Université Paris 1 Panthéon-Sorbonne – Antoine Mandel October 25, 2023

Microeconomics 1 – MMMMEF/ QEM

Midterm Exam (90 mins)

No mobile phone or calculator. One sheet containing personal notes authorised.

We consider throughout a consumer with consumption set $X = \{x = (x_1, x_2) \in \mathbb{R}^2_+ \mid x \geq c\}$ where $c = (c_1, c_2) \in \mathbb{R}^2_+$ is a subsistence consumption bundle. Furthermore, the consumer has preferences $\succsim$ on X represented by an utility function of the form $u(x) = (x_1 - c_1)^{\alpha_1}(x_2 - c_2)^{\alpha_2}$, with $\alpha_1, \alpha_2 > 0$. The resulting demand is known as the *linear expenditure system* (LES) and is due to Stone (1954).

Part 1, Graphical study. In this part we consider $\alpha_1 = \alpha_2 = \frac{1}{2}$ and $c_1 = c_2 = 1$.

1. (i) What is the utility level at $(3, \frac{3}{2})$? (ii) Do we have that $(\frac{3}{2}, 3) \sim (3, \frac{3}{2})$?
2. Trace the indifference curves corresponding to utility levels (i) $u^* = 1$, (ii) $u^* = 0$.
3. Compute $\nabla u(3, \frac{3}{2})$.
4. Let $p = (\frac{1}{4}, 1)$ and $w = \frac{9}{4}$, determine the demand of the consumer graphically.

Part 2. Analytical study. In this part we revert to the general case where $\alpha_1, \alpha_2 > 0$ and $c_1, c_2 \geq 0$.

1. (i) When is a function u said to represent a preference relation $\succsim$? (ii) Show that if u represents $\succsim$ and f is a strictly increasing function, then $f \circ u$ also represents $\succsim$. (iii) Conclude that we can set $\alpha_1 + \alpha_2 = 1$ in the LES.
2. Show that $\succsim$ is (i) continuous, (ii) monotone, and (iii) convex on X. Let $p \gg 0$ and $w > 0$. (iv) Why should we expect the demand of the consumer to satisfy Walras' law? (v) Why should we expect the demand of the consumer to be unique (i.e, a singleton)?
3. (i) Using 1.(ii), explain why we can solve the utility maximization problem for $v(x) = \ln u(x)$ instead of $u(x)$. (ii) Determine the demand $x(p, w)$ of the consumer using the Kuhn-Tucker conditions. (*Computational hint:* Use that $p \cdot \bar{x} = w \implies p \cdot (\bar{x} - c) = w - p \cdot c$.) (iii) Compute the marginal rate of substitution $MRS_{12}(x) = -\frac{\partial v / \partial x_1}{\partial v / \partial x_2}(x)$. (iv) Evaluate MRS_{12} at $\bar{x} \in x(p, w)$. Why was this result expected?

4. (i) Why can we normalize $p_2 = 1$ without losing generality? (ii) State the WARP in the framework of the demand. (iii) Verify that the LES satisfies the WARP in the simpler case when $\alpha_1 = \alpha_2 = \frac{1}{2}$ and $c_1 = c_2 = 0$. (*Hints:* Use the fact that $x + \frac{1}{x} \geq 2$ for all $x > 0$, with equality iff $x = 1$.)

5. (bonus) How is the demand affected by (i) an equal percentage change in all prices and the wealth, (ii) separate changes in the wealth and each price? How is the indirect utility function (i.e., the utility level at the demand) affected by (iii) an equal percentage change in all prices and the wealth, (iv) separate changes in the wealth and each price?