Microeconomics 1. $\boxed{TD4.}$ ROBINSON CRUSOE.

(12) $t = 0, 1.$

$u(x) = \ln(x_0) + \beta \ln(x_1).$ $\beta \in]0,1[.$

$\omega = (z_0, 0)$ initial endowment.

$f(z) = A z^{\alpha}.$ prod. fct.

1) find the competitive eq.
2) find the firm profit.
3) find Pareto set.

1C1f.

$x_1 = q.$
$x_0 = z_0 - z.$

1). Walrasian eq.

(x^*, y^*, p^*). s.th. x^* sol° UMP.
+ y^* sol° PMP.
+ market clearing.

$$\begin{cases} \max u(x) \\ p^* \cdot x \leq p^* \cdot \omega + p^* \cdot y^* \end{cases}$$

UMP

suppose $\vec{p} = (w, p)$

price of x_0 price of x_1

$$p \cdot x \leq \underbrace{p_{x_0}(k_0 - x_0) + p_{x_1} x_1}$$

$$= p \cdot \omega + p \cdot y$$

PMP

$$\begin{cases} \max p^* \cdot y \\ F(y) \leq 0 \end{cases}$$

and MC market clearing

$$x^* = \omega + y^*$$

UMP. KT.

$$\longrightarrow \begin{cases} \nabla_x u(x^*) = \lambda p^* & \lambda \geq 0. \\ \lambda\,(p^* \cdot (x^* - \omega - y^*)) = 0. \end{cases}$$

monotone

$\Rightarrow \lambda \neq 0 \longrightarrow p \cdot x^* = p(\omega + y^*)$. on Budget line Walras' law.

$$\longrightarrow \boxed{MRS_{12}(x^*) = \frac{p_1^*}{p_2^*}}$$

PMP. KT.

$$\longrightarrow \begin{cases} p^* = \lambda \nabla_y F(y^*) & \lambda \geq 0. \\ \lambda(F(y^*)) = 0. \end{cases}$$

monotone

$\longrightarrow \lambda \neq 0 \longrightarrow F(y^*) = 0$ on production frontier.

$$\boxed{\frac{p_1^*}{p_2^*} = \frac{\frac{\partial F(y^*)}{\partial y_1}}{\frac{\partial F(y^*)}{\partial y_2}} = MRT_{12}(y^*)}$$

So we need to solve.

(✱)

$$MRS_{12}(x^*) = MRT_{12}(y^*) = \frac{p_1^*}{p_2^*}$$

and. $x^* = \omega + y^*$.

← choices, . .

and. $p^*.x^* = p^*.(\omega + y^*)$.

and. $\boxed{x_1^* = f(q^*)}$

q^* ← the production function. $= f(k_0 - x_0^*) = A(k_0 - x_0^*)^{\alpha}$.

$$\boxed{MRS_{12}(x^*)} = \frac{\frac{\partial u}{\partial x_0}(x^*)}{\frac{\partial u}{\partial x_1}(x^*)} = \frac{\frac{1}{x_0^*}}{\frac{\beta}{x_1^*}} = \boxed{\frac{x_1^*}{x_0^*}\,\frac{1}{\beta}}.$$

$u(x) = \ln(x_0) + \beta \ln(x_1)$

$$MRT_{12}(y^*) = \frac{\frac{\partial F}{\partial y_1}(y^*)}{\frac{\partial F}{\partial y_2}(y^*)}.$$

with. $F(y) = q - f(z)$.

$$= q - Az^\alpha$$

$f(z) = Az^\alpha$.

$y_2 = q$ $y_1 = -z$

$\Rightarrow$

$$\frac{\partial F}{\partial y_1}(y) = \underbrace{\frac{\partial z}{\partial y_1}}_{=-1} \frac{\partial F}{\partial z}(y) = -\frac{\partial F}{\partial z}(y).$$

chain rule.

$$= \frac{\partial f}{\partial z}(z)$$

$$= \alpha A z^{\alpha-1} = \frac{\alpha}{z} f(z).$$

and. $$\frac{\partial f}{\partial y_2}(y) = \frac{\partial f}{\partial q}(y) = \underline{\underline{1}}.$$

So. $$\boxed{MRT_{12}(y^\star) = \frac{\alpha}{z^\star} f(z^\star) \underset{\uparrow}{=} \frac{\alpha\, q^\star}{z^\star}.}$$

$y^\star = (-z^\star, q^\star)$

on production frontier.

$$\boxed{MRT_{12}(y^\star) = \frac{\alpha\, x_1^\star}{k_0 - x_0^\star}.}$$

$q^\star = x_1^\star$

and. $x_0^\star = k_0 - z^\star$.

(*)

1.

$$\frac{x_1^*}{x_0^*}\frac{1}{\beta} = \frac{p_1^*}{p_2^*} = \frac{\alpha x_1^*}{k_0 - x_0^*}$$

$$x^* = \omega + y^*.$$

i.e. $\begin{pmatrix} x_0^* \\ x_1^* \end{pmatrix} = \begin{pmatrix} k_0 \\ 0 \end{pmatrix} + \begin{pmatrix} -z^* \\ q^* \end{pmatrix}$

$$x_1^* = q^* = f(z^*) = f(k_0 - x_0^*) = A(k_0 - x_0^*)^\alpha.$$

$$\frac{x_1^*}{x_0^*}\frac{1}{\beta} = \frac{\alpha x_1^*}{k_0 - x_0^*} \Longrightarrow \frac{k_0}{x_0^*} - 1 = \alpha\beta \cdot \frac{\cancel{x_1^*}}{\cancel{x_1^*}}$$

$$\boxed{x_0^* = \frac{k_0}{1+\alpha\beta}}$$

donc $y_2^* = -z^* = -(k_0 - x_0^*)$

$$\Downarrow$$

$$\boxed{y_2^* = -\frac{\alpha\beta}{1+\alpha\beta} k_0}$$

$$\longrightarrow \quad x_1^* = f(k_0 - x_0^*)$$

$$= A\,(k_0 - x_0^*)^{\alpha}.$$

$$k_0 - x_0^* = \frac{(\cancel{1} + \alpha\beta)\,k_0 - \cancel{k_0}}{1 + \alpha\beta} = \frac{\alpha\beta}{1+\alpha\beta}\,k_0 .$$

also.

$$\longrightarrow \quad \boxed{x_1^* = A\left(\frac{\alpha\beta}{1+\alpha\beta}\,k_0\right)^{\alpha}} = y_2^*$$

$$\frac{p_1^*}{p_2^*} = \frac{\alpha x_1^*}{k_0 - x_0^*} = A\alpha\,(k_0 - x_0^*)^{\alpha - 1}.$$

$$\boxed{\left.\frac{p_1}{p_2}\right|^* = \alpha A \cdot \left(\frac{\alpha\beta}{1+\alpha\beta}\,k_0\right)^{\alpha-1}}$$

2). $\vec{p}^{*} \cdot y(\vec{p}^{*})$.

$$= p_1^* y_1^* + p_2^* y_2^*.$$

$$= \alpha A \left(\frac{\alpha\beta}{1+\alpha\beta} l_0\right)^{\alpha-1} \times \left(-\frac{\alpha\beta}{1+\alpha\beta} l_0\right)$$

normalize $p_2^* = 1$.

$$+ 1 \times A \left(\frac{\alpha\beta}{1+\alpha\beta} l_0\right)^{\alpha}.$$

$$\longrightarrow \boxed{\Pi(p^*) = (1-\alpha)\, A \left(\frac{\alpha\beta}{1+\alpha\beta} l_0\right)^{\alpha}.}$$

so the equilibrium profit is positive.

iff $0 \leq \alpha \leq 1$.

i.e. the prod. fct. is CONCAVE.

3). The Pareto soln is all soltn for.

$$\begin{cases} \max u(x) \\ \text{s.th. } x = \omega + y & \text{ie market clears.} \quad \text{or } \leq \ldots \\ \text{and } F(y) \leq 0 & \text{production is feasible.} \end{cases}$$

ie. there's only one consumer here, so the pb. simply consists in maximizing their utility under CONSTRAINTS of feasibility.

(without reference to prices !!?)

ie. $$\begin{cases} \max u(x) \\ \text{s.th } f(x-\omega) \leq 0. \end{cases}$$

KT cond θ.

$\hat{x} >> 0$ interior sol θ.

$$\begin{cases} \boxed{D_x u(x^*) = \lambda \underline{\underline{D_x}} F(x^* - \omega)} & (\star) \\ \lambda \left(F(x^* - \omega) \right) = 0 \end{cases}$$

manc $\longrightarrow$ $\lambda \neq 0 \longrightarrow F(y^*) = 0$.

ie on production frontier

ie $q^* = f(z^*)$,

ie. $x_2^* = f(k_0 - x_0^*)$.

$$\boxed{D_x F(x - \omega) = D_{\underline{y}} F(y)}$$

y

$\underline{x = \omega + \underline{\underline{y}}}$

$\longrightarrow$ So $(\star)$ implies $\boxed{MRS_{12}(x^*) = MRT_{12}(y^*)}$

When we solved for (x_0^*, x_1^*), in 1), we didn't use the equality of MRS and MRT with $p_1/p_2|^*$ . . . .

idea here: $MRS_{11} = \dfrac{x_1^*}{x_0^*}\dfrac{1}{\beta} = MRT_{12} = \dfrac{\alpha x_1^*}{l_0 - x_0^*}$

and efficient production $\Rightarrow$ $x_1^* = A(l_0 - x_0^*)^\alpha$!

$\Rightarrow$ we find the same x^* and y^* !! $\Leftarrow$ $y^* = x^* - \omega$

So in the 1c 1p case, ie ROBINSON CRUSOE econ.

$$\boxed{\text{Walrasian eq } \underline{\text{iff}} \text{ Pareto optimum}}$$!!

(11). MWG. 15.C2.

1c l.p.
compute the eq. prices, profits and consumptions when

$$f(z) = \sqrt{z}. \qquad u(x) = \ln x_1 + \ln x_2.$$

and total endowment of leisure is $\bar{L} = 1$.

in private ownership econ. ~~the~~ profits are distributed to consumers - stakeholders.

→ so the total wealth is:

$$w = \underbrace{p \cdot \omega}_{\text{initial endowment}} + p \cdot y.$$

production plan.

the budget constraint is

$$p \cdot x \leq p \cdot w = p \cdot (\omega + y).$$

Def

Walrasian eq.

consumption demand

$$(p^*, x^*, y^*).$$

price

production plan.

equiv $u(x^*) \geq u(x)$.

s.th.

UMP $\begin{cases} x^* \succsim x' \ \forall x' \in B(p^*). \\ \text{i.e. } p^* \cdot x' \leq p^* \cdot (\omega + y^*). \end{cases}$

⊕ AND

PMP $\begin{cases} p^* \cdot y^* \geq p^* \cdot y' \\ \forall y' \in Y \end{cases}$

production set.

$\{(-z, q) ; z \geq 0 \text{ and } f(z) \geq q\}.$

AND

Market clearing. $\begin{cases} \boxed{x^* = \omega + y^*.} \end{cases}$

demand

initial endowment.

production

OCO

market clearing imposes:

$$x = \omega + y.$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} t \\ 0 \end{pmatrix} + \begin{pmatrix} -z \\ q \end{pmatrix}.$$

where good 1 is leisure, with $z = -y_1$ is labor provided!

$$\longrightarrow \boxed{z = t - x_1.}$$

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the WALRASIAN EQ. (p^*, x^*, y^*) must satisfy, as we've seen,

derived from KT conds.:

$$\boxed{MRS_{12}(x^*) = MRT_{12}(y^*) = \frac{p_1}{p_2}\Big|_*} \quad \text{FOCs.}$$

$$\begin{cases} p \cdot x^* = p^* \cdot (\omega + y^*) \qquad x^* = \omega + y^* \\ F(y^*) = 0. \end{cases}$$

market clearing

efficient production.

* $u(x) = \ln x_1 + \ln x_2$.

$$\longrightarrow \boxed{MRS_{12} = \frac{1/x_1}{1/x_2} = \frac{x_2}{x_1}}.$$

* $f(z) = \sqrt{z}$. $\longrightarrow$ $F(y) = q - f(z)$.

$$= y_2 - f(-y_1).$$

$$\longrightarrow \boxed{MRT_{12} = \frac{\partial F/\partial y_1}{\partial F/\partial y_2} = \frac{\frac{\partial F(-y_1)}{\partial(-y_1)}}{1} = \frac{\partial f}{\partial z} = \frac{1}{2\sqrt{z}}}.$$

from $MRS_{12} = MRT_{12}$ and market clearing i.e. $x = \omega + y$.

we can determine the PARETO set!

$$x = \omega + y \longrightarrow \begin{cases} x_2 = q \\ x_1 = t - z. \end{cases}$$

efficient prod. $\longrightarrow$ $x_2 = q = f(z) = \sqrt{z}$.

prod. function.

So. $MRS_{12} = MRT_{12}$.

$$\iff \frac{x_2}{x_1} = \frac{\sqrt{z}}{t-z} = \frac{1}{2\sqrt{z}}.$$

$$\iff \frac{t}{z} - 1 = 2$$

$$\Rightarrow \boxed{z^* = \frac{t}{3}} \Rightarrow \boxed{q^* = \sqrt{z} = \sqrt{\frac{t}{3}}}$$

$$\Rightarrow \boxed{x_2^* = \sqrt{\frac{t}{3}}} = q. \Rightarrow x_1 = 2\sqrt{z}\, x_2.$$

$$= 2z. \quad \boxed{x_1^* = \frac{2}{3}t}$$

as in (12) it is no surprise to find a unique PARETO optimum independent of the eq. price system in the Robinson Crusoe case.

Finally, $\frac{p_1}{p_2} = MRS_{12} = MRT_{12} \Rightarrow$

$$\boxed{\left.\frac{p_1}{p_2}\right|_{*} = \frac{1}{2}\sqrt{\frac{3}{t}}}$$ ← only the price ratio matters.

The Walrasian eq. is:

$$\left(p^*,\ \underbrace{x^* = (x_1^*, x_2^*)}_{\text{demand}},\ \underbrace{y^* = (-z^*, q^*)}_{\text{production plan}} \right).$$

The eq. profit is: $p^* \cdot y^* = \pi(p^*)$.

$$= -p_1^* z^* + p_2^* q^*$$

normalizing $p_2 = 1$.

$$= \frac{1}{2}\sqrt{\frac{3}{t}}\,\frac{t}{3} + \sqrt{\frac{t}{3}} = \frac{3}{2}\sqrt{\frac{t}{3}}$$

$$\boxed{\pi(p^*) = \frac{1}{2}\sqrt{3t}}$$