Exercice 1: **2020 MIDTERM**

1). (i) Leontief. $u(x) = \min\{x_1, x_2\}$.

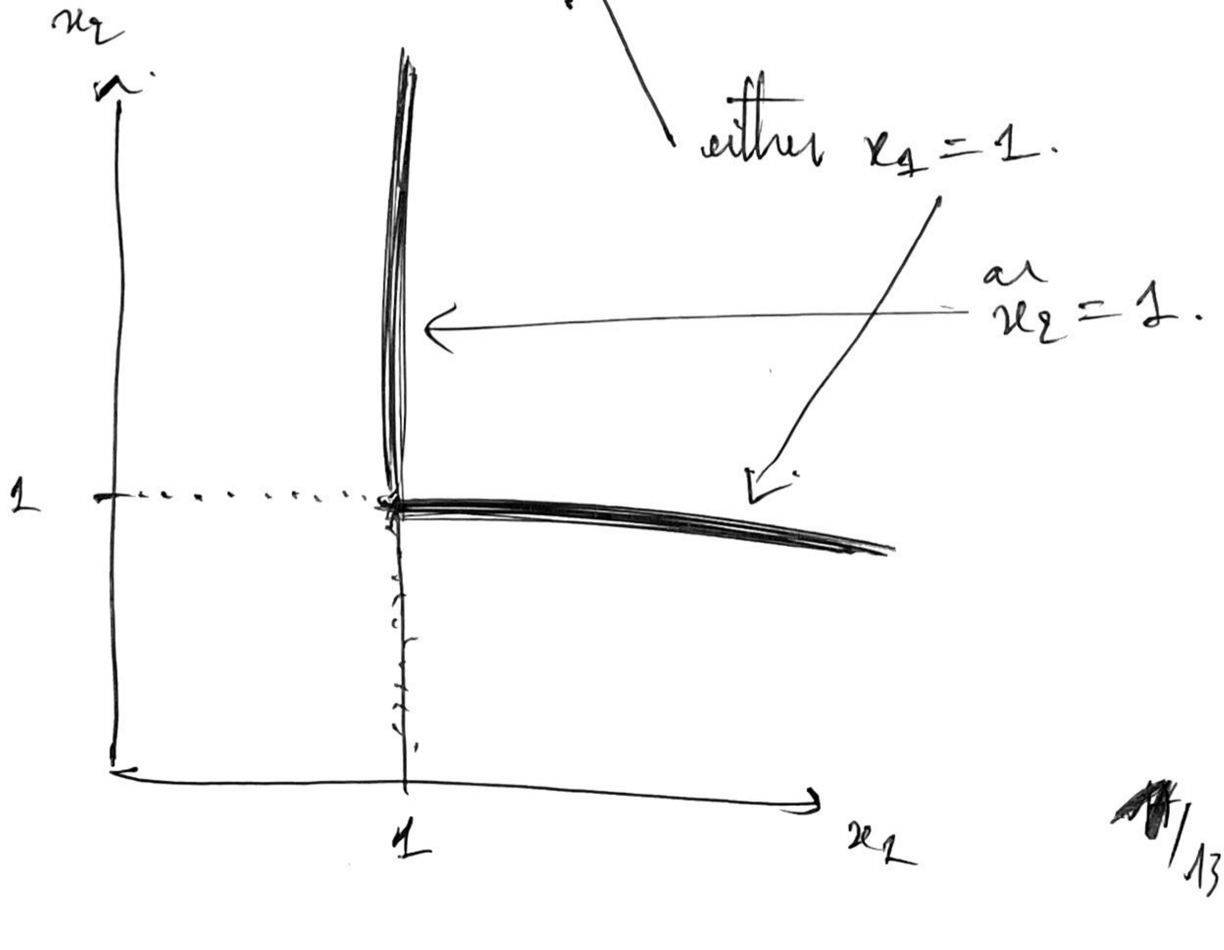
(ii). Cobb-Douglas — $u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.

(iii) Linear. $u(x) = \alpha_1 x_1 + \alpha_2 x_2$.

2). indifference curve utility level
$u^* = 1$.

$\Downarrow$

Leontief $1 = \min\{x_1, x_2\}$.



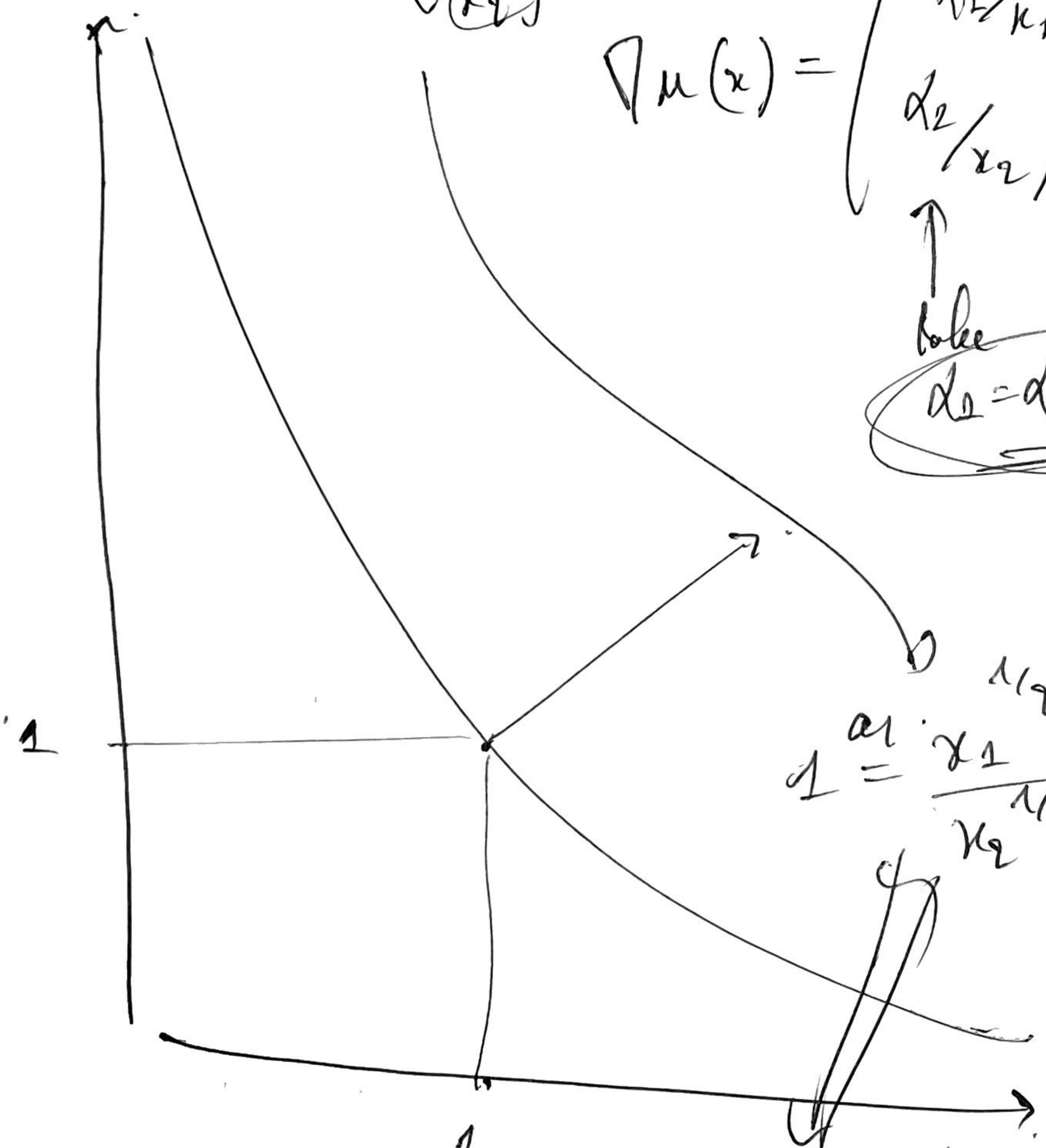
~~$\tilde{\ }$~~ are unchanged under
~~both~~ increasing tf

$\Rightarrow u(x) = \cancel{\alpha_1 \ln x_1 + \alpha_2 \ln x_2}$
$\not\equiv x_1^{\alpha_1} x_2^{\alpha_2}$

$\Rightarrow \nabla u(x) = \begin{pmatrix} \frac{\alpha_1}{x_1} \\ \frac{\alpha_2}{x_2} \end{pmatrix} \times u(x)$

$\nabla u(x) = \begin{pmatrix} \alpha_1/x_1 \\ \alpha_2/x_2 \end{pmatrix}$

take $\alpha_1 = \alpha_2 = \frac{1}{2}$



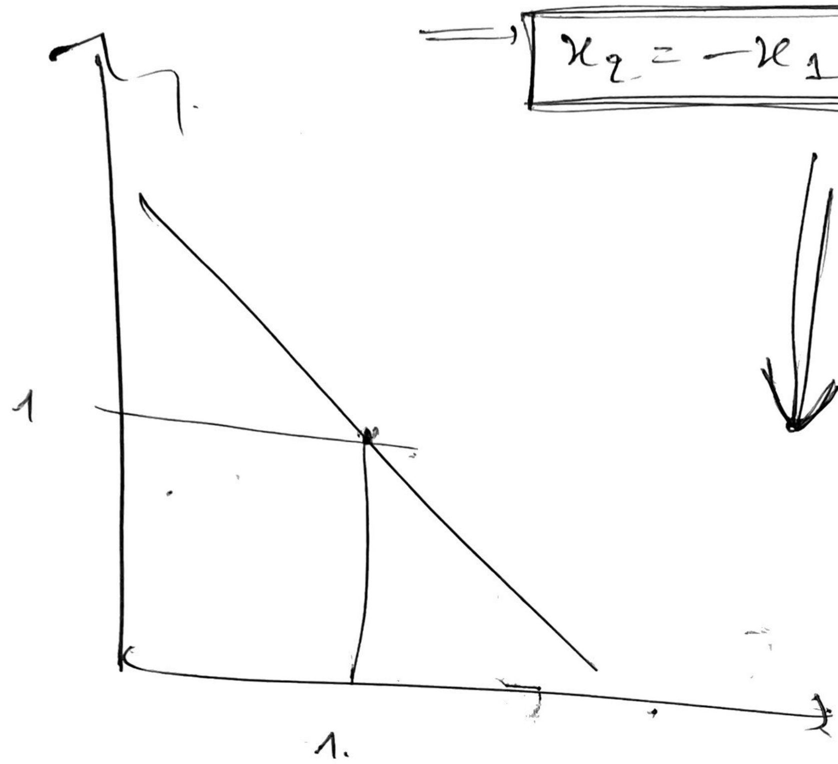
$1 = \frac{x_1^{1/2}}{x_2^{1/2}}$

$\boxed{x_2 = \frac{1}{x_1}}$

linear.

$$1 = \underbrace{\frac{1}{2}x_1}_{} + \underbrace{\frac{1}{2}x_2}_{}.$$

$$\Longrightarrow \boxed{x_2 = -x_1 + 2}$$



Exercice 2: $u(x) = x_1^{0.2} x_2^{0.5} x_3^{0.3}$

1). Let $p \gg 0$, $w > 0$.
The demand of the consumer $x(p,w)$ is the set of solutions to the UMP

$$\begin{cases} \max u(x). \\ p \cdot x \leq w. \end{cases}$$

The demand is unchanged under strictly increasing transformations of the utility fct. u representing $\succsim$ the preferences of the consumer.

(because $\succsim$ is preserved under strictly increasing tf of u. rep. $\succsim$!).

we can therefore consider instead.

$$v(x) = \ln u(x) = 0.2 \ln x_1 + 0.5 \ln x_2 + 0.3 \ln x_3 .$$

$\bar{x} \in x(p,w)$ iff.

$$\begin{cases} \nabla v(\bar{x}) = \lambda p . \\ \text{and} \quad \lambda \underbrace{(p \cdot \bar{x} - w)}_{=0} = 0 . \end{cases}$$

Kuhn-Tucker necessary conditions.

<u>Walras' law ??</u>

$$\nabla v(\bar{x}) = \underbrace{\begin{pmatrix} \frac{0.2}{x_1} \\ \frac{0.5}{x_2} \\ \frac{0.3}{x_3} \end{pmatrix}}_{\gg 0 \text{ if. } \bar{x} \text{ is an interior solution.}} = \lambda \underbrace{\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}}_{\gg 0.} .$$

$\Rightarrow$ So $\lambda > 0$ so

$$p \cdot \bar{x} = w \quad \text{ie Walras' law.}$$

~~[illegible]~~

$$\Rightarrow 0.2 + 0.5 + 0.3 = 1.$$
$$= \lambda (p \cdot \bar{x}) = \lambda w.$$

$$\Rightarrow \lambda = \frac{1}{w}.$$

$$\Rightarrow \boxed{\bar{x}_i = \frac{w \alpha_i}{p_i}}$$

$$\boxed{\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} = w \begin{pmatrix} \frac{0.2}{p_1} \\ 0.5/p_2 \\ 0.3/p_3 \end{pmatrix}.}$$

is the solution.

(We could have deduce that it would be unique from the fact that Cobb-Douglas is monotone and strictly quasiconcave.)

2). $$\frac{\partial x_1}{\partial p_2} = \frac{\partial}{\partial p_2}\left(\frac{0.2\, w}{p_1}\right) = 0.$$

So the demand of the consumer for good 1 x_1 is <u>not</u> affected by changes in the price p_2 of the good 2.

3). The <u>indirect utility</u> is defined as the utility level at $x(p,w)$.

$$v(x(p,w))$$

$$= \sum_i \alpha_i \ln x_i(p,w)$$

$$= \sum_i \alpha_i \ln \frac{\alpha_i w}{p_i}$$

$$= \ln \alpha_i w - \ln p_i$$

$$\boxed{\frac{\partial v}{\partial p_2}(x(p,w)) = -\frac{\alpha_2}{p_2}}$$ < 0.

So the indirect utility of the consumer is (negatively) affected by a rise in the price of good 2.

4). the demand. $x(p,w)$ is homogeneous of deg 0,

i.e we can verify that $\forall t$.

$$\underline{x(tp, tw) = x(p,w).}$$

So any proportional ~~changes~~ t in all prices and the agent's wealth does not affect their demand.

~~It~~ is also true for an increase of 1% in all prices and w.

<u>Exercice 3.</u> d satisfies Walras' law.

$$d_1(p,w) = \frac{w}{p_1+p_2}$$

1. Walras' law $\Leftrightarrow p.d = w$.

$$p_1 d_1 + p_2 d_2 = w.$$

$$\Longrightarrow d_2 = \frac{w - p_1 d_1}{p_2}.$$

$$d_2 = \frac{w}{p_2} - \frac{p_1 w}{(p_1+p_2)p_2} = \frac{\overbrace{(p_1+p_2)w - p_1 w}^{\cancel{p_2} w}}{\cancel{p_2}(p_1+p_2)}.$$

$$\Longrightarrow d_2 = \frac{w}{p_1+p_2} = d_1 .$$

we recognize the demand corresponding to Leontief pref ?? $u(x) = \min(x_1, x_2)$.

2). WARP in the framework of the demand.

$d(x,p) \equiv$ the demand of the consumer, satisfies the weak axiom of revealed preference.

iff.

$$\boxed{\begin{array}{c} p' \cdot d(p, w) \leq w' \text{ and } d(p, w) \neq d(p', w') \\ \Longrightarrow p \cdot d(p', w') > w . \end{array}}$$

You should remember its meaning:

$p' \cdot d(p,w) \leq w'$ means that both

$$d(p,w) \text{ and } d(p',w') \subset B_{p,w}.$$
$$\neq$$

and $d(p,w) = C(B_{p,w})$.

so $d(p,w) \succ^{*} d(p',w')$.

Ξ revealed strictly preferred to $d(p',w')$

$$d(p,w) \succsim^* d(p',w') \iff \begin{cases} d(p,w) \succsim^* d(p',w') \\ \text{and} \\ d(p',w') \not\succsim^* d(p,w) \end{cases}$$

$\Rightarrow$ So the WARP

means that when "evaluated" over any other sets $\supset d(p,w)$ and $d(p',w')$.

$C(\cdot)$ can not reveal $d(p',w') \succsim^* d(p,w)$.

and by def. $d(p',w') = C(B_{p',w'})$.

So it means $d(p',w') \succsim^*$ for any subsets in $B_{p',w'}$.

so $B_{p',w'}$ cannot include $d(p,w)$.

$\iff \underline{p' \cdot d(p,w) > w}$.

3.) Show that d satisfies WARP.

$$d_{i=1,2}(p,w) = \frac{w}{p_1+p_2}.$$

let. $p \cdot d(p', w') \leq w$.

i.e. $p_1 \frac{w'}{p_1'+p_2'} + p_2 \frac{w'}{p_1'+p_2'} \leq w$.

$\Longleftarrow$ and. $d(p,w) \neq d(p',w')$.

$$\Downarrow$$

$$\frac{w'}{p_1'+p_2'} \leq \frac{w}{p_1+p_2}.$$

that we cannot have the equality.

$$\Downarrow$$

$$\frac{w'}{p_1'+p_2'} < \frac{w}{p_1+p_2}.$$

So it does satisfy WARP.

$\longmapsto$ $\boxed{p' \cdot d(p,w) > w'}$

$\square$.

Exercice 4: $\{A, B, C\}. =: X$

* Alice : $A \succ_A B \succ_A C$.
* Bob : $B \succ_B C \succ_B A$.
* Lucy : $C \succ_L A \succ_L B$.

1). Show each pref $\succsim_{i = A, B, L}$ can be represented by a utility fct.

⟶ it can because the set of alternatives is <u>finite</u> ...
We can build three fcts :

$$a : \begin{cases} A \longmapsto 3 \\ B \longmapsto 2. \\ C \longmapsto 1. \end{cases}$$

$$b : \begin{cases} A \longmapsto 1. \\ B \longmapsto 3 \\ C \longmapsto 2. \end{cases} \qquad l : \begin{cases} A \longmapsto 2. \\ B \longmapsto 1. \\ C \longmapsto 3 \end{cases}.$$

s.th. fct i would rep. pref. $\succsim_i$.

2). $\boxed{\text{rational} \equiv \text{transitive and complete}}$ i.e all elements can be compared pairwise.

So the pref. of each agent would be complete and transitive requiring that.

$A \succ_A C$; $B \succ_B A$ and $C \succ_L B$.

(perhaps quickly remind the def.

* $\succsim$ complete iff. $\forall x, y \in X$
  either $x \succsim y$, $x \precsim y$ or both.

* $\succsim$ transitive iff. $\forall x, y, z$.
  $x \succsim y$ and $y \succsim z \Longrightarrow x \succsim z$.)

The instructions of the exercise are quite ambiguous ...

Consider $\{\{A, B\}, \{B, C\}, \{C, A\}\} = \mathcal{B}$.
collective choice $c(\cdot) \equiv$ the alternative that is preferred by the majority is chosen.

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3) Determine the choice mode in each situation.

$c(\{A, B\}) = \{A\}$ because. $A \succ_A B$. $B \succ_B A$ and. $A \succ_L B$.

i.e 2 against 1.

Likewise.

$c(\{B, C\}) = \{B\}$.

and. $c(\{C, A\}) = \{C\}$.

4) <u>Does the choice structure $(\mathcal{B}, c(\cdot))$ satisfies WARP ?</u>

$c(\cdot)$ reveals. $A \succ^* B$.

$B \succ^* C$ and. $C \succ^* A$.

it does not "reveal" any inconsistencies so it. does satisfy the Weak Axiom of Revealed Pref.

5). <u>Does there exist a pref. rel. that rationalizes c ?</u>

BUT it <u>cannot be rationalized</u>, because. $\succsim^*$ is not <u>transitive</u>.

$\square$