Microeconomics 1A. T 10 October 18 2023

32 - 33 AND 36 are all PAST EXAMS.

+ 34 = insurance.

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professions: (A) (B) (C)

schools

(1) → (C) with certainty

(2) → (A) "

(3) → (B) with prob. 0.1
(A) — 0.9

We assume $L(1) \sim L(3)$ and $L(1), L(3) \succ L(2)$

The "schools" here are lotteries and the "professions" are outcomes.

$L_1 = (0, 0, 1)$
$L_2 = (1, 0, 0)$
$L_3 = (0.9, 0.1, 0)$

We assume $\succsim$ of the decision maker is continuous s.th. it is representable by a utility fct.

+ we assume $\succsim$ satisfies the independence axiom s.th. its utility fct has the expected utility form.

$$\longrightarrow U(L) = u_A p_A + u_B p_B + u_C p_C$$

* $\boxed{\text{1) Give a representation of the utility fct of this decision-maker.}}$

$$L_1 \sim L_3 \iff \underset{\displaystyle u_C}{\underset{\parallel}{U(L_1)}} = \underset{\displaystyle 0.9\,u_A + 0.1\,u_B}{\underset{\parallel}{U(L_3)}}$$

$$\boxed{\cancel{u_C} = 0.9\,u_A + 0.1\,u_B}$$

and

$$L_1, L_3 \succ L_2 \iff \underset{\displaystyle u_A}{\underset{\parallel}{U(L_2)}} < U(L_1), U(L_3).$$

$$\Updownarrow$$

$$\boxed{u_A < u_C} = 0.9\,u_A + 0.1\,u_B$$

$$\iff 0.1\,u_A < 0.1\,u_B$$

$$\iff \boxed{u_A < u_B}$$

2) $\boxed{\text{school (4)} \longrightarrow L_4 = \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right) \text{ how would school (4) rank compared with (1–3)?}}$

$$U(L_4) = \frac{1}{3}\left(u_A + u_B + u_C\right)$$

$$\Longrightarrow \quad U(L_4) - U(L_1) = U(L_4) - U(L_3).$$

$$\parallel$$

$$\frac{1}{3}\left(u_A + u_B + u_C\right) - \left(\frac{9}{10}u_A + \frac{1}{10}u_B\right).$$

$$\parallel$$

$$\frac{1}{3}\left(u_A + u_B + u_C\right) - u_C = \frac{1}{3}\left(u_A + u_B\right) - \frac{2}{3}u_C$$

$$0.1\,u_B = u_C - 0.9\,u_A$$

$$\boxed{u_B = 10u_C - 9u_A}$$

$$= \frac{1}{3}\left(\underbrace{u_A + 10u_C - 9u_A - 2u_C}_{= -8u_A + 8u_C.}\right)$$

$$\Longrightarrow \boxed{U(L_4) - U(L_{1,3}) = \frac{8}{3}\left(u_C - u_A\right).}$$

$$\boxed{> 0 \text{ since } u_A < u_C}$$

So $\Longrightarrow \boxed{L_4 \succ L_{1,3}}$

By transitivity, $\boxed{L_1 \succ L_2.}$

3). Compound lottery.

$$\boxed{L_\alpha = \alpha L_1 + (1-\alpha) L_2.}$$

We are asked to determine α_c s.th.

$$U(L_{\alpha_c}) = U(L_1) \qquad \left[\text{for } \alpha > \alpha_c, \implies L_\alpha \succ L_{1,3} \right]$$

" linearity.

$$\alpha_c U(L_1) + (1-\alpha_c) \underset{\parallel}{U(L_2)} = U(L_1) = U(L_3)$$

$$\iff \frac{1}{3}\alpha_c \left(u_A + u_B + u_C \right) + (1-\alpha_c) u_A - u_C = 0$$

$$\iff \frac{1}{3}\alpha_c \left(u_B + u_C - 2u_A \right) \quad (u_C - u_A) = 0 \ldots$$

$$u_B + u_C - 2u_A = -9u_A + 10u_C \ldots = 11u_C - 11u_A.$$

$$\Rightarrow \frac{11}{3}\alpha_c\,(u_C - u_A) - (u_C - u_A) = 0.$$

$$\Leftrightarrow \underbrace{(u_C - u_A)}_{>0}\left[\frac{11}{3}\alpha_c - 1\right] = 0. \quad \text{iff}$$

$$\alpha_c = \frac{3}{11} \longleftarrow U(L_{\alpha_c}) = U(L_{1,3})$$

$$\text{if } \alpha \underset{<}{>} \alpha_c \longrightarrow U(L_\alpha) \underset{<}{>} U(L_{1,3})$$

$\square$

(33) $\underline{u(x) = x^a}$ $\quad a < 1.$ "Bernoulli" utility fct ??

1. X uniform dist. over $[0, 1]$.

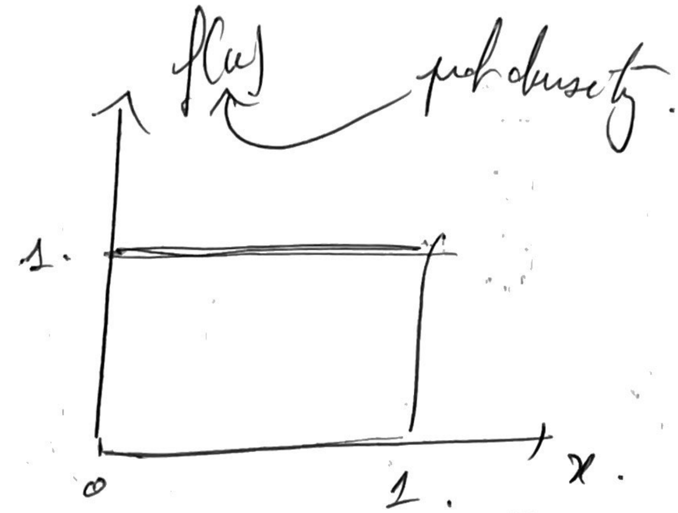
$$E(u(X)) = \int u(x)\, dF(x)$$

$$= \int_0^1 u(x)\, dx \quad \text{uniform dist.}$$

$$= \int_0^1 x^a\, dx, \quad = \left[\frac{x^{a+1}}{a+1}\right]_0^1$$

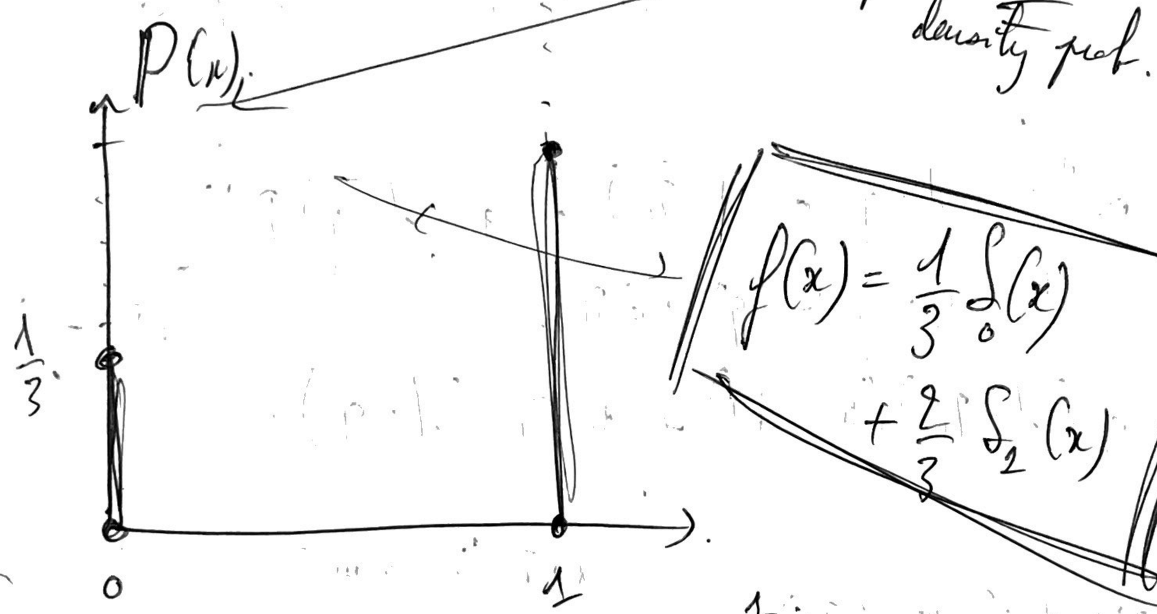
$$\boxed{E(u(X)) = \frac{1}{1+a}.}$$

uniform dist.



$$F(x) = \int_0^{x \le 1} \underset{1}{f(t)}\,dt = \underline{x}.$$

"cumulative dist."



$$\boxed{f(x) = \frac{1}{3}\delta_0(x) + \frac{2}{3}\delta_2(x)}$$

$$E(u(Y)) = \int_0^1 u(x) f(x)\,dx$$

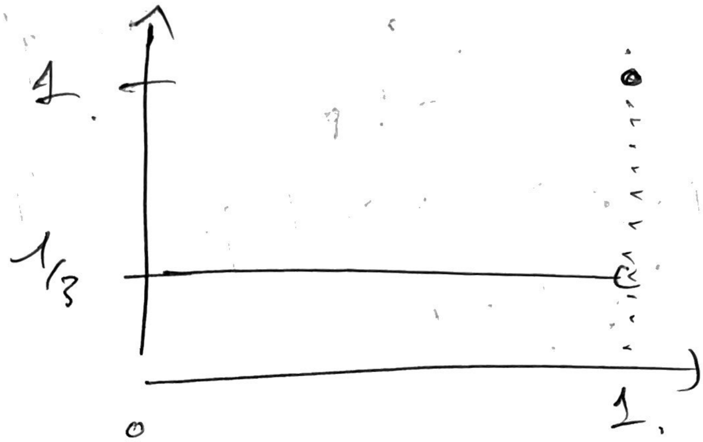
$$= \frac{1}{3}u(0) + \frac{2}{3}u(1) = \frac{2}{3}$$

$$F(x) = \int_0^x f(x)\,dx.$$

$$\frac{1}{3}\delta_0(x) + \frac{2}{3}\delta_1(x).$$

$$\boxed{F(x) = \begin{cases} \frac{1}{3} \ ; & 0 \le x < 1. \\ 1. \ ; & x = 1. \end{cases}}$$

$\longrightarrow$ yields the same result.



3). indifferent btw X and Y ? $\longrightarrow$ $\boxed{X \sim Y.}$

Von Neumann-Morgenstern utility.

$$\Longleftrightarrow U(X) = U(Y)$$

Bernoulli utility.

$$\Longleftrightarrow E(u(X)) = E(u(Y))$$

$$\Longleftrightarrow \frac{1}{1+a^*} = \frac{2}{3}.$$

$$\Longleftrightarrow a^* = \frac{3}{2} - 1 \Longleftrightarrow \boxed{a^* = \frac{1}{2}}$$

4) If $a > a^*$, then

$$E(u(X)) = \frac{1}{1+a} < \frac{1}{1+a^*} = \frac{2}{3} = E(u(Y)).$$

$$\Longleftrightarrow \quad U(X) < U(Y)$$

$$\Longleftrightarrow \quad \boxed{X \prec Y}$$

5) ARROW-PRATT coefficient of absolute risk aversion.

$$\boxed{r_A(x) = -\frac{u''(x)}{u'(x)}}$$

$$u(x) = x^a \longrightarrow \quad u'(x) = a x^{a-1}.$$

$$u''(x) = a(a-1)\, x^{a-2}.$$

$$\Longrightarrow \boxed{r_A(x) = \underbrace{(1-a)}_{>0} \frac{1}{x}.}$$

(B4). ie. MWG 6.C.1.

The decision-maker has wealth w.
and probability π of a loss D.

ie. is facing the lottery

$$L = \{(w, w-D), p = (1-\pi, \pi)\}.$$

They can insure against possible loss at price q.
ie. buying α units of insurance cost αq and gives right to α coverage if the loss D happens.

The new lottery, as a fct of the amount of insurance purchased α, is thus

$$L_\alpha = \{(w - \alpha q, w - D - \alpha q + \alpha), p = (1-\pi, \pi)\}.$$

We consider that the decision maker has Bernoulli utility $u(x) = \ln x$.

1) Show that if the price of the insurance is actuarially fair, ie $q = \pi$, then the decision-maker insures completely.

2) Show that if the price is not actuarially fair, i.e. $q > \pi$, then they do not insure completely.

3) Find the price for which they do no insure at all.

Def: The price q of an INSURANCE is ACTUARIALLY FAIR iff it is equal to the expected cost of the insurance.

i.e. iff the insurance makes no loss or gain on average.

Prop: The price q is ACTUARIALLY FAIR iff

$$q = \pi$$

(in the example above).

Proof: From the point of view of the insurer they will gain αq if the loss does not occur, i.e. with probability $1-\pi$ and $\alpha q - \alpha$ if the loss occur, i.e. with prob. π.

So the expected gain of the insurer is

$$\mathbb{E}(X) = \alpha q (1-\cancel{\pi}) + (q\alpha - \alpha)\pi$$

$$= \alpha (q - \pi)$$

So $E(X) = 0$ iff $\boxed{q = \pi}$

To solve 1), 2) and 3), we must know what the decision of the decision-maker is, as a fct of the price of the insurance q.

I.e, given q, how much α does the decision-maker choose to purchase?

Suppose the decision-maker is a preference/utility-maximizer, suppose their preference is representable by $U(.)$ of the expected utility form, and its Bernoulli utility distribution (i.e the assignment of a utility value to each outcome) is $u(x) = \ln x$.

Then the problem can be stated as a UMP: the decision-maker will buy α^* units of insurance s.th.

$$\alpha^* \text{ is solution to } \begin{cases} \max U(L_\alpha) \\ \alpha \geq 0. \end{cases}$$

→ This is an unconstrained UMP, so its solution must verify.

$$\frac{\partial}{\partial \alpha} U(L_{\alpha^*}) = 0,$$

(1st order necessary condition)

$$U(L_\alpha) = u(w-\alpha q)(1-\pi) + u(w-D+\alpha(1-q))\,\pi.$$

$$\frac{\partial}{\partial\alpha}U(L_\alpha) = \frac{-q(1-\pi)}{w-\alpha q} + \frac{(1-q)\pi}{w-D+(1-q)\alpha}.$$

given $u(x) = \ln x$.

so. $\frac{\partial}{\partial\alpha}U(L_{\alpha^*}) = 0$ iff.

$$q(1-\pi)(w-D+(1-q)\alpha^*) = (1-q)\pi(w-\alpha^* q)$$

$$\Longleftrightarrow \alpha^*\left[(1-q)q(1-\cancel{\pi}) + \cancel{q(1-q)\pi}\right] = -q(1-\pi)(\cancel{w}-D) + w\pi(1-\cancel{q})$$

$$\Longrightarrow \quad q(1-q)\,\alpha^* = q(1-\pi)D + w(\pi - q)$$

$$\Longrightarrow \quad \boxed{\alpha^* = \frac{(1-\pi)}{(1-q)} D + w\,\frac{\left(\frac{\pi}{q} - 1\right)}{(1-q)}}$$

with $q < 1$, otherwise it is clear that they won't insure.

1) So if the price is actuarially fair, ie $q = \pi$,

then $\underline{\alpha^* = D}$. ie the decision maker insures completely.

2) if $q > \pi$ then $\frac{\pi}{q} - 1 < 0$.

and $\frac{1-\pi}{1-q} > 1$. $\therefore$ conclusion is ambiguous.

Sign of $\alpha^* - D$?

$$\alpha^* - D = \frac{1-\pi}{1-q} D + \frac{w\left(\frac{\pi}{q} - 1\right)}{1-q} - \frac{(1-q)D}{1-q}$$

$$= \left[(q-\pi) D + w\left(\frac{\pi}{q} - 1\right) \right] \frac{1}{1-q}$$

$$= \underbrace{\left(D - \frac{w}{q}\right)}_{} \underbrace{(q-\pi)}_{>0 \text{ iff } q > \pi} \underbrace{\frac{1}{1-q}}_{>0 \text{ because } 0<q<1.}$$

so everything depends on the sign of that one.

If D is sufficiently small as compared to w. (i.e. $D < \frac{w}{q}$).

then.

$$q > \pi \Longrightarrow \alpha^* - D < 0.$$

i.e. the decision maker does <u>not</u> insure completely.

3) Finally, find q^* s.th.

$$\alpha^*(q^*) = 0$$ i.e. the decision maker does not insure at all

i.e.

$$(1-q^*)\,\alpha^* = (1-\pi)D + w\left(\frac{\pi}{q^*} - 1\right).$$

i.e. $$q^{*2}\alpha^* + q\left((1-\pi)D + w - \alpha^*\right) + w\pi = 0.$$

$$\Delta = \left((1-\pi)D + w - \alpha^*\right)^2 - 4\alpha^* w\pi$$

$$q^*_\pm = \frac{-\left((1-\pi)D + w - \alpha^*\right) \pm \sqrt{\Delta}}{2\alpha^*}.$$

assuming. $\Delta > 0$.

(36) BERNOUILLI $u(x)$. $L = \{(0,4,9); (\frac{1}{6}, \frac{1}{2}, \frac{1}{3})\}$.

(1) Def of CERTAINTY EQUIVALENT.

Given a BERNOUILLI utility fct $u(\cdot)$, the CERTAINTY EQUIVALENT of $F(\cdot)$, $c(F,u)$ is defined by

$$\boxed{u(c(F,u)) = E(u(X)) = \int u(x)\,dF(x)}$$

"same lottery"

$$u(c(L,u)) = \frac{u(0)}{6} + \frac{u(4)}{2} + \frac{u(9)}{3}.$$

2). take $u(x) = \sqrt{x}$.

$$\Rightarrow c(L,u) = \left(\frac{\sqrt{0}}{6} + \frac{\sqrt{4}}{2} + \frac{\sqrt{9}}{3}\right)^2.$$

$$= \left(\frac{2}{2} + \frac{3}{3}\right)^2 = \boxed{4.}$$

$$\Longrightarrow \boxed{c(L,u) = 4}$$

3) Arrow-Pratt coeff of ABSOLUTE RISK AVERSION.

$$\boxed{r_A(x)} = -\frac{u''(x)}{u'(x)} = \boxed{\frac{1}{2x}.}$$

$u(x) = \sqrt{x}$.

$\hookrightarrow u'(x) = \frac{1}{2\sqrt{x}}$.

$\hookrightarrow u''(x) = -\frac{1}{4x^{3/2}}$.

4) Assume $w = 12$. and owns the lottery.
* the minimum price they accept to sell the price, p, is. given by.

$$u(12+p) = \mathbb{E}(u(12+L)) = u(c(12+L,u)).$$

$\iff$

$$\sqrt{12+p} = \frac{\sqrt{12}}{6} + \frac{\sqrt{16}}{2} + \frac{\sqrt{21}}{3}.$$

$\Longrightarrow p = \ldots$

5)

$$p = -w + u^{-1}u(c(w+L,u)).$$

$$\boxed{p = -w + c(w+L,u).}$$

???

it is an alternative definition of the risk premium.

We can show that. $x - c_x$ decreasing fct of the wealth

$\iff r_A(x)$ decreasing fct of x.