Université Paris 1 Panthéon-Sorbonne – Antoine Mandel October 23, 2024

Microeconomics 1 – MMMEF / QEM

Midterm Exam (90 mins)

No mobile phone or calculator. One sheet containing personal notes authorised.

We consider a consumer with Cobb-Douglas (1928) preference relation $\succsim$ on the consumption set $X = \mathbb{R}^2_+$. That is, the consumer's preference relation $\succsim$ is represented by the utility function $u(x) = x_1^{\alpha} x_2^{1-\alpha}$, with $\alpha \in]0, 1[$.

1. Recall the definitions of the *(i)* continuity, *(ii)* strong monotonicity, and *(iii)* strict convexity of a preference relation $\succsim$.

2. Show that the Cobb-Douglas preference relation $\succsim$ is *(i)* continuous and *(ii)* strongly monotone.

Definition 1 *A monotone preference relation $\succsim$ on $X = R^L_+$ is homothetic if all indifference sets are related by proportional expansion along rays; that is, if $x \sim y$, then $\beta x \sim \beta y$ for any $\beta \geq 0$.*

3. (Bonus) Show that the Cobb-Douglas preference relation $\succsim$ is homothetic.

4. Let $\alpha = \frac{1}{2}$. *(i)* Trace the indifference curves corresponding to utility levels $u^* = 1$ and $u^* = 2$. *(ii)* (Bonus) Use the two indifference curves to graphically illustrate the homothetic property of $\succsim$.

5. *(i)* Show that the Cobb-Douglas preference relation can also represented by the utility function $w(x) = \alpha \ln(x_1) + (1 - \alpha) \ln(x_2)$. *(ii)* Show that w is strictly concave and use it to conclude that the Cobb-Douglas preference relation is strictly convex.

Definition 2 *Let $\mathcal{B}$ a family of nonempty subsets of X. Recall that one can associate to a preference relation $\succsim$ on X, the choice rule defined for all $B \in \mathcal{B}$ by $C_{\succsim}(B) = \{x \in B \mid x \succsim y, \ \forall y \in B\}$.*

6. *(i)* Recall the definition of the *weak axiom of revealed preference* (WARP) in the framework of the choice-based approach. *(ii)* Show that the rationality of $\succsim$ implies that the choice rule $C_{\succsim}$ satisfy the WARP. *(iii)* Conclude that the demand of Cobb-Douglas preference-maximizer satisfies the WARP.

* Let us now consider the consumer is facing a price system $p = (p_1, p_2) \gg 0$ and has wealth $w > 0$.

7. *(i)* State the utility maximization problem (UMP) for the consumer under consideration. *(ii)* Explain why it has a single solution.

8. Let $\bar{x} \gg 0$ a solution to the UMP. *(i)* State the Kuhn-Tucker/first-order (necessary) conditions satisfied by $\bar{x}$. *(ii)* Use the KT conditions to find the demand $x(p, w)$ of a consumer with Cobb-Douglas preference.

9. Verify that the demand of the consumer $x(p, w)$ is *(i)* homogeneous of degree 0 and *(ii)* satisfies Walras' law.

10. The *indirect utility function* is defined as $v(p, w) := u(x(p, w))$. *(i)* Show that we have $\frac{\partial v(p,w)}{\partial w} = \frac{v(p,w)}{w}$. *(ii)* (Bonus) Why was it expected?

11. Let $\alpha = \frac{1}{2}$, $p = \bar{p} := (1, 4)$, and $w = \bar{w} := 8$. *(i)* What is the utility level at the solution? *(ii)* Superimpose a graphical representation of the solution to the UMP to the figure drawn in question 4), i.e., draw the consumer's budget set, the solution, and the gradient of the utility function at the solution.

12. (Bonus) Still on the same figure: *(i)* Assume that w varies. Draw the associated wealth-consumption curve, $E_{\bar{p}} = \{x(\bar{p}, w) \mid w > 0\}$. *(ii)* Assume that p_2 varies. Draw the associated offer curve, $O_{\bar{p_1}, \bar{w}} = \{x((\bar{p_1}, p_2)), \bar{w}) \mid p_2 > 0\}$.