$\boxed{\text{Micro 1. Midterm 2024.}}$

Cobb-Douglas. $\succsim$ rep. by.

$$u(x) = x_1^{\alpha} x_2^{1-\alpha} \qquad \alpha \in]0;1[.$$

$$x \in X = \mathbb{R}_+^2$$

1). Recall the def of (i) continuity,
ii) strong monotonicity,
iii) strict convexity of $\succsim$

1) i). $\succsim$ is <u>continuous</u> iff

$\forall \{x_n\}, \{y_n\}$. $x_n \succsim y_n \ \forall n$.

$x_n \longrightarrow x$
and $y_n \longrightarrow y$ $\Longrightarrow x \succsim y$.

(i.e. the preference relation is "preserved under limits") ii). $\succsim$ is strongly monotone iff.

$$\forall x, y. \quad \underbrace{x > y}_{} \Longrightarrow x \succ y.$$

$$\text{ie} \begin{pmatrix} \forall i \;\; x_i \geq y_i \\ \text{and } \exists j \;\; x_j > y_j \end{pmatrix}.$$

iii) $\succsim$ is strictly convex iff. $\forall x, y, z$, $x \neq y$.
and. $\forall \lambda \in]0, 1[$, $x \succsim z$ and $y \succsim z$.

$$\Longrightarrow$$

$$\lambda x + (1-\lambda) y \succ z$$

or, equivalently, $\succsim$ is strictly convex iff
u representing $\succsim$ is strictly quasi-concave.

ie. $\forall x, y.$ $x \neq y$. $\lambda \in]0, 1[$.

$$u(\lambda x + (1-\lambda) y) > \min\{u(x), u(y)\}$$

$\boxed{\text{2) Show that } \succsim \text{ is i) continuous and ii) strongly monotone.}}$

2) i). Let $\{x_n\}, \{y_n\}$ s.th.

$$x_n \longrightarrow x \qquad \text{and. } x_n \succsim y_n \quad \forall n.$$
$$y_n \longrightarrow y$$

We have.

$$x_n \succsim y_n \quad \forall n.$$

$$\Longleftrightarrow \quad u(x_n) \geq u(y_n) \quad \forall n. \qquad \text{(because } u \text{ represents } \succsim\text{.)}$$

$$\Longrightarrow \quad u(x) \geq u(y) \qquad \left(\text{using that } u : x \longmapsto x_1^{\alpha} x_2^{1-\alpha} \text{ is continuous}\right).$$

$\left(\text{ie. } \lim_{n \to \infty} u(x_n) = u(x)\right)$ $\forall \{x_n\}$ s.th $x_n \to x$.

$$\Longleftrightarrow \quad x \succsim y. \qquad \square$$

ii) Let $x > y$.

ie $x = y + \varepsilon$ with $\varepsilon > 0$.

ie $\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$ s.th $\forall i = 1,2 \ \ \varepsilon_i \geq 0$ and $\exists j \ \ \varepsilon_j > 0$ $\}$, We have.

$$u(x) = x_1^{\alpha} x_2^{1-\alpha}.$$

$$= (y_1+\varepsilon_1)^{\alpha} (y_2+\varepsilon_2)^{1-\alpha}.$$

$$= \underbrace{y_1^{\alpha} y_2^{1-\alpha}}_{\substack{= \\ u(y).}} \left(1+\frac{\varepsilon_1}{y_1}\right)^{\alpha} \left(1+\frac{\varepsilon_2}{y_2}\right)^{1-\alpha}.$$

either $\left(1+\frac{\varepsilon_1}{y_2}\right)$ is > 1.

or $\left(1+\frac{\varepsilon_2}{y_2}\right)$ is > 1,

and both are ≥ 1.

So in all cases we find that.

$$u(x) > u(y)$$

$$\Longleftrightarrow x \succ y \qquad A.$$

Equivalently, we could have shown that.

$$\nabla u(x) = \begin{pmatrix} \frac{\alpha}{x_1} u(x) \\ \frac{1-\alpha}{x_2} u(x) \end{pmatrix} \gg 0 . \quad \forall x \gg 0.$$

so. u is strictly increasing

so $\succsim$ is strongly monotone.

3) Show that $\succsim$ is homothetic.

Def 1: $\succsim$ is <u>homothetic</u> iff. $\forall x, y$.

$$x \sim y \implies \beta x \sim \beta y \quad \forall \beta \geq 0.$$

So, suppose. $x \sim y$.

$$\Updownarrow$$

$$u(x) = u(y)$$

$$\iff \beta u(x) = \beta u(y) \quad \forall \beta \geq 0.$$

$$\iff \beta x_1^{\alpha} x_2^{1-\alpha} = \beta y_1^{\alpha} y_2^{1-\alpha} \quad \forall \beta \geq 0.$$

$$\iff (\beta x_1)^{\alpha} (\beta x_2)^{1-\alpha} = (\beta y_1)^{\alpha} (\beta y_2)^{1-\alpha} \quad \forall \beta \geq 0$$

$$\iff u(\beta x) = u(\beta y) \quad \forall \beta \geq 0.$$

$$\iff \beta x \sim \beta y \quad \forall \beta \geq 0.$$

so $\succsim$ is homothetic $\square$.

we could have said that u is homogeneous of deg 1.

4) let $\alpha = \frac{1}{2}$.

(i) Trace $I(u^* = 1)$ and $I(u^* = 2)$.

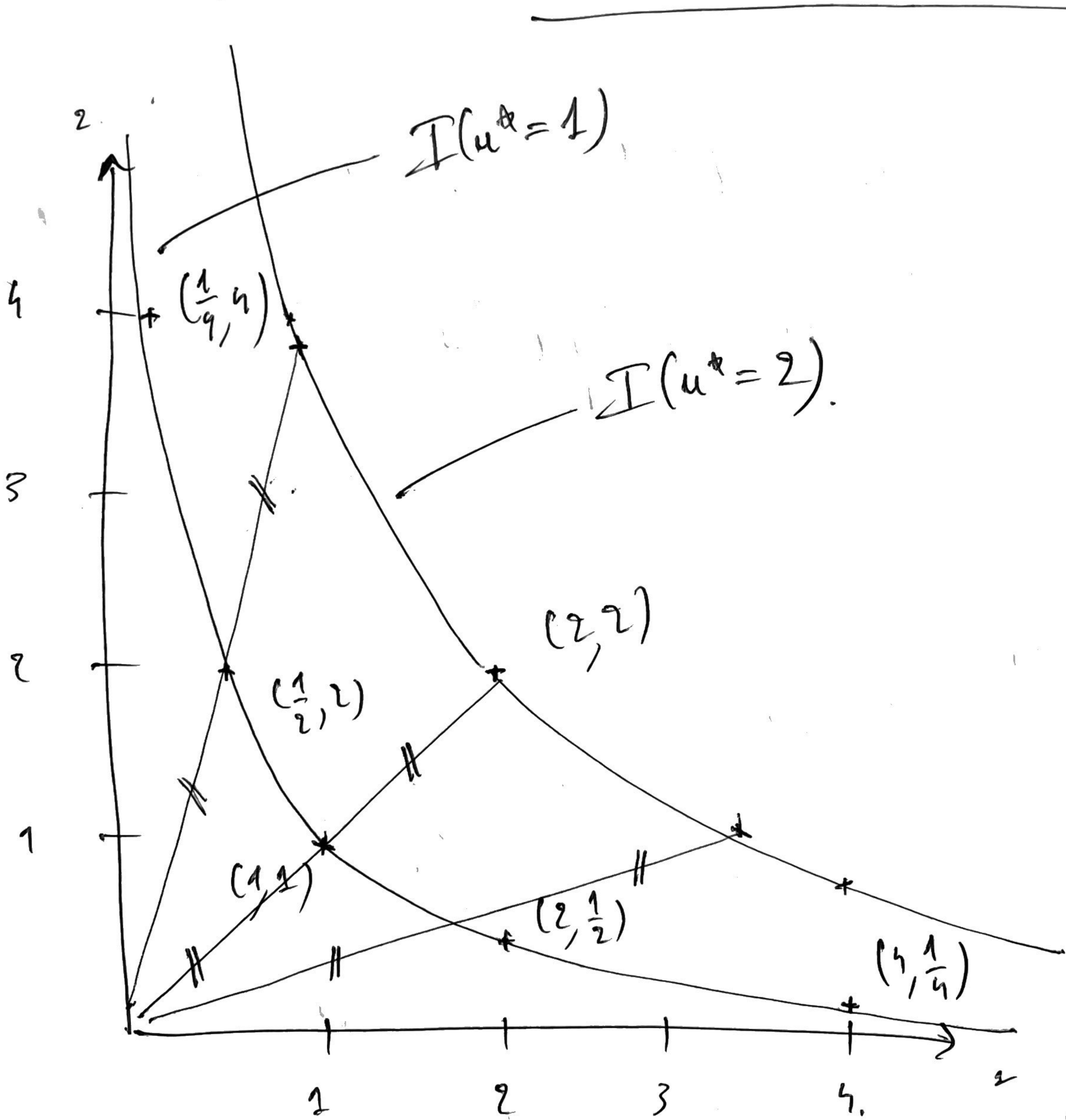
(ii) Use them to illustrate the homothetic prop. of $\succsim$.

4) i) ii)

$$I(u^* = 1) = \{x = (x_1, x_2) \; ; \; u(x) = 1\}.$$

$$= \{x \; ; \; x_1^{\alpha} x_2^{1-\alpha} = 1\}.$$

so for $\alpha = \frac{1}{2} \implies I(u^* = 1) = \{x \; ; \; x_2 = \frac{1}{x_1}\}$



i) To trace $I(u^* = 1) = \{x \,;\, x_2 = \frac{1}{x_1}\}$.

I used that.

$(1,1), (2, \frac{1}{2}), (\frac{1}{2}, 2), (4, \frac{1}{4})$

and $(\frac{1}{4}, 4)$ all $\in I(u^* = 1)$

ii) To trace $I(u^* = 2)$ you could also simply find its equation, as we did for $u^* = 1$,

but here the idea was to use the fact that $\succsim$ is <u>homothetic</u>.

Indeed it follows from Def 1. that it is possible to deduce the consumer's entire preference relation from a single indifference set.

Noting that $u(2,2) = 2^{1/2}\, 2^{1/2} = 2$.

$\Longrightarrow (2,2) \in I(u = 2^*)$.

and $(2,2) = 2(1,1)$ with $(1,1) \in I(u^* = 1)$

It follows that $\forall (x_1, x_2) \in I(u^* = 1)$.

i.e. s.th. $(x_1, x_2) \sim (1,1)$.

$\Longrightarrow$ we have that $2(x_1, x_2) \sim (2,2)$

i.e. $2(x_1, x_2) \in I(u^* = 2)$

5) i) Show that $\succsim$ can also be represented by $w(x) = \alpha \ln x_1 + (1-\alpha) \ln x_2$.

ii) Show that w is strictly concave and that it implies that $\succsim$ is strictly convex.

5) i) We have $w(x) = \ln u(x) \quad \forall x$.

u represents $\succsim$ i.e.

$$\forall x,y. \quad x \succsim y \iff u(x) \geq u(y).$$

$$\Updownarrow$$

$$w(x) \geq w(y)$$

using that $\ln$ is a strictly increasing function.

Thus $x \succsim y \iff w(x) \geq w(y) \quad \forall x,y$.

i.e. w also represents $\succsim$ $\square$.

ii) $\nabla w(x) = \begin{pmatrix} \alpha/x_1 \\ 1-\alpha/x_2 \end{pmatrix}$.

$$Hw(x) = \begin{pmatrix} -\alpha/x_1^2 & 0 \\ 0 & -\frac{(1-\alpha)}{x_2^2} \end{pmatrix}.$$

so $Hw(x)$ is negative definite. $\forall\, x \gg 0$.
(because both its eigenvalues are strictly negative)

Thus w is strictly concave.

$\Longrightarrow$ w is strictly quasi-concave.

$\Longleftrightarrow$ $\succsim$ is strictly convex.

(using that w rep. $\succsim$). $\square$.

6) i) Recall the definition of the WARP in the framework of the choice-based approach.

ii) Show that $\succsim$ rational $\Longrightarrow$ $C_{\succsim}$ satisfies WARP

iii) Conclude that the demand of a Cobb-Douglas maximizer satisfies WARP.

6) i) The choice structure $(\mathcal{B}, C(\cdot))$ satisfies the WARP iff

$$\left(\exists B \in \mathcal{B} \;\; x, y \in B \text{ and } x \in C(B)\right)$$

$$\Longrightarrow \left(\nexists B' \in \mathcal{B} \;\; x, y \in B', \; y \in C(B') \text{ and } x \notin C(B')\right).$$

Said in the language of revealed preference:

$x \succsim^{*} y$ (i.e. x revealed at least preferred to y).

$\Downarrow$

$y \not\succ^{*} x$ (i.e. y cannot be revealed strictly preferred to x).

ii)

Def 2: Let $\mathcal{B}$ a family of nonempty subsets of X.
Recall that one can associate to a preference relation $\succsim$ on X, the choice rule defined for all $B \in \mathcal{B}$ by.

$$C_{\succsim}(B) = \{x \in B ;\ x \succsim y \quad \forall y \in B\}.$$

Suppose $\succsim$ is rational, i.e $\succsim$ is complete and transitive.

Suppose $\exists B \in \mathcal{B}$ s.th. $x, y \in B$ and

$$x \in C_{\succsim}(B) = \{t \in B ;\ t \succsim z \quad \forall z \in B\}$$

Suppose $\exists B' \in \mathcal{B}$ s.th. $x, y \in B'$ and

$$y \in C_{\succsim}(B') = \{t \in B' ;\ t \succsim z \quad \forall z \in B'\}.$$

From $x, y \in B$ and $x \in C_{\succsim}(B)$.

we know that $x \succsim y$

Therefore, by transitivity,

$$\left.\begin{array}{l} y \succsim z \quad \forall z \in B' \\ \text{and } x \succsim y. \end{array}\right\} \Longrightarrow x \succsim z \quad \forall z \in B'$$

$$\Longleftrightarrow \quad x \in C(B').$$

$\square$.

(s.th.
$(x, y \in B'$ and $y \in C(B') \Longrightarrow x \in C(B'))$.
i.e. $\nexists B'$, $x, y \in B'$, $y \in C(B')$ and $x \notin C(B')$)),

(and the completeness simply ensured that all elements of any B subsets of X could be compared).

iii) We know that if $\succsim$ is represented by some utility function then $\succsim$ is rational.

<u>Proof</u>: Suppose $u : X \longrightarrow \mathbb{R}$. represents $\succsim$.

* $\forall x, y$. either $u(x) \geq u(y)$. or $u(x) \leq u(y)$ or both.

$\Longleftrightarrow$ $\forall x, y$. either $x \succsim y$ or. $x \precsim y$ or both.

so $\succsim$ is <u>complete</u>.

* Suppose. $x \succsim y$ and $y \succsim z$.

$\Longleftrightarrow$ $u(x) \geq u(y)$ and $u(y) \geq u(z)$

$\Longrightarrow$ $u(x) \geq u(z)$

$\Longleftrightarrow$ $x \succsim z$. so $\succsim$ is <u>transitive</u>.

The Cobb-Douglas preference relation $\succsim$ is represented by $u(x) = x_1^{\alpha} x_2^{1-\alpha}$.

therefore it is rational,

Therefore, from q. ii), the demand of a Cobb-Douglas preference-maximizer $C_{\succsim}$ satisfies WARP.

7) i) State the UMP for the consumer under consideration.

ii) Explain why it has a single solution.

7) i) The UMP is

$$\begin{cases} \max u(x) \\ \text{s.th. } \underbrace{p \cdot x \leq w}_{} \end{cases}$$

with $p \gg 0$ and $w > 0$.

i.e.

$x \in B_{p,w}$, the Walrasian budget set.

ii) We know that $\succsim$ strictly convex.

$\iff$ u strictly quasiconcave.

$\implies$ the solution to the UMP is unique.

(It also requires that the constraint set is convex, here $B_{p,w}$ is always convex.)

Proof: Suppose $x, y \in x(p,w)$ with $x \neq y$ are both solutions to the UMP. $\implies$ $u(x) = u(y)$.

By strict quasiconcavity, we have.

$$u(x) \geq u(y) \text{ and } x \neq y.$$

$$\implies \underbrace{\nabla u(y)}_{= \lambda p} \cdot (x - y) > 0.$$

λp by the Kuhn-Tucker conditions.

But by local non-satiation both $p \cdot x = p \cdot y = w$. (Walras' law).

so $0 > 0$ i.e contradiction

so the solution to the UMP is unique. $\square$.

16/

8) Let $\bar{x} \gg 0$ solution to the UMP.
i) State the Kuhn-Tucker conditions.
ii) Use it to find the demand of the consumer $x(p,w)$

8) i) Let $\bar{x} \gg 0$ solution to the UMP.
According to the KT conditions, $\bar{x}$ must satisfy:

$$(1) \quad \nabla u(\bar{x}) = \lambda p \quad \text{for some } \lambda \geq 0.$$

and

$$(2) \quad \underline{\lambda(p.\bar{x} - w) = 0.}$$

i.e. either $\lambda = 0$ or Walras' law.

ii). Actually, the UMP is invariant w.r.t. the utility function representing the preference of the consumer, therefore we could solve the UMP.

$$\begin{cases} \max w(x) \\ x \in B_{p,w}. \end{cases}$$

In this case the conditions are: $\bar{x} \gg 0$ solution of:

(1) $\nabla w(\bar{x}) = \lambda p$ for some $\lambda \geq 0$,

and $\lambda(p \cdot \bar{x} - w) = 0$. (2).

We have:

$$\nabla w(x) = \begin{pmatrix} \alpha / x_1 \\ \\ \dfrac{1-\alpha}{x_2} \end{pmatrix} \gg 0.$$

and $p \gg 0$ thus $\lambda \neq 0$.

thus, $p \cdot \bar{x} = w$ $(*)$

$$(1) \Longleftrightarrow \frac{\partial w}{\partial x_1}(\bar{x}) = \lambda p_1 = \alpha / \bar{x}_1$$

and $\dfrac{\partial w}{\partial x_2}(\bar{x}) = \lambda p_2 = \dfrac{1-\alpha}{\bar{x}_2}$.

$$\Longrightarrow \lambda \underbrace{(p_1 \bar{x}_1 + p_2 \bar{x}_2)}_{= p \cdot \bar{x} \underset{(*)}{=} w} = \alpha + (1-\alpha) = 1.$$

so. $\lambda = 1/w$.

and injecting back into (1) we get.

$$x(p,w) = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \end{pmatrix} = \begin{pmatrix} \alpha w / p_1 \\ (1-\alpha) \frac{w}{p_2} \end{pmatrix}.$$

9) Verify that the demand of the consumer is.
(i) homogeneous of degree 0.
ii) satisfies Walras' law.

9) i). $$x(\lambda p, \lambda w) = \begin{pmatrix} \alpha \frac{\cancel{\lambda} w}{\cancel{\lambda} p_1} \\ (1-\alpha) \frac{\cancel{\lambda} w}{\cancel{\lambda} p_2} \end{pmatrix}.$$

$$= \begin{pmatrix} \alpha w / p_1 \\ (1-\alpha) \frac{w}{p_2} \end{pmatrix} = x(p,w) \quad \forall \lambda > 0.$$

so. $x(p,w)$ is homogeneous of degree 0.

ii) We already know from solving the KT conditions that $p\cdot\bar{x} = w$, but let's verify it...

$$p\cdot x(p,w) = \cancel{p_1}\frac{\alpha w}{\cancel{p_1}} + (1-\alpha)\frac{w}{\cancel{p_2}}\cancel{p_2}.$$

$$= w.$$

Δ.

10) The indirect utility function is defined as $v(p,w) := u(x(p,w))$.

i) Show that $\frac{\partial v}{\partial w}(p,w) = \frac{v(p,w)}{w}$

ii) Why was it expected ?

10) i) $v(p,w) = u(x(p,w))$

$$= \left(\frac{\alpha w}{p_1}\right)^{\alpha}\left(\frac{(1-\alpha)w}{p_2}\right)^{1-\alpha}$$

$$v(p,w) = w \left(\frac{\alpha}{p_1}\right)^{\alpha} \left(\frac{1-\alpha}{p_2}\right)^{1-\alpha}.$$

so. $$\frac{\partial v(p,w)}{\partial w} = \left(\frac{\alpha}{p_1}\right)^{\alpha} \left(\frac{1-\alpha}{p_2}\right)^{1-\alpha}$$

$$= \frac{v(p,w)}{w}.$$

ii) It was expected because it is a general result that the Lagrange multiplier λ solution to the UMP corresponds to the marginal utility of wealth. (*)

i.e. $$\frac{\partial w(x(p,w))}{\partial w} = \lambda = \frac{1}{w}$$

$$\frac{\partial \ln v(p,w)}{\partial w} = \frac{1}{v(p,w)} \frac{\partial v(p,w)}{\partial w}.$$

Proof of $(*)$:

$$\frac{\partial v}{\partial w}(p,w) = \frac{\partial u(x(p,w))}{\partial w}$$

$$= \frac{\partial x(p,w)}{\partial w} \cdot \nabla u(x(p,w)).$$

$$= \frac{\partial x(p,w)}{\partial w} \cdot \lambda p.$$ because of the KT conditions since $x(p,w)$ is solution.

$$= \lambda \frac{\partial(p \cdot x(p,w))}{\partial w}.$$ because $\lambda p \cdot x(p,w) = \lambda w$. still by the KT conditions.

$$= \lambda \frac{\partial w}{\partial w} = \lambda.$$

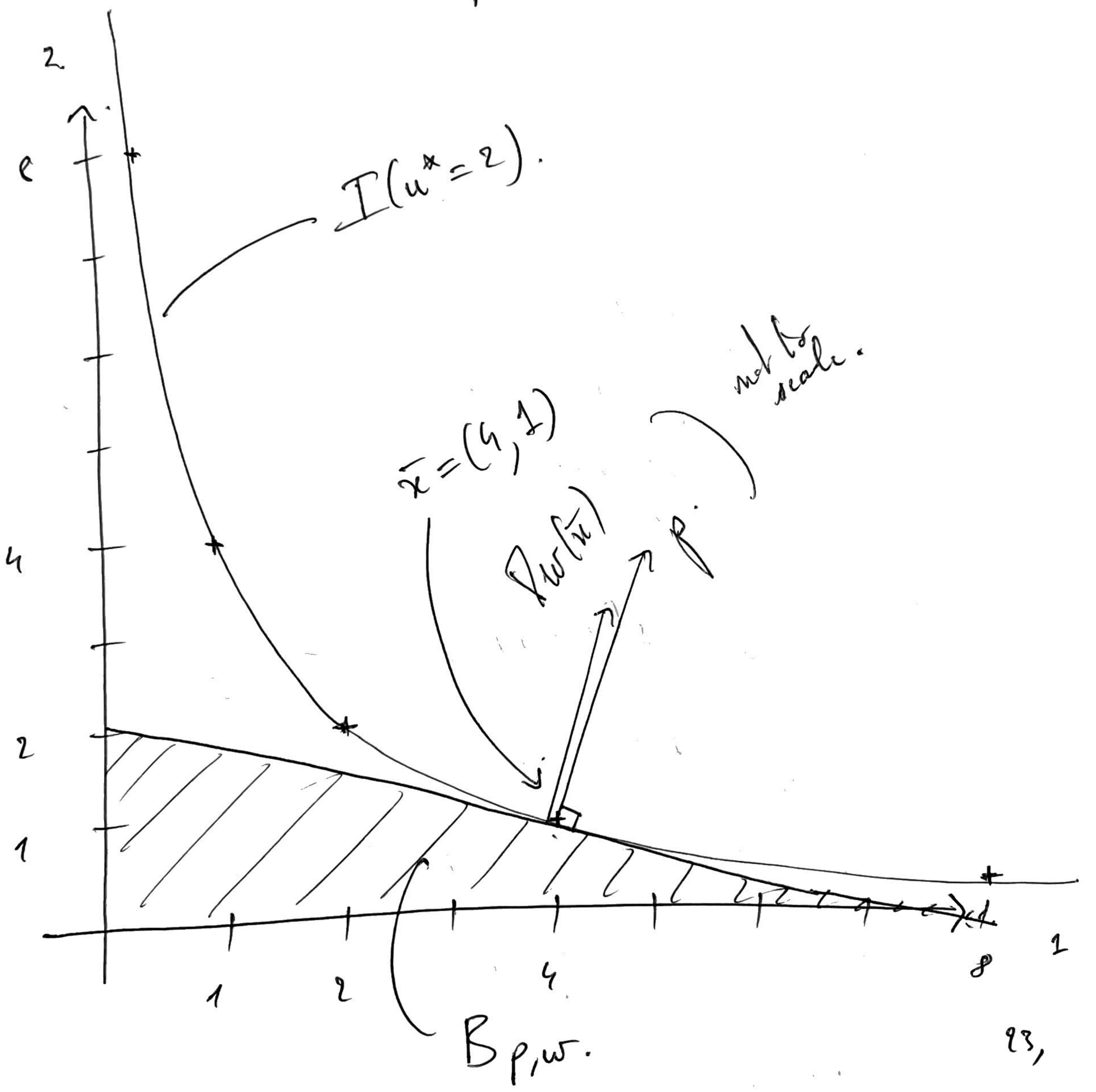
11) Let $\alpha = \frac{1}{2}$, $p = \bar{p} = (1,4)$
$w = \bar{w} = 8$.

i) $u(x(\bar{p},\bar{w})) = ?$ ii) Represent the solution graphically.

11) i) $x(\bar{p},\bar{w}) = \begin{pmatrix} \alpha\bar{w}/\bar{p}_1 \\ (1-\alpha)\frac{\bar{w}}{\bar{p}_2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\frac{8}{1} \\ \frac{1}{2}\frac{8}{4} \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

$\Rightarrow$ $u(x(\bar{p},\bar{w})) = 4^{1/2} 1^{1/2} = 2$.

ii) so. $\bar{x} = x(\bar{p},\bar{w}) \in I(u^* = 2)$.



The equation of the Budget line is

$$p \cdot x = w.$$

i.e. $$x_2 = -\frac{p_1}{p_2} x_1 + \frac{w}{p_2}.$$

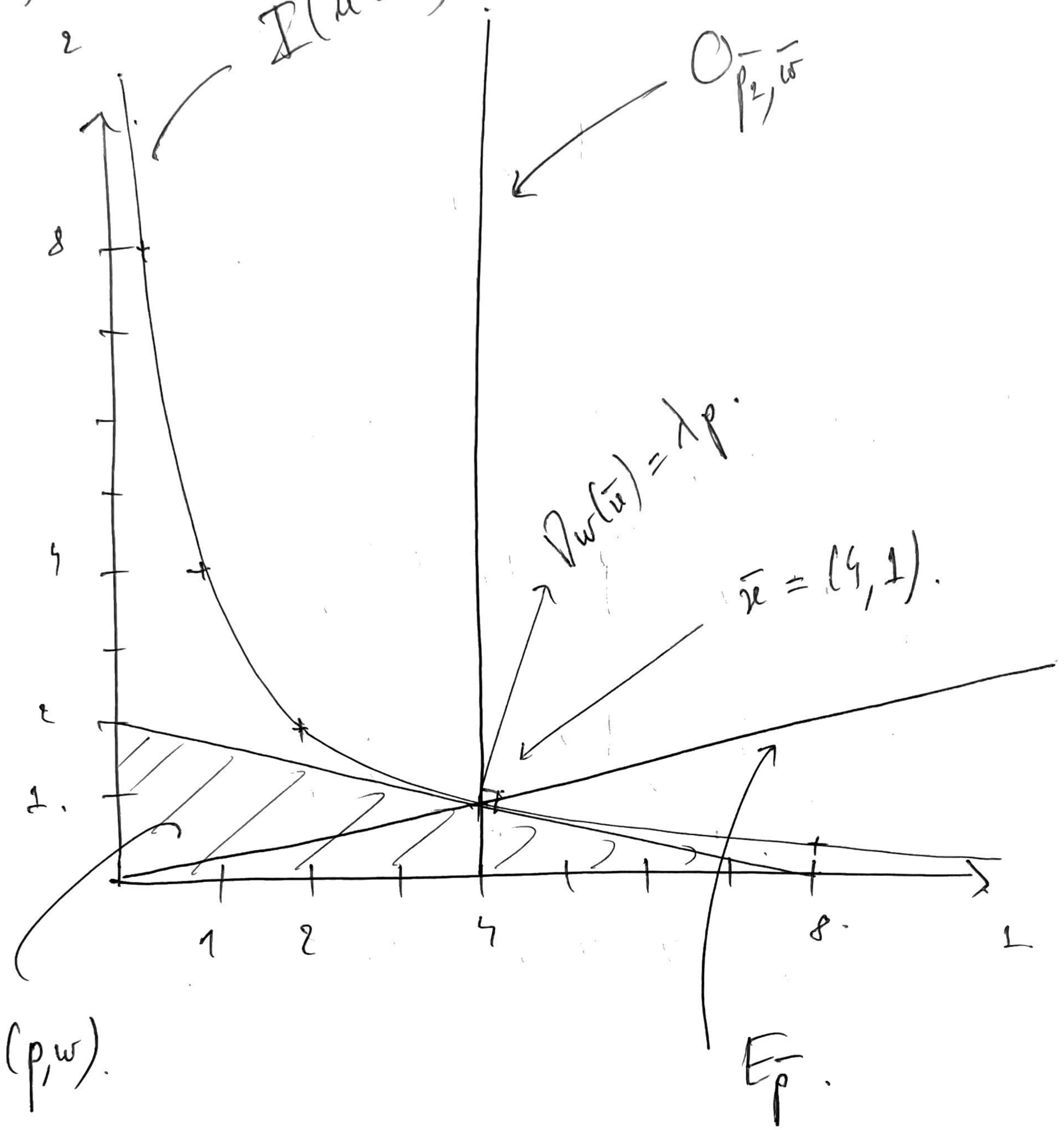
$$= -\frac{1}{4} x_1 + 2$$

or we knew that $\bar{x}$ was on the budget line, from 9) ii), and we know that its slope is $-p_1/p_2$.

$$D_w(\bar{x}) = \lambda p = \frac{1}{8}\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}1/8\\1/2\end{pmatrix}.$$

$$\boxed{\begin{array}{l}\text{12). i) Assume } w \text{ varies. Draw the wealth-consumption curve.}\\ \quad E_{\bar{p}} = \{x(\bar{p}, w),\ w > 0\}\\ \text{ii) Assume } p_2 \text{ varies. Draw the offer curve } O_{\bar{p}_1, \bar{w}} = \{x((\bar{p}_1, p_2), \bar{w}),\ p_2 > 0\}.\end{array}}$$

2) i) ii)



ii) The wealth-consumption curve is given by.

$$E_{\bar{p}} = \{ x(\bar{p}, w) \,;\, w > 0 \}.$$

$$= \left\{ \left(\frac{\alpha w}{\bar{p}_1}, (1-\alpha)\frac{w}{\bar{p}_2} \right) ; \; w > 0 \right\}$$

$$E_{\bar{p}} = \{ \lambda \bar{x} \ ; \ \lambda > 0 \}.$$

i.e. the whole half-line generated by the direction given by $\bar{x}$.

ii) The offer curve is given by

$$O_{\bar{p}_1, \bar{w}} = \{ x((\bar{p}_1, p_2), \bar{w}) \ ; \ p_2 > 0 \}$$

$$= \left\{ \left(\alpha \frac{\bar{w}}{\bar{p}_1} , (1-\alpha) \frac{\bar{w}}{p_2} \right) \ ; \ p_2 > 0 \right\}$$

$$= \left\{ x = (\bar{x}_1, x_2) \ ; \ x_2 > 0 \right\}.$$

i.e. $x_1(p, w)$ is independant of p_2.

Université Paris 1 Panthéon-Sorbonne – Antoine Mandel October 23, 2024

Microeconomics 1 – MMMEF / QEM

Midterm Exam (90 mins)

No mobile phone or calculator. One sheet containing personal notes authorised.

We consider a consumer with Cobb-Douglas (1928) preference relation $\succsim$ on the consumption set $X = \mathbb{R}^2_+$. That is, the consumer's preference relation $\succsim$ is represented by the utility function $u(x) = x_1^{\alpha} x_2^{1-\alpha}$, with $\alpha \in]0,1[$.

1. Recall the definitions of the *(i)* continuity, *(ii)* strong monotonicity, and *(iii)* strict convexity of a preference relation $\succsim$.

2. Show that the Cobb-Douglas preference relation $\succsim$ is *(i)* continuous and *(ii)* strongly monotone.

Definition 1 *A monotone preference relation $\succsim$ on $X = R^L_+$ is homothetic if all indifference sets are related by proportional expansion along rays; that is, if $x \sim y$, then $\beta x \sim \beta y$ for any $\beta \geq 0$.*

3. (Bonus) Show that the Cobb-Douglas preference relation $\succsim$ is homothetic.

4. Let $\alpha = \frac{1}{2}$. *(i)* Trace the indifference curves corresponding to utility levels $u^* = 1$ and $u^* = 2$. *(ii)* (Bonus) Use the two indifference curves to graphically illustrate the homothetic property of $\succsim$.

5. *(i)* Show that the Cobb-Douglas preference relation can also represented by the utility function $w(x) = \alpha \ln(x_1) + (1-\alpha)\ln(x_2)$. *(ii)* Show that w is strictly concave and use it to conclude that the Cobb-Douglas preference relation is strictly convex.

Definition 2 *Let $\mathcal{B}$ a family of nonempty subsets of X. Recall that one can associate to a preference relation $\succsim$ on X, the choice rule defined for all $B \in \mathcal{B}$ by $C_{\succsim}(B) = \{x \in B \mid x \succsim y,\ \forall y \in B\}$.*

6. *(i)* Recall the definition of the *weak axiom of revealed preference* (WARP) in the framework of the choice-based approach. *(ii)* Show that the rationality of $\succsim$ implies that the choice rule $C_{\succsim}$ satisfy the WARP. *(iii)* Conclude that the demand of Cobb-Douglas preference-maximizer satisfies the WARP.

* Let us now consider the consumer is facing a price system $p = (p_1, p_2) \gg 0$ and has wealth $w > 0$.

7. *(i)* State the utility maximization problem (UMP) for the consumer under consideration. *(ii)* Explain why it has a single solution.

8. Let $\bar{x} \gg 0$ a solution to the UMP. *(i)* State the Kuhn-Tucker/first-order (necessary) conditions satisfied by $\bar{x}$. *(ii)* Use the KT conditions to find the demand $x(p,w)$ of a consumer with Cobb-Douglas preference.

9. Verify that the demand of the consumer $x(p,w)$ is *(i)* homogeneous of degree 0 and *(ii)* satisfies Walras' law.

10. The *indirect utility function* is defined as $v(p,w) := u(x(p,w))$. *(i)* Show that we have $\frac{\partial v(p,w)}{\partial w} = \frac{v(p,w)}{w}$. *(ii)* (Bonus) Why was it expected?

11. Let $\alpha = \frac{1}{2}$, $p = \bar{p} := (1,4)$, and $w = \bar{w} := 8$. *(i)* What is the utility level at the solution? *(ii)* Superimpose a graphical representation of the solution to the UMP to the figure drawn in question 4), i.e., draw the consumer's budget set, the solution, and the gradient of the utility function at the solution.

12. (Bonus) Still on the same figure: *(i)* Assume that w varies. Draw the associated wealth-consumption curve, $E_{\bar{p}} = \{x(\bar{p},w) \mid w > 0\}$. *(ii)* Assume that p_2 varies. Draw the associated offer curve, $O_{\bar{p_1},\bar{w}} = \{x((\bar{p_1},p_2)),\bar{w}) \mid p_2 > 0\}$.