// Microeconomics 1. $\boxed{\text{TD } 12}$

(P6). $\boxed{f(z) = \alpha z \qquad \alpha > 0.}$

a). The PMP writes.

$$\begin{cases} \max \bar{p}^t \cdot y \\ y \in \mathcal{Y} \end{cases}$$

where $\pi(\bar{p}, y) = \bar{p}^t \cdot y$ is the profit (net) given the production plan y.

$y \in \mathcal{Y} \iff f(y) \leq 0.$

$\iff (z \geq 0 \text{ and } q \leq f(z)).$

for $(-z, q) = y$. in the case of a single output technology.

So let p denote the price of the output, w the price vector of the inputs, ie $\bar{p} = (w, p)$.

we have. $\pi(y) = \bar{p}^t \cdot y = pq - w \cdot z$

$\leq pf(z) - w \cdot z.$

so the PMP writes. $\boxed{\max_{z \geq 0} pf(z) - w \cdot z.}$

b) $Y = \{ y \in \mathbb{R}^L ; F(y) \le 0 \}$.

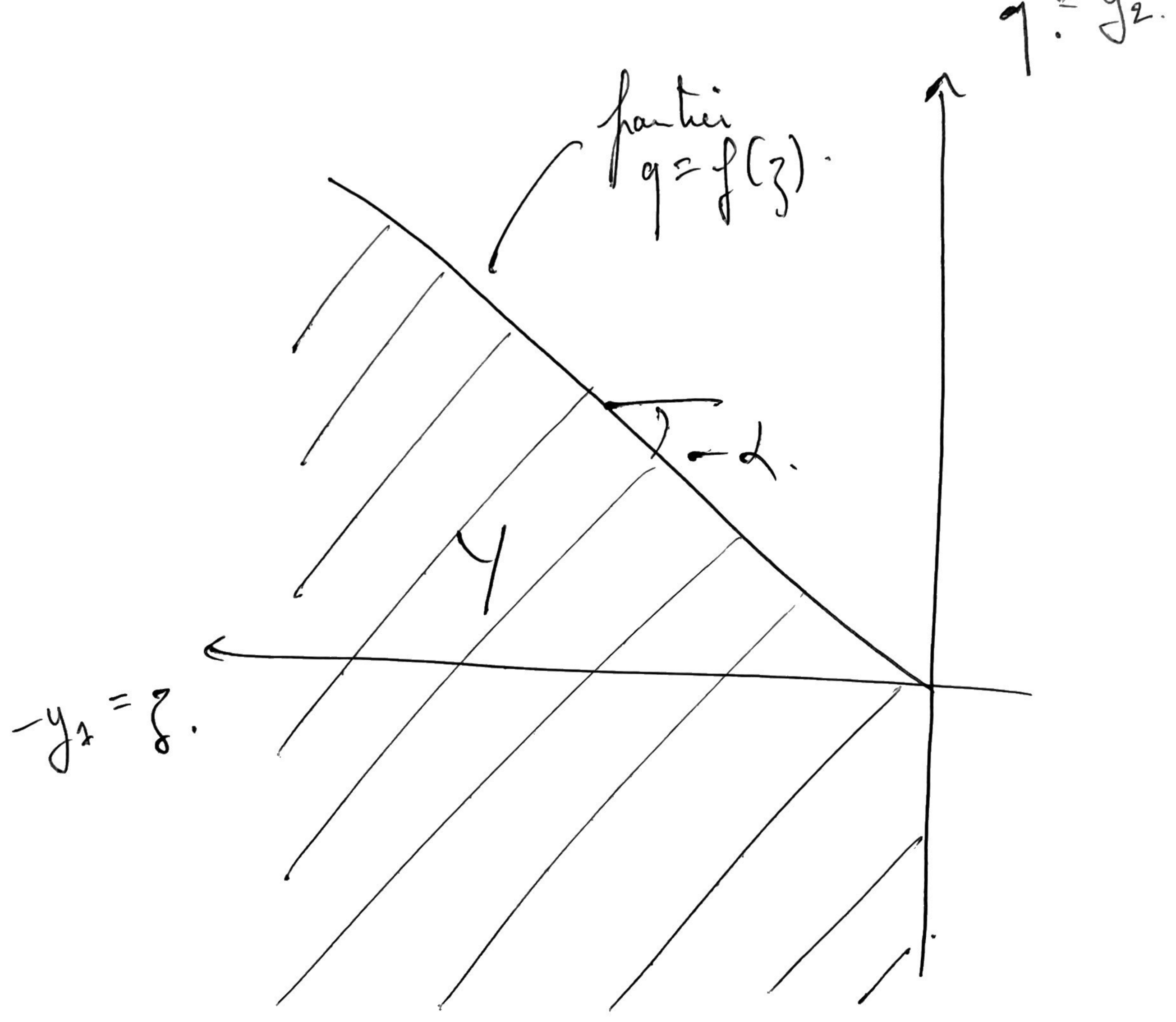
written in terms of the trf fct.

$Y = \{ (-z, q) ; q - f(z) \le 0$ and $z \ge 0 \}$.

production set.

written in terms of the prod. fct.

production frontier is $\bar{Y} - \dot{Y} = \{ (-z, q) ; q = f(z)$ and $z \ge 0 \}$.



c). Determine the profit fct of this firm (graphically).

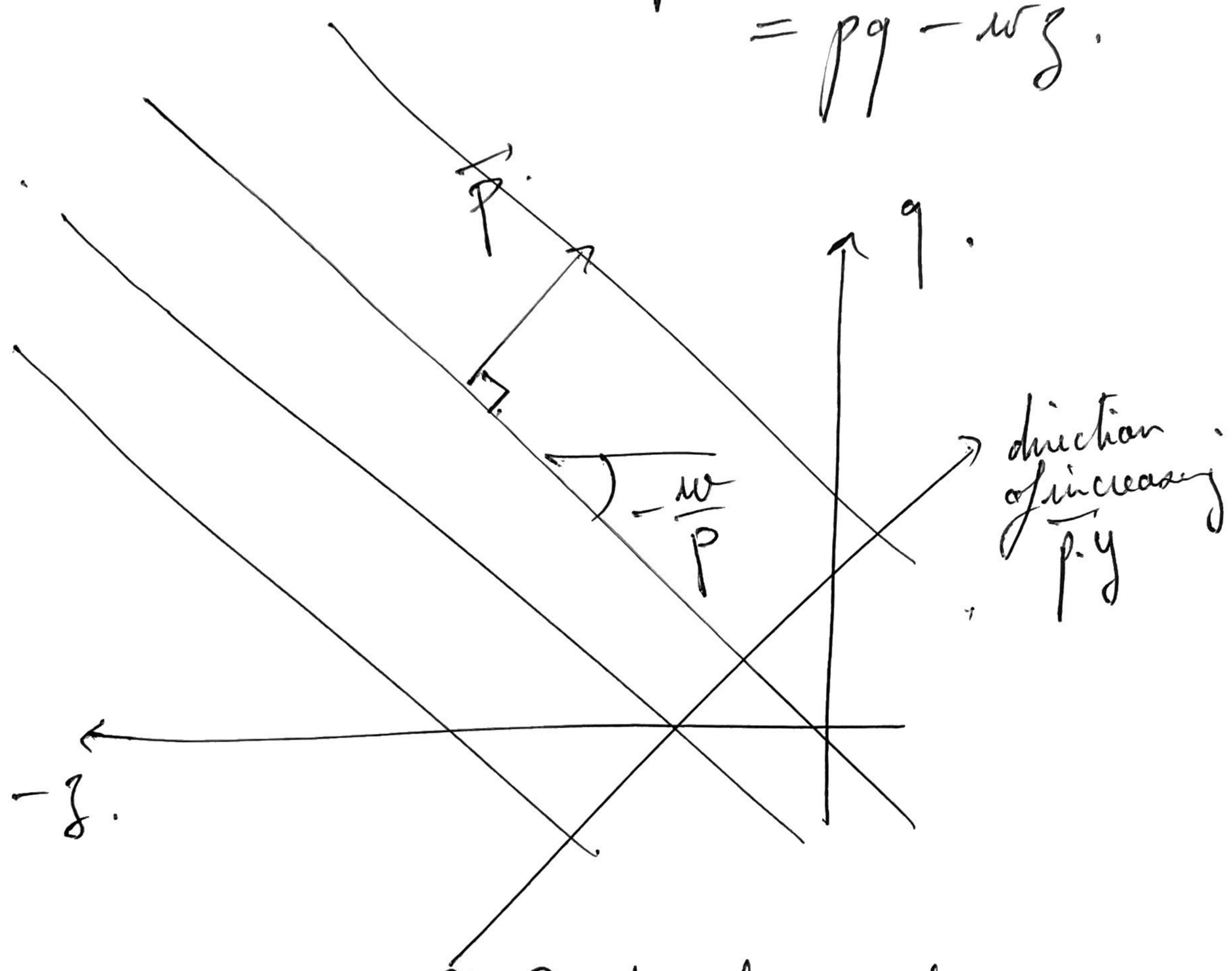
let. $\bar{p}' = (w, p)$. and. $y = (-z, q)$.

w: price of input. p: price of output.

The isoprofit lines verify. $\bar{p}' \cdot y = \text{cste}$. ("$p \cdot y'$").

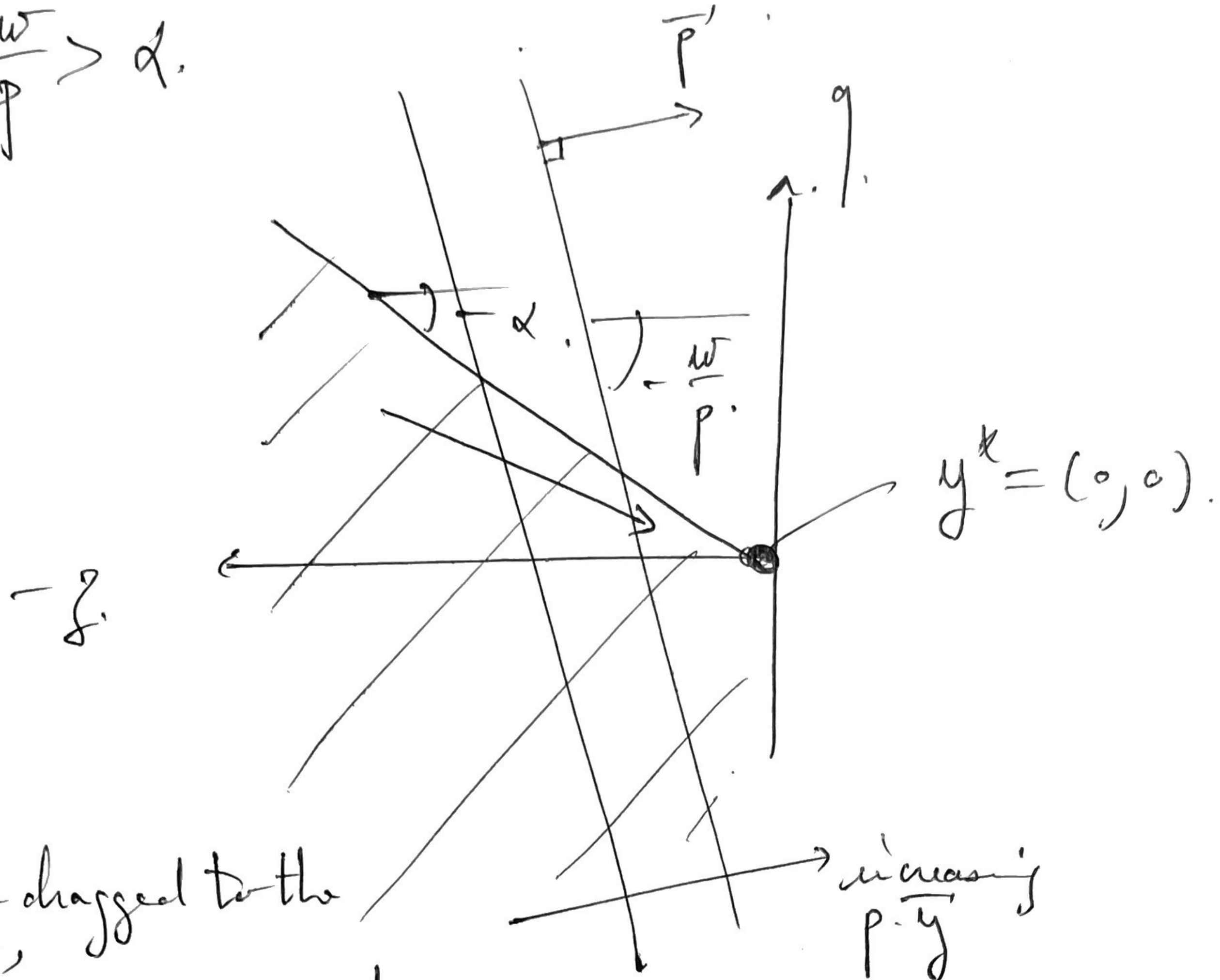
$= pq - wz$.

ie.



So the solution of the PMP depends on the comparison of α with $\frac{w}{p}$.

* if $\frac{w}{p} > \alpha$,



we are dragged to the right,

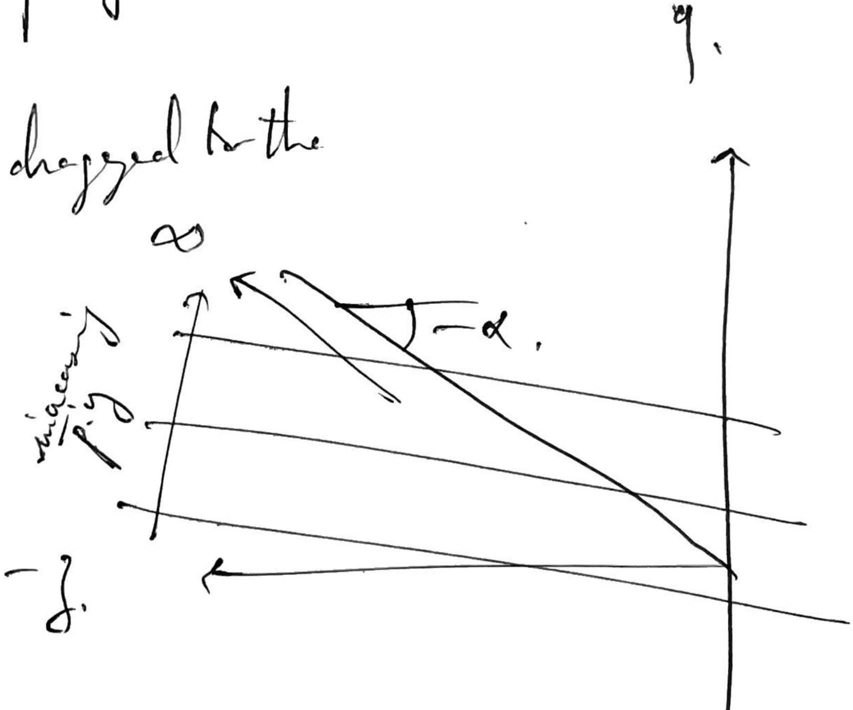
$y^* = (0,0)$ and

$y(p) = y^*$, $\quad \Pi(p) = \bar{p}' \cdot y^* = 0$.

* if $\frac{w}{p} < \alpha$, we are dragged to the left, ∞

"$y(p) = (-1, \alpha) \times \infty$"

and $\Pi(p) = +\infty$.



* if $\frac{w}{p} = \alpha$.

$y(p) = \{(-1, \alpha)z \,;\, z \geq 0\}$. i.e the whole production frontier,

and $\Pi(p) = 0$ $\blacksquare$.

(P7). $Y = \{ y \in \mathbb{R}^L ; f(y) \leq 0 \}$.

a). PMP.

b) Show f continuous and strictly quasiconvex

$\Longrightarrow$ PMP has unique solution.

a) The profit maximization problem (PMP) writes; for $p \gg 0$.

$\bar{y} \in y(p)$: supply fct / corr.

iff $\bar{y}$ solution to $\begin{cases} \max\limits_{y \in Y} p \cdot y \end{cases}$

i.e. $\begin{cases} \max\limits_{y \in \mathbb{R}^L} p \cdot y \\ \text{s.th. } f(y) \leq 0 \end{cases}$

b). By def, $\bar{y} \in y(p)$ iff

$\forall y \in y(p)$. $p \cdot \bar{y} \geq p \cdot y$.

the KT conditions are .

$\bar{y} \in y(p)$. ie $\bar{y}$ sol⁰ PMP.
$\underline{\underline{>> 0}}$.

$\longrightarrow$ $\begin{cases} p = \lambda \nabla f(\bar{y}) & \lambda \geq 0. \\ \text{and.} & \\ \lambda(f(\bar{y})) = 0. & \end{cases}$

so . $p >> 0.$ $\longrightarrow$ $\lambda > 0.$

$\longrightarrow$ $\underline{\underline{f(\bar{y}) = 0.}}$! ie. sol⁰ found on the production frontier.

So ... suppose. y^* and $\bar{y} \in y(p)$.
$y^* \neq \bar{y}$.

we have. $p \cdot \bar{y} = p \cdot y^* = \pi(p)$.

$\longrightarrow$ so let $y'' = \alpha \bar{y} + (1-\alpha) y^*$. for $\alpha \in]0,1[$

we have. $p \cdot y'' = \alpha \pi(p) + (1-\alpha)\pi(p) = \pi(p)$

so $y'' \in y(p)$ ie is also sol⁰.

But if F is f is strictly quasiconvex,

we have.

$$\bar{y} \neq y^* \Longrightarrow F(y'') < \max\{\underbrace{F(\bar{y})}_{=0}, \underbrace{F(y^*)}_{=0}\}$$

so $\underline{f(y'') < 0}$. contradiction with KT!

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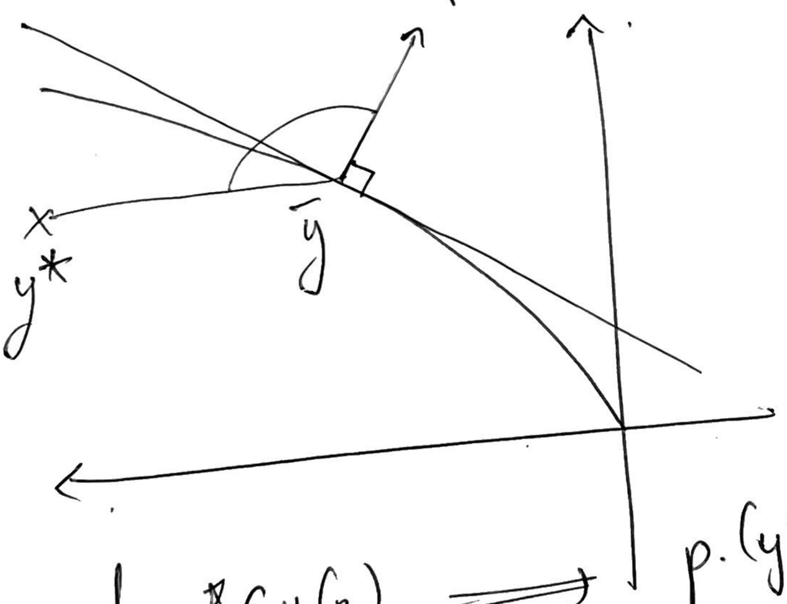
alternative proof:

$p = \lambda \nabla f(\bar{y})$.



Suppose $\bar{y} \in y(p)$.

$$\Downarrow$$

$$p = \lambda \nabla f(\bar{y}).$$

$$\Downarrow$$

$$p \cdot (y^* - \bar{y}) = \lambda \nabla f(\bar{y}) \cdot (y^* - \bar{y})$$

so if $y^* \in y(p) \Longrightarrow p \cdot (y^* - \bar{y}) = 0$.

but from f strictly quasiconvex, we have. $y^* \neq \bar{y} \Longrightarrow \nabla f(\bar{y}) \cdot (y^* - y) < 0$

$\Longrightarrow$ CONTRADICTION!

P8 $\boxed{f(z) = \alpha\sqrt{z} \qquad \alpha > 0}$

The transformation fct writes:

$$F(y) = q - f(z). \qquad y = (-z, q).$$

PMP. $\begin{cases} \max \vec{p} \cdot y. \\ F(y) \leq 0. \end{cases} \Longrightarrow \begin{cases} \max p f(z) - w \cdot z. \\ z \geq 0 \end{cases}$

The KT condθ write

$\bar{y} \gg 0$ solθ.

$\longrightarrow \begin{cases} p = \lambda \nabla F(\bar{y}), \quad \lambda \geq 0 \\ \text{and} \quad \lambda F(\bar{y}) = 0. \end{cases}$

$\underline{\underline{\text{or}}}$ $\underline{\underline{p \nabla f(\bar{z}) - w = 0}}$ $(*)$

(and we had: $\bar{q} = f(\bar{z})$).

(because: $\vec{p} \cdot y$ increasing!)

both will yield the same results...

$(*) \Longrightarrow \quad p\dfrac{\partial f}{\partial z}(\bar{z}) = w.$

$$\text{i.e.} \quad p\frac{\alpha}{2\sqrt{z}} = w.$$

$$\Longrightarrow \quad \bar{z} = \left(\frac{p\alpha}{2w}\right)^2 \,/\!/$$

and $\bar{q} = \alpha\sqrt{\bar{z}} \Longrightarrow \quad \bar{q} = \dfrac{\alpha^2 p}{2w}. /\!/$

So the profit fct is

$$\pi(p) = \vec{p}\cdot y(p) = p\bar{q} - w\bar{z}.$$
$$= p\frac{\alpha^2 p}{2w} - w\left(\frac{p\alpha}{2w}\right)^2 = \frac{(p\alpha)^2}{w}\left(\frac{1}{2}-\frac{1}{4}\right).$$

$$\Longrightarrow \quad \pi(p) = \frac{(\alpha p)^2}{4w}. /\!/$$

$\textcircled{P10}$ $\boxed{z \in \mathbb{R}_+^{L-1} \quad f(z)}$

a) <u>cost fct.</u> $c(w,q) = \min_{z \geq 0} w \cdot z$

s.th. $f(z) \geq q$.

c). $\boxed{\text{Show } f \text{ concave} \Longrightarrow c(w,q) \text{ convex in } q.}$

assume $\begin{cases} c(w,q) = w \cdot z^* \text{ and.} \\ c(w,q') = w \cdot z^{*'} \end{cases}$

$c(w,q)$ convex in q.

$\Longleftrightarrow \quad c(w, \underbrace{\alpha q + (1-\alpha) q'}_{= q^\alpha}) \leq \alpha c(w,q) + (1-\alpha) c(w,q').$

$\forall \alpha \in [0,1]$.

$c(w,q^\alpha) = \min_{f(z) \geq q^\alpha} w \cdot z$.

f concave $\Longrightarrow f(z'') \geq \alpha f(z) + (1-\alpha) f(z')$.

So.

$$\alpha c(w,q) + (1-\alpha) c(w,q')$$
$$= w.(\alpha z^* + (1-\alpha) z^{*'}). \quad (\|).$$

Using the concavity of f, we have.

$$f(\alpha z^* + (1-\alpha) z^{*'}) \geq \alpha f(z^*) + (1-\alpha) f(z^{*'})$$
$$\geq \alpha q + (1-\alpha) q' =: q''.$$

and. $c(w,q'') = \min_{f(z) \geq q''} w.z$.

MINIMUM.

$\downarrow$

So, by def. $c(w,q'') = w.z''^* \leq$
$$w.(\alpha z^* + (1-\alpha) z^{*'}).$$
$$= \alpha c(w,q) + (1-\alpha) c(w,q').$$

i.e. $c(\cdot,q)$ convex in q.

$\square$.

$\boxed{\text{1). Show } c(w,q) \text{ is CONCAVE. w.r.t. } w.}$

i.e. show:

$$c(w'',q) \geq \alpha\, c(w,q) + (1-\alpha)\, c(w',q).$$

$$w'' = \alpha w + (1-\alpha) w' \quad \forall \alpha \in [0,1].$$

$$c(w'',q) = w'' \cdot z(w'',q).$$

solB to CMP.

$$\begin{cases} \min w'' \cdot z \\ z \geq 0 \text{ and } f(z) \geq q. \end{cases}$$

$$= \alpha \underbrace{w \cdot z(w'',q)}_{\geq c(w,q)} + (1-\alpha) \underbrace{w' \cdot z(w'',q)}_{\geq c(w',q)}.$$

because by def $c(w,q)$ is the minimum value of $w \cdot z$. $\forall z$ s.th. $f(z) \geq q$.

likewise.

so:

$$c(w'',q) \geq \alpha\, c(w,q) + (1-\alpha)\, c(w',q). \quad \square.$$

(PM) $f(z) = z_1^a + z_2$. $\quad 0 < a < 1$. $\quad p \quad (w_1, w_2) = w$.

a). Determine the conditional input demand, profit and supply fct!

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CMP. $\begin{cases} \min\limits_{z \geq 0} w \cdot z \\ \text{s.th. } f(z) \geq q \end{cases}$

$\Rightarrow \begin{cases} w = \lambda \nabla f(z^*) \\ \text{and } \lambda (f(z^*) - q) = 0 \end{cases}$

$\Rightarrow$ (1) $w_1 = \lambda . a z_1^{a-1}$. so. $\dfrac{w_1}{w_2} = a z_1^{*\,a-1}$.

(2) $w_2 = \lambda$

(3) $q = z_1^{*\,a} + z_2^*$.

$\Rightarrow \boxed{z_1^* = \left(\dfrac{w_1}{w_2}\dfrac{1}{a}\right)^{\frac{1}{a-1}}}$ and. $z_2^* = q - z_1^{*\,a}$

$\boxed{z_2^* = q - \left(\dfrac{w_1}{w_2}\dfrac{1}{a}\right)^{\frac{a}{a-1}}}$

$\Longrightarrow$ The profit fct is now obtained by maximizing.

$$\begin{cases} \max\limits_{q \geq 0} \; pq - c(w,q) \end{cases}$$

The cost fct. was thus.

$$c(w,q) = \max_{f(z) \geq q} w.z$$

$$= w.z^* = w_1 z_1^* + w_2 z_2^*$$

$$= w_1 \left(\frac{w_2}{w_2} \frac{1}{a} \right)^{\frac{1}{a-1}} + w_2 \left(q - \left(\frac{w_1}{w_2} \frac{1}{a} \right)^{\frac{a}{a-1}} \right)$$

So.

$$pq - c(w,q) \ldots$$

(linear in q, not bounded from above.

$\Longrightarrow \pi(\bar{p}) = +\infty$.

in all cases! i.e. indep of a!

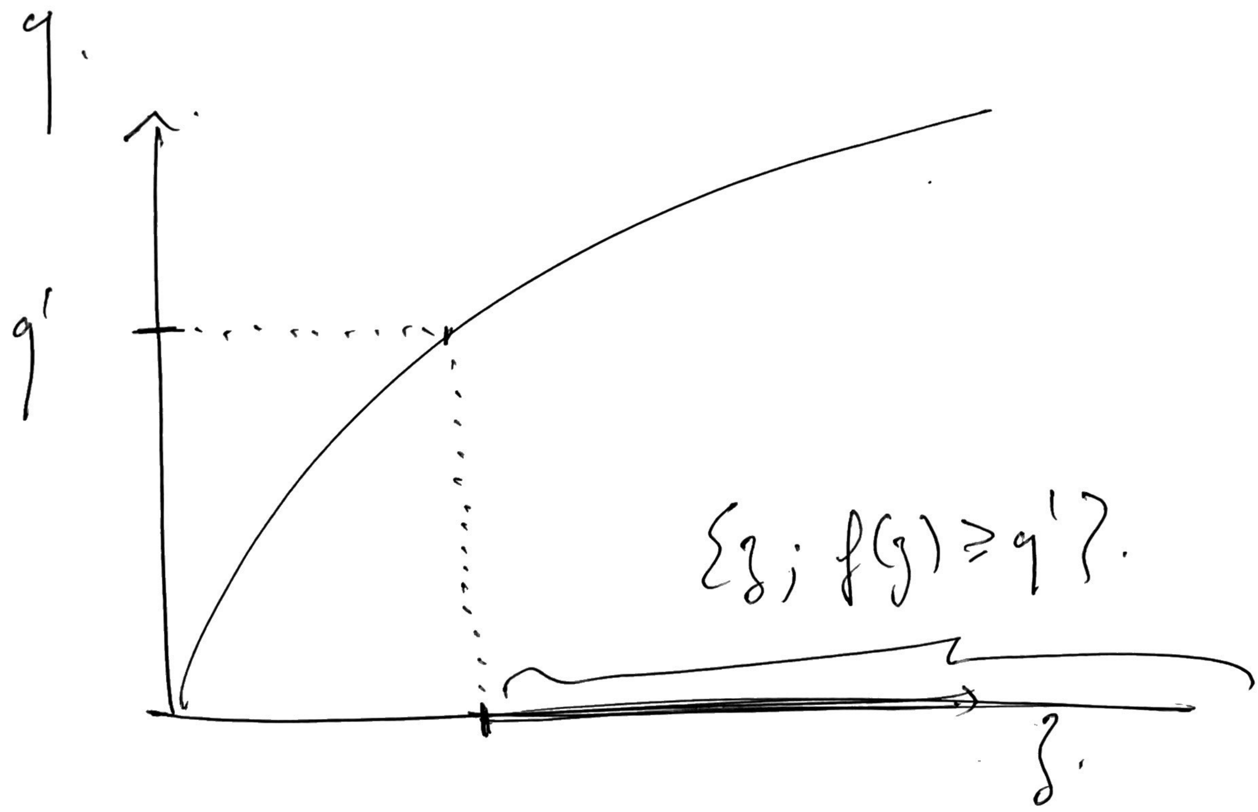
maximized for. $\frac{\partial}{\partial q}\left(pq - c(w,q)\right) = 0$.

i.e. $p = \frac{\partial c}{\partial q}(w,q) = w_2 \ldots$

(12). $f(z) = \alpha\sqrt{z}$.

→ cost fct.
conditional demand for inputs
profit fct.
supply fct.

CMP. $\begin{cases} \min wz \\ \text{s.th. } f(z) \geq q \end{cases}$



$\Longrightarrow z^* \in z(w, q).$

$\Longrightarrow f(z^*) = q = \alpha\sqrt{z^*}.$

$\Longrightarrow \boxed{z^* = \left(\frac{q}{\alpha}\right)^2}$ CONDITIONAL DEMAND.

$\Longrightarrow$ cost fct:

$$\boxed{c(w, q) = w \cdot z^* = w\left(\frac{q}{\alpha}\right)^2}.$$

$\Longrightarrow$ the profit fct is obtained by solving and supply curve:

$$\boxed{\left\{ \max_{q \geq 0} \; pq - c(w, q). \right.}$$

$\Longrightarrow q^*$ s.th. $p = \frac{\partial}{\partial q}(c(w, q^*)) = \frac{2w q^*}{\alpha^2}.$

$\Longrightarrow \boxed{q^* = \frac{\alpha^2 p}{2w}}$ and $z^* = \frac{\alpha^4 p^2}{4w^2} \frac{1}{\alpha^2}.$

$$\boxed{z^* = \frac{\alpha^2 p^2}{4w^2}}$$

$$\boxed{y(p,w) = \left(-\frac{\alpha^2 p^2}{4w^2} ; \frac{\alpha^2 p}{2w}\right)}$$

SUPPLY CURVE.

is it what we obtained in P8? YES!!

finally, the PROFIT fct is.

$$\pi(p,w) = \bar{p}' \cdot y(p,w).$$

$$= -w\left(\frac{\alpha^2 p^2}{4w^2}\right) + p\,\frac{\alpha^2 p}{2w} = \frac{\alpha^2 p^2}{4w}.$$

$$\boxed{\pi(p) = \frac{(\alpha p)^2}{4w}}$$

also same as in P8!

P9

$$\boxed{\begin{array}{l}\text{Solve the PMP.}\\ \text{for } f(z) = \sqrt{z_1 + z_2}\end{array}} \quad (a)$$

PMP. $\left\{ \begin{array}{l} \max\limits_{z \geq 0} \; p f(z) - w \cdot z \\ \qquad (\text{and } q = f(z)) \end{array} \right.$

so $\bar{z} \gg 0$ solθ. $\longrightarrow$

KT condθ. $\quad p\,Df(\bar{z}) - w = 0.$

i.e. $p \begin{pmatrix} \dfrac{1}{2\sqrt{z_1+z_2}} \\ \dfrac{1}{2\sqrt{z_1+z_2}} \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}.$

$\longrightarrow$ so a solθ. $\bar{z} \gg 0$ is possible iff $\underline{w_1 = w_2}$.

if. $w_1 \neq w_2$, we have to write and solve the KT conditions for the general case of $z \geq 0$, possibly $\bar{z}_1 = 0$ or $\bar{z}_2 = 0 \ldots$

PMP $\iff$ $\begin{cases} \max \ p f(z) - w \cdot z. \\ z \geq 0. \end{cases}$

i.e. $-z_1 \leq 0$
and $-z_2 \leq 0$.

so $\bar{z} \geq 0$ sol.

$\longrightarrow$ KT conditions:

$$\begin{cases} p Df(\bar{z}) - w \overset{(1)}{=} \lambda_1 \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \\ \text{and} \\ \lambda_1 \bar{z}_1 = 0. \quad (2) \\ \text{and.} \\ \lambda_2 \bar{z}_2 = 0. \quad (3) \end{cases} \qquad \text{with. } \lambda_{1,2} \geq 0.$$

Suppose. $\underline{w_1 > w_2}$.

we know that a solution. $\bar{z}_1$ and $\bar{z}_2 > 0$. is no possible.

Try. $\bar{z}_1 = 0. \overset{(2)}{\Longrightarrow} \lambda_1 \neq 0$

in this case $\bar{z}_2 = 0$ is excluded.

$\Longrightarrow \bar{z}_2 \neq 0 \underset{(3)}{\Longrightarrow} \lambda_2 = 0$

so the second line of (1) writes:

$$\frac{p}{2\sqrt{\bar{z}_1+\bar{z}_2}} - w_2 = 0 \qquad (\alpha)$$

$$\Longrightarrow \sqrt{\bar{z}_1+\bar{z}_2} \underset{\substack{\uparrow \\ \text{because} \\ \bar{z}_1 = 0}}{=} \sqrt{\bar{z}_2} = \frac{p}{2w_2}.$$

and the 1st line of (1) writes:

$$\underbrace{\frac{p}{2\sqrt{\bar{z}_1+\bar{z}_2}}}_{\overset{(\alpha)}{=}\, w_2} - w_1 = -\lambda_1.$$

$$\Longrightarrow \lambda_1 = w_1 - w_2 > 0.$$ ok!

so the solution is indeed acceptable.

$\Longrightarrow$ so in this case, $\boxed{\bar{z}_1 = 0 \text{ and } \bar{z}_2 = \left(\frac{p}{2w_2}\right)^2.}$

and. $y(\vec{p}) = (-\bar{z}_1, -\bar{z}_2, \bar{q})$.

with $\bar{q} = f(\bar{z}) = \sqrt{\bar{z}_1 + \bar{z}_2} = \frac{p}{2w_2}$.

$$\longrightarrow \boxed{y(\vec{p}) = \left(0, -\left(\frac{p}{2w_2}\right)^2, \frac{p}{2w_2}\right)}.$$

supply fct.

and the profit function:

$$\Pi(\vec{p}) = \vec{p} \cdot y = p\bar{q} - w \cdot \bar{z}$$

$$= \frac{p^2}{2w_2} - w_2 \left(\frac{p}{2w_2}\right)^2.$$

$$\longrightarrow \boxed{\Pi(\vec{p}) = \frac{p^2}{4w_2}.}$$

By symmetry, under $1 \longleftrightarrow 2$.

if $\underline{w_2 > w_1}$,

then:

$$\boxed{\begin{aligned} y(\bar{p}) &= \left(-\left(\frac{p}{2w_2}\right)^2, 0, \frac{p}{2w_1}\right). \\ \text{and } \pi(\bar{p}) &= p^2/4w_1. \end{aligned}}$$

Finally, if $\underline{w_1 = w_2}$. $=: w$,

we have. $\frac{1}{2\sqrt{\bar{z}_1 + \bar{z}_2}}\, p = w$

$\longrightarrow \sqrt{\bar{z}_1 + \bar{z}_2} = \bar{q} = \frac{p}{2w}$. $(*)$

$$\begin{aligned} \Longrightarrow w \cdot \bar{z} &= w_1 \bar{z}_1 + w_2 \bar{z}_2 \\ &= w(\bar{z}_1 + \bar{z}_2) \underset{(*)}{=} w\left(\frac{p}{2w}\right)^2 = \frac{p^2}{4w} \end{aligned}$$

so. $\pi(\bar{p}') = p\bar{q} - w.\bar{z}$

$$\Longrightarrow \quad \boxed{\pi(\bar{p}') = \frac{p^2}{4w}}.$$

and.

$$\boxed{y(p) = \left\{ \left(-\bar{z}_1, -\bar{z}_2, \frac{p}{2w}\right); \text{ with. } \bar{z}_1 + \bar{z}_2 = \left(\frac{p}{2w}\right)^2, \ \bar{z}_1 \geq 0 \text{ and } \bar{z}_2 \geq 0 \right\}}.$$

(h) $f(z) = \sqrt{\min\{z_1, z_2\}}$ is not differentiable, so we cannot use the FOCs ....

(c). instead of CES, let's do it for Cobb–Douglas ...

i.e.

$$\boxed{f(z) = z_1^{\alpha_1} z_2^{\alpha_2}}.$$

The best way is to solve the CMP before going back to the PMP.

The CMP writes
$$\begin{cases} \min\ w \cdot z \\ \text{s.th.}\ f(z) \geq q \\ z \geq 0 \end{cases}$$

$\bar{z} \gg 0$ sol.

$\Longrightarrow$ KT.
$$\begin{cases} w = \lambda \nabla f(\bar{z}) \quad (1) \quad \lambda \geq 0. \\ \text{and} \\ \lambda (f(\bar{z}) - q) = 0 \quad (2) \end{cases}$$

so. $w \gg 0 \Longrightarrow \lambda > 0$.
$\Longrightarrow f(\bar{z}) = q$ i.e. production is <u>efficient</u>

and. (1) $\Longleftrightarrow$

$$\begin{cases} w_1 = \lambda \dfrac{\alpha_1}{z_1} f(z) \\ \text{and} \\ w_2 = \lambda \dfrac{\alpha_2}{z_2} f(z) \end{cases}$$

$$\Longrightarrow \boxed{\frac{w_1}{w_2} = \frac{\alpha_1}{\alpha_2}\frac{z_2}{z_1}} \quad (*).$$

$$q = f(z) = z_1^{\alpha} z_2^{\alpha} \underset{(*)}{=} z_1^{\alpha_1+\alpha_2} \left(\frac{\alpha_2}{\alpha_1}\frac{w_1}{w_2}\right)^{\alpha_2}.$$

so the conditional demand for inputs $z(q)$ is $z(q) = (\bar{z}_1, \bar{z}_2)$

with

$$\Longrightarrow \boxed{\begin{array}{l} \bar{z}_1(q) = q^{\frac{1}{\alpha_1+\alpha_2}} \left(\dfrac{\alpha_2}{\alpha_1}\dfrac{w_2}{w_2}\right)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} \\ \text{and} \\ \bar{z}_2(q) = q^{\frac{1}{\alpha_1+\alpha_2}} \left(\dfrac{\alpha_1}{\alpha_2}\dfrac{w_2}{w_1}\right)^{-\frac{\alpha_1}{\alpha_1+\alpha_2}}. \end{array}}$$

by symm.

The cost fct is.

$$c(w,q) = w \cdot z(q).$$

PMP. $\left\{ \max_{q \geq 0} \; pq - c(w,q). \right.$

if. $\underline{\alpha_1 + \alpha_2 = 1}$

$$pq - c(w,q) = q \Bigg[\underbrace{p - w_1 \left(\frac{\alpha_2}{\alpha_1} \frac{w_1}{w_2} \right)^{-\alpha_2} - w_2 \left(\frac{\alpha_1}{\alpha_2} \frac{w_2}{w_1} \right)^{-\alpha_1}}_{=: \, \delta} \Bigg].$$

linear in q!

$\Longrightarrow$ so the result depends on the sign of δ.

if $\delta < 0$. then $\bar{q}$ sol° is $\bar{q} = 0$.

$$\longrightarrow \quad \boxed{y(\vec{p}\,') = (0, 0, 0) \text{ ie inaction and } \pi(\vec{p}\,') = 0.}$$

if. $\delta > 0$. $\longrightarrow$ then. $\boxed{\pi(\vec{p}\,') = +\infty \text{ and } y(\vec{p}\,') = \emptyset}$

if $\delta = 0$, then any $\bar{q}$ works.

$$\longrightarrow \quad \boxed{\pi(\vec{p}\,') = 0. \text{ and } y(\vec{p}\,') = \left\{ q \left(\left(\frac{\alpha_2}{\alpha_2} \frac{w_1}{w_2}\right)^{-\alpha_2}, \left(\frac{\alpha_1}{\alpha_2} \frac{w_2}{w_1}\right)^{-\alpha_1}, 1 \right); \ q \geq 0 \right\}.}$$

if. $\alpha_1 + \alpha_2 > 1$. , $pq - c(w, q)$.

is <u>CONVEX in q</u>. so: $\pi(\vec{p}\,') = +\infty$. and $y(\vec{p}\,') = \emptyset$, if $\alpha_1+\alpha_2<1$.

$$g(q) := pq - c(w,q).$$

$$= pq - q^{\frac{1}{\alpha_1+\alpha_2}} \underbrace{\left[w_1 \left(\frac{\alpha_2}{\alpha_1}\frac{w_1}{w_2}\right)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} + w_2\left(\frac{\alpha_1}{\alpha_2}\frac{w_2}{w_1}\right)^{-\frac{\alpha_1}{\alpha_1+\alpha_2}}\right]}_{=:\,\delta}$$

$$g(q) = pq - q^{\frac{1}{\alpha_1+\alpha_2}}\delta.$$

$\longrightarrow$ $\bar{q}$ sol. must verify

$$\frac{\partial}{\partial q} g(\bar{q}) = 0$$

ie $$p = \frac{\delta}{\alpha_1+\alpha_2}\,\bar{q}^{\frac{1}{\alpha_1+\alpha_2}-1}$$

$$\Longrightarrow \quad \bar{q} = \left(\frac{(\alpha_1 + \alpha_2)\, p}{\delta} \right)^{\frac{1}{\frac{1}{\alpha_1+\alpha_2} - 1}}$$

which you can input in $z(\bar{q})$ the conditional demand.

to get $y(\bar{p}') = (-z(\bar{q}), \bar{q})$.

then compute $\pi(\bar{p}') = \bar{p}' \cdot y(\bar{p}')$.

$$\boxed{(c) \quad f(z) = (z_1^{\rho} + z_2^{\rho})^{1/\rho}.}$$

CMP.
$$\begin{cases} \min w \cdot z. \\ \text{s.th. } f(z) \geq q. \\ z \geq 0. \end{cases}$$

$\bar{z} >> 0$ sol.

$\Longrightarrow$ KT.
$$\begin{cases} w = \lambda \nabla f(\bar{z}) \quad \lambda \geq 0. & (1) \\ \text{and.} \\ \lambda (f(\bar{z}) - q) = 0. & (2). \end{cases}$$

$w >> 0 \quad \Longrightarrow \quad \lambda > 0 \quad \Longrightarrow \quad f(\bar{z}) = q$. as always..

$$\nabla f(\bar{z}) = \begin{pmatrix} \frac{\partial}{\partial z_1} f(\bar{z}) \\ \frac{\partial}{\partial z_2} f(\bar{z}) \end{pmatrix}.$$

$$\frac{\partial}{\partial z_i} f(z) = \frac{1}{\cancel{\rho}} \cancel{\rho} z_i^{\rho-1} \left(z_1^{\rho} + z_2^{\rho}\right)^{\frac{1}{\rho}-1}$$

$$\forall i = 1, 2.$$

no.

$$\rightarrow \boxed{\frac{w_1}{w_2} = \frac{\bar{z}_1^{\,\rho-1}}{\bar{z}_2^{\,\rho-1}}} \rightarrow \bar{z}_2 = \bar{z}_1 \left(\frac{w_2}{w_1}\right)^{\frac{1}{\rho-1}}$$

$\rightarrow$

$$q = f(\bar{z}) = \left(z_1^{\rho} + z_2^{\rho}\right)^{1/\rho}$$

$$q = z_1 \left(1 + \left(\frac{w_2}{w_1}\right)^{\frac{\rho}{\rho-1}}\right)^{1/\rho}.$$

$\rightarrow$ The conditional demand for input $z(q)$ is:

$$z(q) = (\bar{z}_1, \bar{z}_2) \quad \text{with.}$$

$$\bar{z}_1(q) = \frac{q}{\left(1 + \left(\frac{w_2}{w_1}\right)^{\rho/(\rho-1)}\right)^{1/\rho}}.$$

and.

$$\bar{z}_2(q) = \frac{q}{\left(1 + \left(\frac{w_1}{w_2}\right)^{\rho/(\rho-1)}\right)^{1/\rho}}.$$

obtained by symm. under $1 \leftrightarrow 2$.

The. cost fct. is.

$$c(w, q) = w \cdot z(q).$$

The PMP is.
$$\begin{cases} \max\limits_{q \geq 0} \; p \cdot q - w \cdot z(q). \end{cases}$$

$$p \cdot q - w \cdot g(q).$$

$$= q \cdot \underbrace{\left[p - \overbrace{\left(\frac{w_1}{\left(1+\left(\frac{w_2}{w_1}\right)^{\frac{e}{e-1}}\right)^{1/e}} + \frac{w_2}{\left(1+\left(\frac{w_1}{w_2}\right)^{\frac{e}{e-1}}\right)^{1/e}} \right)}^{=(*)} \right]}.$$

The result will depend on the sign of this term...

$$(*) = \frac{\overbrace{w_1 + w_2 \left(\frac{w_2}{w_1}\right)^{\frac{1}{e-1}}}^{(**)}}{\left(\left(1+\left(\frac{w_2}{w_1}\right)^{\frac{e}{e-1}}\right)\right)^{1/e}}$$

$$(**) = w_1 \left(1 + \underbrace{\frac{w_2}{w_1}\left(\frac{w_2}{w_1}\right)^{\frac{1}{e-1}}}_{= \left(\frac{w_2}{w_1}\right)^{\frac{e}{e-1}}} \right).$$

$\Longrightarrow$

$$(\star) = w_1 \cdot \frac{\left(1 + \left(\frac{w_2}{w_1}\right)^{\frac{e}{e-1}}\right)}{\left(1 + \left(\frac{w_2}{w_2}\right)^{\frac{e}{e-1}}\right)^{1/e}} = \frac{e-1}{e}$$

$$= w_1 \cdot \left(1 + \left(\frac{w_2}{w_1}\right)^{\frac{e}{e-1}}\right)^{1-\frac{1}{e}}$$

$$\boxed{\left(w_1^{e/e-1} + w_2^{e/e-1}\right)^{\frac{e-1}{e}}} = (\star)$$

$$\text{if} \quad p > \left(w_1^{e/e-1} + w_2^{e/e-1}\right)^{\frac{e-1}{e}}$$

then $\pi(\bar{p}') = +\infty$ and $y(\bar{p}') = \emptyset$.

if $p^{e/(e-1)} = w_1^{e/(e-1)} + w_2^{e/(e-1)}$,

then $\pi(\bar{p}) = 0$

and $y(\bar{p}) = \left\{ \left(-\left(1+\left(\frac{w_2}{w_1}\right)^{\frac{e}{e-1}}\right)^{-1/e}, -\left(1+\left(\frac{w_1}{w_2}\right)^{\frac{e}{e-1}}\right)^{1/e}, 1 \right) q ;\ q \geq 0 \right\}$.

and if $p^{e/(e-1)} < w_1^{e/(e-1)} + w_2^{e/(e-1)}$,

then $\pi(\bar{p}) = 0$,

$y(\bar{p}) = (0, 0, 0)$