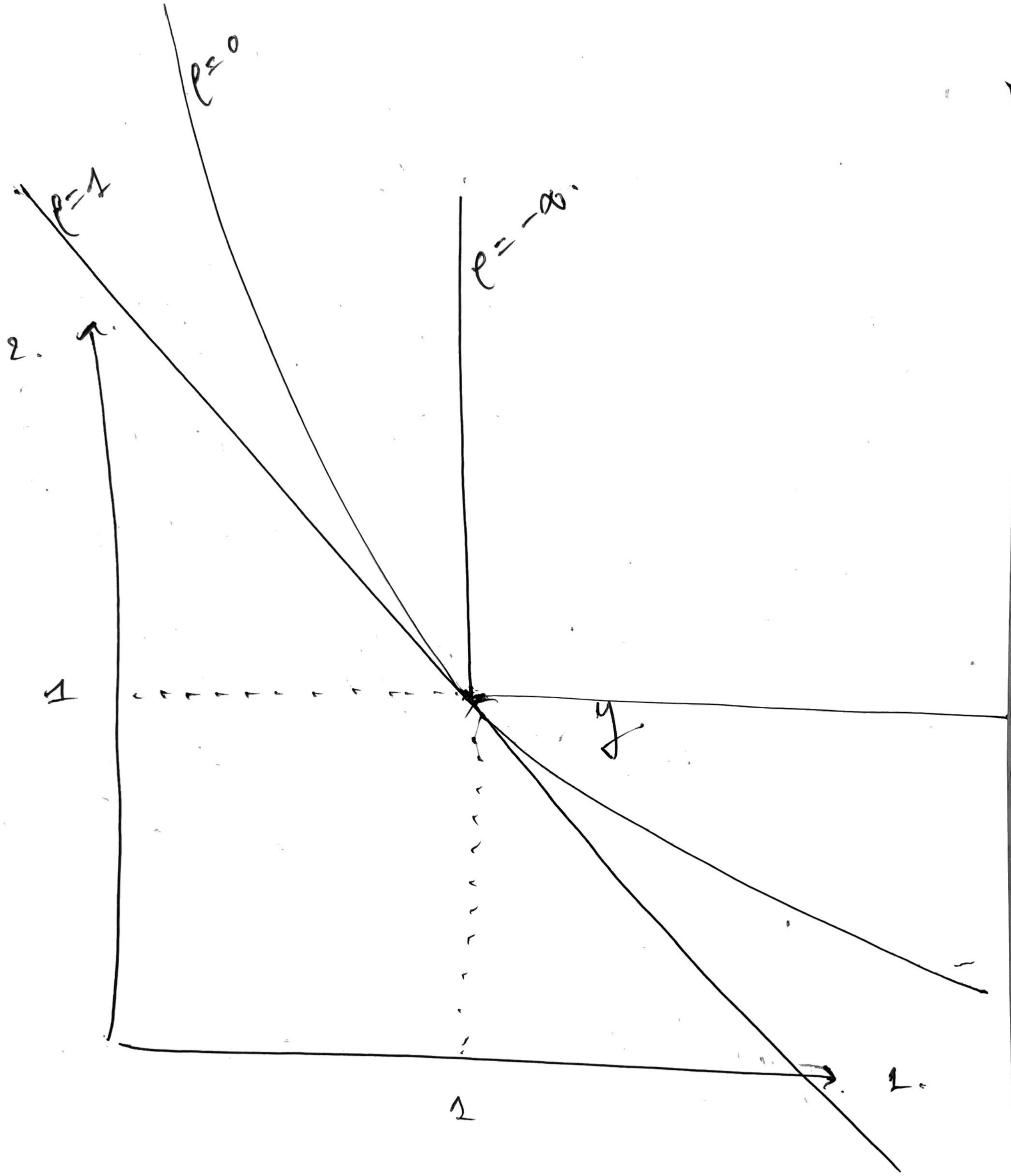
(because ~ asymptote).

$I(y) = \{x, \ x_1^{\alpha_1} x_2^{\alpha_2} = C \}$.

$1^{1/2} 1^{1/2} = 1.$

$= \{x, \ x_2 = 1/x_1 \}$.

(*) remember $y \in I(y)$



On wednesday, we'll solve C? and C8, we'll spend sometime ~~studying~~ the properties of the linear and Central prof

EXTRA:

$$\boxed{\text{Prove. } \succsim \text{ continuous AND monotonous} \implies \succsim \text{ representable.}}$$

See MWG.
Prop. 3C.1 p. 47.

$e = (1, \ldots, 1)$.

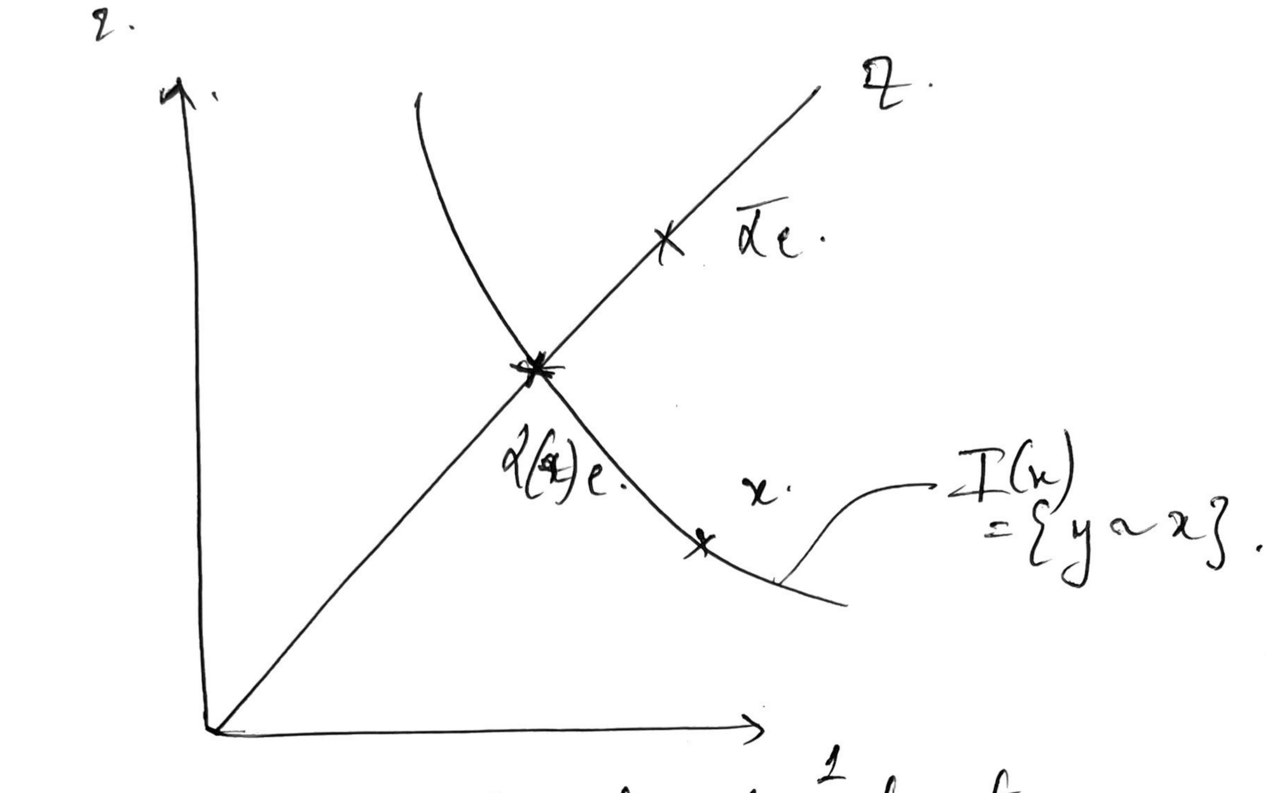
$\alpha e \in Z \ \forall \alpha \geq 0$. $\rightsquigarrow$ defines Z.

$\succsim$ monotonous iff
$\forall x \gg_{\mathbb{R}^\ell} y \implies x \succ_X y$

$\implies$ So. $\forall x \in \mathbb{R}^\ell_+ \quad x \succ \underline{0}$ $(0, \ldots, 0)$

$\forall \bar{\alpha}$ s.th. $\bar{\alpha} e \gg x$,
we have. $\bar{\alpha} e \succ x$.

1/.



$\Longrightarrow$ from monotonicity and continuity

locally nonsatiated

indifference curves are **NOT THICK!**

how to use it?

we can show that there is a unique $\alpha(x)$ s.th.

$$\boxed{\alpha(x)e \sim x}$$

WHERE DO WE USE THE FACT THAT IT IS CONTINUOUS ??

$\succsim$ CONTINUOUS.

$\iff \forall y$ $U(y)$ and $L(y)$ are <u>closed</u>.

AND, remember.

the whole set.

$X = U(y) \cup L(y)$.

$I(y) = U(y) \cap L(y)$.

$\Longrightarrow$ so ...

$I(y) = \{x \sim y\}$.

how can I prove the hit closes

hence the sets.

$\{\alpha;\ \alpha \succsim x\}$

and $\{\alpha;\ \alpha \precsim x\}$

are non empty ??

to use the completeness of $\succsim$.

Micro 1A. $\boxed{\text{COMPUTING LIMITS.}}$ September 27, 2023.

Quick note on the notation I wanted to use, because it is very handy

$\boxed{\text{NOTATIONS}}$

We write $f(x) \underset{x \to c}{\sim} g(x)$ and we say f is "equivalent" to g (when $x \to c$.)

to say that $\displaystyle\lim_{x \to c} \frac{f(x)}{g(x)} = 1.$

Example: $x + x^2 \underset{x \to 0}{\sim} x$

because $\displaystyle\frac{x + x^2}{x} = 1 + x \xrightarrow[x \to 0]{} 1$

We write $f(x) \underset{x \to c}{=} o(g(x))$ so f is "neglectable" w.r.t. g (when $x \to c$).

to say that $\displaystyle\lim_{x \to c} \frac{f(x)}{g(x)} = 0.$

(or, equivalently, $\displaystyle\lim_{x \to c} \frac{g(x)}{f(x)} = \pm\infty.$

i.e. g grows (in absolute value) much faster than f when $x \to c$.

or f "decays" much faster than g when $x \to c$.

$$\boxed{\begin{array}{l}\text{We write } f(x) = O(g(x)) \\ \text{to say that } \lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = M\end{array}}$$

i.e f is "dominated" by g when $x \to c$.

$\Updownarrow$

with M finite.
$\in \mathbb{R} \setminus \{\pm\infty\}$.

equivalently, we can write

$$f(x) \underset{x\to c}{\sim} M g(x)$$

⊕ note that f dominated by g
$\implies$ g dominated by f.

$$\boxed{\begin{array}{l}\text{The "equivalence" } f(x) \underset{x\to c}{\sim} g(x). \\ \text{can also be written} \quad (\text{or } f(x) \text{ equivalently}) \\ f(x) \underset{x\to c}{=} g(x) + \underbrace{o(\cancel{g}(x))}_{\text{neglectable w.r.t. } g(x) \text{ when } x\to c}\end{array}}$$

with we indeed have

$$\lim_{x\to c} \frac{f(x)}{g(x)} = 1.$$

USAGE. These concepts and notations (of equivalence $\sim$, dominance $o(\cdot)$, and neglectability $O(\cdot)$) are very useful to compute limits easily, using approximations.

Consider, for instance, Taylor series.

$$f(x) = \underbrace{\sum_{k \geq 0}}_{\text{i.e. } \sum_{k=0}^{+\infty}} \frac{f^{(k)}(a)}{k!} (x-a)^k$$

We can write.

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(a)(x-a)^k + \underbrace{o\left((x-a)^n\right)}.$$

i.e. all the rest of the series which is neglectable w.r.t. the "smallest" term of the left hand side series $(x-a)^n$ as $x \to a$.

or.

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k + \underbrace{O\left((x-a)^{n+1}\right)}.$$

i.e all the rest of the series, $\sum_{k=n+1}^{+\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$ which is dominated by $(x-a)^{n+1}$ when $x \to a$.

We all these LIMITED series expansions

For instance, you <u>do not have to know</u>, e.g., the "full" series expansion of

$$x \longmapsto \exp(x),\ \ln(1+x),\ \cos x,\ \sin x.,$$

which are:

$$e^x = \sum_{k \geq 0} \frac{x^k}{k!} \; ; \qquad \ln(1+x) = \sum_{k\geq 1} (-1)^{n+1} \frac{x^n}{n} \; ;$$

$$\cos x = \sum_{k\geq 0} (-1)^k \frac{x^{2k}}{(2k)!} \; ; \qquad \sin x = \sum_{k\geq 0} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

(although you can easily find those by computing $f', f'', \dots, f^{(n)}$ and guessing the form of the series by INDUCTION)

BUT you really <u>do want to know</u> their "LIMITED" expansion, up to a few terms, at least the first one, ie ...

$$\boxed{e^x \underset{x\to 0}{=} 1 + x + O(x^2)} \underset{x\to 0}{=} 1 + O(x)$$

$$\underset{x\to 0}{=} 1 + x + \frac{x^2}{2} + O(x^3).$$

$$\boxed{\ln(1+x) \underset{x\to 0}{=} x + O(x^2)} \underset{x\to 0}{=} x - \frac{x^2}{2} + O(x^3).$$

$$\boxed{\cos x \underset{x\to 0}{=} 1 + O(x^2)} \underset{x\to 0}{=} 1 - \frac{x^2}{2} + O(x^4)$$

$$\boxed{\sin x \underset{x\to 0}{=} x + O(x^3)} \underset{x\to 0}{=} x - \frac{x^3}{6} + O(x^5).$$

Exercice : you might also want to know the limited expansion :

$$\boxed{(1+x)^{\alpha} \underset{x \to 0}{=} 1 + \alpha x + O(x^2).}$$

We can show it using the limited expansions of. exp. and $\ln(1+x)$.

$$(1+x)^{\alpha} = e^{\alpha \ln(1+x)} \underset{x \to 0}{=} e^{\alpha(x + O(x^2))}$$
$$\underset{x \to 0}{=} 1 + \alpha x + O(x^2)$$

$\square$.

(You certainly want to know the "full" expansion of the geometric series. $\frac{1}{1-x} = \sum_{k=0}^{+\infty} x^k$).

APPLICATIONS.

Going back to exercices. C16 b) and c),

We could for instance compute C16 b) without knowing L'Hôpital's rule !

C16 h).

Show. CES $\xrightarrow[\rho \to 0]{}$ Cobb-Douglas.

$u(x) = \left(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho\right)^{1/\rho}$ $\qquad$ $u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.

with $\alpha_1 + \alpha_2 = 1$.

We could write

$\left(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho\right)^{1/\rho}$

using the "LIMITED" expansion of exp.

$$\alpha_i x_i^\rho = \alpha_i e^{\rho \ln x_i} \underset{\rho \to 0}{=} \alpha_i \left(1 + \rho \ln x_i + O(\rho^2)\right).$$

for both $i = 1, 2$.

So.

$$\left(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho\right)^{1/\rho} \underset{\rho \to 0}{=} \Big(\underbrace{\alpha_1 + \alpha_2}_{=1} + \underbrace{\left(\alpha_1 \ln x_1 + \alpha_2 \ln x_2\right)}_{= \ln\left(x_1^{\alpha_1} x_2^{\alpha_2}\right)} \rho + O(\rho^2) \Big)^{1/\rho}$$

$$\underset{\rho \to 0}{=} \left(1 + \ln\left(x_1^{\alpha_1} x_2^{\alpha_2}\right) \rho + O(\rho^2)\right)^{1/\rho}$$

$$\underset{\rho \to 0}{=} \exp\left(\frac{1}{\rho} \ln\left(1 + \ln\left(x_1^{\alpha_1} x_2^{\alpha_2}\right) \rho + O(\rho^2)\right)\right).$$

$$\underset{\rho \to 0}{=} \exp\left(\frac{1}{\rho}\left(\rho \ln\left(x_1^{\alpha_1} x_2^{\alpha_2}\right) + O(\rho^2)\right)\right).$$

using the "LIMITED" expansion of $\ln(1+x)$.

$$\underset{\rho \to 0}{=} \exp\Big(\ln\left(x_1^{\alpha_1} x_2^{\alpha_2}\right) + \underbrace{O(\rho)}_{\underset{\rho \to 0}{=} o(1)}\Big).$$

$$\underset{\rho \to 0}{=} x_1^{\alpha_1} x_2^{\alpha_2} + \underbrace{O(\rho)}_{\underset{\rho \to 0}{=} \underbrace{o(1)}_{\text{i.e. something that tends to } 0}} \qquad \square.$$

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Likewise for (C16 c).

Show:

$$\text{CES} \xrightarrow[\rho \to -\infty]{} \text{Leontief} \quad u(x) = \min\{x_1, x_2\}.$$

We have:

$$\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho} = \begin{cases} \alpha_1 x_1^{\rho}\left(1 + \dfrac{\alpha_2}{\alpha_1}\underbrace{\left(\dfrac{x_1}{x_2}\right)}_{<1}^{-\rho}\right) \underset{\rho\to-\infty}{\sim} \alpha_1 x_1^{\rho} & \text{if } x_1 < x_2 \\ \text{or} & \\ \alpha_2 x_2^{\rho}\left(\dfrac{\alpha_1}{\alpha_2}\underbrace{\left(\dfrac{x_2}{x_1}\right)}_{<1}^{-\rho} + 1\right) \underset{\rho\to-\infty}{\sim} \alpha_2 x_2^{\rho} & \text{if } x_2 < x_1 \end{cases}$$

So...

$$\lim_{\rho\to-\infty}\left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{1/\rho} = \begin{cases} x_1 & \text{if } x_1 < x_2 \\ x_2 & \text{if } x_2 < x_1. \end{cases}$$

(using that $\alpha_i^{1/\rho} \xrightarrow[\rho\to-\infty]{} 0$, $i = 1, 2$.)

(and, for $x_1 = x_2$, we have

$$u(x) = \left(\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}\right)^{1/\rho} = \left(\underbrace{(\alpha_1 + \alpha_2)}_{=1} x_{1\,(2)}^{\rho}\right)^{1/\rho} = x_{1\,(2)} \quad (\forall \rho.))$$

So... $\lim_{\rho\to-\infty} u(x) = \min\{x_1, x_2\}$

$\square$. 7/7