(C3) – $(\mathcal{B}, C(\cdot))$.

with $\mathcal{B} = \{\{x,y\}, \{x,y,z\}\}$

and. $C(\{x,y\}) = \{x\}$.

Prove that if $(\mathcal{B}, C(\cdot))$ satisfies WARP, then it must hold.

$$C(\{x,y,z\}) = \{x\} \text{ or } = \{z\} \text{ or } = \{x,z\}.$$

Reminder. WARP. Weak Axiom of Revealed Preference.

$(\mathcal{B}, C(\cdot))$ satisfies the WARP iff:

if for some. $B \in \mathcal{B}$.
$x, y \in B$ and $x \in C(B)$.
then for any $B' \in \mathcal{B}$ with $x, y \in B'$,
we must also have $x \in C(B')$

⊕ remember that $C(\cdot)$, the "choice rule" assigns a nonempty set.

(T08)

Reminder. The choice structure $(\mathcal{B}, C(\cdot))$ "reveals" a pref. rel. $\succsim^*$ defined by

$$\boxed{x \succsim^* y \iff \exists B \in \mathcal{B} \text{ with } x, y \in B \text{ and } x \in C(B).}$$

x is "revealed preferred to" y (at least).

Cf. y restate the WARP.

x is "revealed strictly preferred to" y if $\exists B \in \mathcal{B}$ with $x, y \in B$, $x \in C(B)$ and $y \notin C(B)$!

$\Downarrow$

WARP = "If $x \succsim^* y$ (i.e. x is revealed (at least) preferred to y) then $y \not\succ^* x$. (i.e. y cannot be revealed strictly preferred to x)."

$$\boxed{\text{WARP} \equiv \quad x \succsim^* y \Longrightarrow y \not\succ^* x.}$$

So assume $\mathcal{B} = \{\{x,y\}, \{x,y,z\}\}$.

and $\boxed{C(\{x,y\}) = \{x\}}$.

$\Downarrow$

this means $\exists B \in \mathcal{B}$ (in this case $\{x,y\}$) with $x, y \in B$, $x \in C(B)$ and $y \notin C(B)$.

$\Longrightarrow$ $x \succ^* y$. (x revealed <u>strictly</u> preferred to y.)

$\Updownarrow$

$(x \succsim^* y$ and $x \not\precsim^* y)$. $(*)$

For the WARP to be satisfied by the choice structure $(\mathcal{B}, C(\cdot))$, i.e. not to "reveal" inconsistencies, we then cannot have

$y \in C(\{x,y,z\})$. (otherwise, we would have $y \succsim^* x$ which would contradict $(*)$.)

But $C(\{x,y,z\})$ cannot be empty,

so we must have

either $x \in C\{x,y,z\}$ or $y \in C(\{x,y,z\}$ or both

$\square$

(4) let $x(p,w)$ = demand of the consumer

s.th. $x_i(p,w) = \dfrac{w}{p_1+p_2}$ $i = 1,2$.

a) prove that this demand is homogeneous of deg 0.
b) prove that it satisfies Walras' law.
c) State the Weak axiom of revealed pref. (WARP) in the framework of the demand.
d) Prove that this demand satisfies WARP.

Proof: (a) let $\alpha > 0$. $\forall i = 1,2$.

$$x_i(\alpha p, \alpha w) = \frac{\cancel{\alpha} w}{\cancel{\alpha} p_1 + \cancel{\alpha} p_2} = \frac{w}{p_1+p_2} = x_i(p,w) \cdot \underbrace{\alpha^0}_{=1}.$$

$\Longrightarrow$ so $x(p,w)$ is homogeneous of deg 0. $\square$

b) $$p \cdot x(p,w) = p_1 x_1(p,w) + p_2 x_2(p,w).$$
$$= \frac{p_1 w}{p_1+p_2} + \frac{p_2 w}{p_1+p_2} = \underline{w}.$$

$\Longrightarrow$ so $x(p,w)$ satisfies Walras' law. $\square$

Prop.

c) Show that the Walrasian demand function $x(p,w)$ satisfies WARP iff $\forall$ (p,w) and (p',w'):

$p \cdot x(p'w') \leq w$ ~~and~~ $x(p',w') \neq x(p,w)$

$$\Longrightarrow p' \cdot x(p,w) > w'$$

Proof: $x(p, w)$ specifies the choice.

$$\underbrace{C(B_{p,w})}_{\substack{=\text{ affordable bundles given} \\ p \gg 0 \text{ and } w > 0}} = x(p, w) = \begin{array}{l}\text{set of } x \text{ in } B_{p,w} \\ \text{which are preferred} \\ \text{to all others.}\end{array}$$

Assume $p \cdot x(p', w') \leq w$,

it means that $\underline{x(p', w') \subset B_{p,w}}$.

Assume $x(p', w') \neq x(p, w)$

$\Longrightarrow$ ~~thus~~ we have, $x(p, w) \succ^* x(p', w')$

$\succ^*$ = revealed strictly preferred to ...

because ($x \succsim^* x'$ and $x' \not\succsim^* x$!)

$x(p', w')$ and $x(p, w) \in B_{p,w}$ and $\Rightarrow$

Thus, by WARP, we cannot have:

$$\boxed{\begin{array}{l} x(p', w') \text{ and } x(p, w) \in \underline{B_{p', w'}} \\ \underline{\text{and}} \quad x(p', w') = C(B_{p', w'}) \end{array}}$$

But $x(p', w') \in B_{p', w'}$ and $x(p', w') = C(B_{p', w'})$ are both TRUE.

so $x(p, w) \in B_{p', w'}$ <u>must be FALSE</u>. $\square$

i.e. $\boxed{\text{we must have } p' \cdot x(p, w) > w'}$

.... c). So the WARP in the framework of the demand is.

$$\boxed{\begin{array}{c} p.x(p',w') \leq w \text{ and } x(p',w') \neq x(p,w). \\ \Longrightarrow p'.x(p,w) > w' \end{array}}$$

d). $x_i(p',w') = \dfrac{w'}{p_1'+p_2'}$. $i=1,2$.

So. $p.x(p',w) \leq w$

$$\Longleftrightarrow p_1 \frac{w'}{p_1'+p_2'} + p_2 \frac{w'}{p_1'+p_2'} \leq w.$$

$$\Longleftrightarrow \boxed{(p_1+p_2)\, w' \leq w.(p_1'+p_2').} \quad (*).$$

And. $x(p,w) \neq x(p',w') \Longleftrightarrow \dfrac{w'}{p_1'+p_2'} \neq \dfrac{w}{p_1+p_2}.$

$\Longrightarrow$. (*) is strict!

i.e. $(p_1+p_2)\, w' < w\,(p_1'+p_2')$.

$$\Longleftrightarrow w' < (p_1'+p_2')\frac{w}{p_1+p_2} = p'.x(p,w).$$

i.e $\boxed{p'.x(p,w) > w'}$.

Π.