TD 15.

17 20 16 25

post exam.

17.

$L = 2 \quad I = 2 \quad J = 1.$

$\omega_a = (4, 1) \quad u_a(x_a) = x_{a1} x_{a2} \quad \theta_1 = 0.$
$\omega_b = (1, 1) \quad u_b(x_b) = x_{b1} + x_{b2}. \quad \theta_2 = 1.$

$Y = \{ y_2 \leq \sqrt{-y_1}, y_1 \leq 0 \}.$
$= \{ (-z, q) ; z \geq 0 \text{ and } q - f(z) \leq 0$ with $f(z) = \sqrt{z} \}.$

1) A competitive eq. $(p^*, x^* = (x_a^*, x_b^*), y^*)$ satisfies.

* utility max for both consumers, i.e. x_i^* solution to.

$$\begin{cases} \max u_i(x_i). \\ \text{with } p^* \cdot x_i \leq p^* \cdot \omega_1 + \theta_i p^* \cdot y^*. \end{cases} \quad \forall i = a, b.$$

* profit max for the firm. i.e. y^* sol to.

$$\begin{cases} \max p^* \cdot y. \\ y \in Y. \end{cases} \underset{f \text{ incr.}}{\iff} \begin{cases} \max\limits_{z \geq 0} p_q^* f(z) - p_z^* z \end{cases}$$

* market clearing for both goods. i.e. $x_a^* + x_b^* = \omega + y^*.$
with $\omega = \omega_a + \omega_b.$

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2) We start by finding y^* as a fct of $\frac{p_q^*}{p_z^*}$.
(since PMP does not depend on $x_i^* = a, b$.)

PMP

$$\frac{\partial\left(p_q^* f(z) - p_z^* \cdot z\right)}{\partial z} = \frac{p_q^*}{2\sqrt{z^*}} - p_z^* = 0.$$

$$\Longrightarrow \boxed{z^* = \left(\frac{p_q^*}{2 p_z^*}\right)^2}.$$

Demand for input.

and.

$$\boxed{q^* = \sqrt{z^*} = \frac{p_q^*}{2 p_z^*}}. \Longrightarrow \pi(p^*) = p^* \cdot y^*.$$

$$= \frac{p_q^{*2}}{2 p_z^*} - \frac{p_q^{*2}}{4 p_z^*}$$

$$\Downarrow$$

$$\boxed{y^* = (-z^*, q^*)}.$$

$$\Longrightarrow \boxed{\pi(p^*) = \frac{p_q^{*2}}{4 p_z^*}}.$$

UMP$_a$. Now, UMP for consumer. a.

$$\begin{cases} \max.\ u_a(x_a) = x_{a1} x_{a2}. \\ \text{s.th.}\ p^* \cdot x_a \le p^* w_a. = p_z^* 4 + p_q^* \end{cases}$$

Walras' law (from LNS).

$$p^* \cdot x_a^* = p^* \cdot \omega_a.$$

$$\Longleftrightarrow \boxed{\frac{p_j^*}{p_q^*} x_{a1}^* + x_{a2}^* = \frac{p_j^*}{p_q^*} 4 + 1.} \quad (*)$$

and KT condition for an interior solution.

$$\nabla u_a(x_a^*) = \lambda p^* .$$

$$\Longleftrightarrow \begin{pmatrix} x_{a2}^* \\ x_{a1}^* \end{pmatrix} = \lambda \begin{pmatrix} p_j^* \\ p_q^* \end{pmatrix}.$$

$$\Longrightarrow \frac{x_{a2}^*}{p_j^*} = \frac{x_{a1}^*}{p_q^*} \Longleftrightarrow \boxed{p_q^* x_{a2}^* = p_j^* x_{a1}^*}$$

$$\Longrightarrow x_{a2}^* = \frac{p_j^*}{p_q^*} x_{a1}^* \longrightarrow \text{injecting in } (*).$$

So,

$$\Longrightarrow 2 \frac{p_j^*}{p_q^*} x_{a1}^* = \frac{p_j^*}{p_q^*} 4 + 1.$$

$$\Longrightarrow \boxed{x_{a1}^* = 2 + \frac{1}{2} \frac{p_q^*}{p_j^*}}$$

and

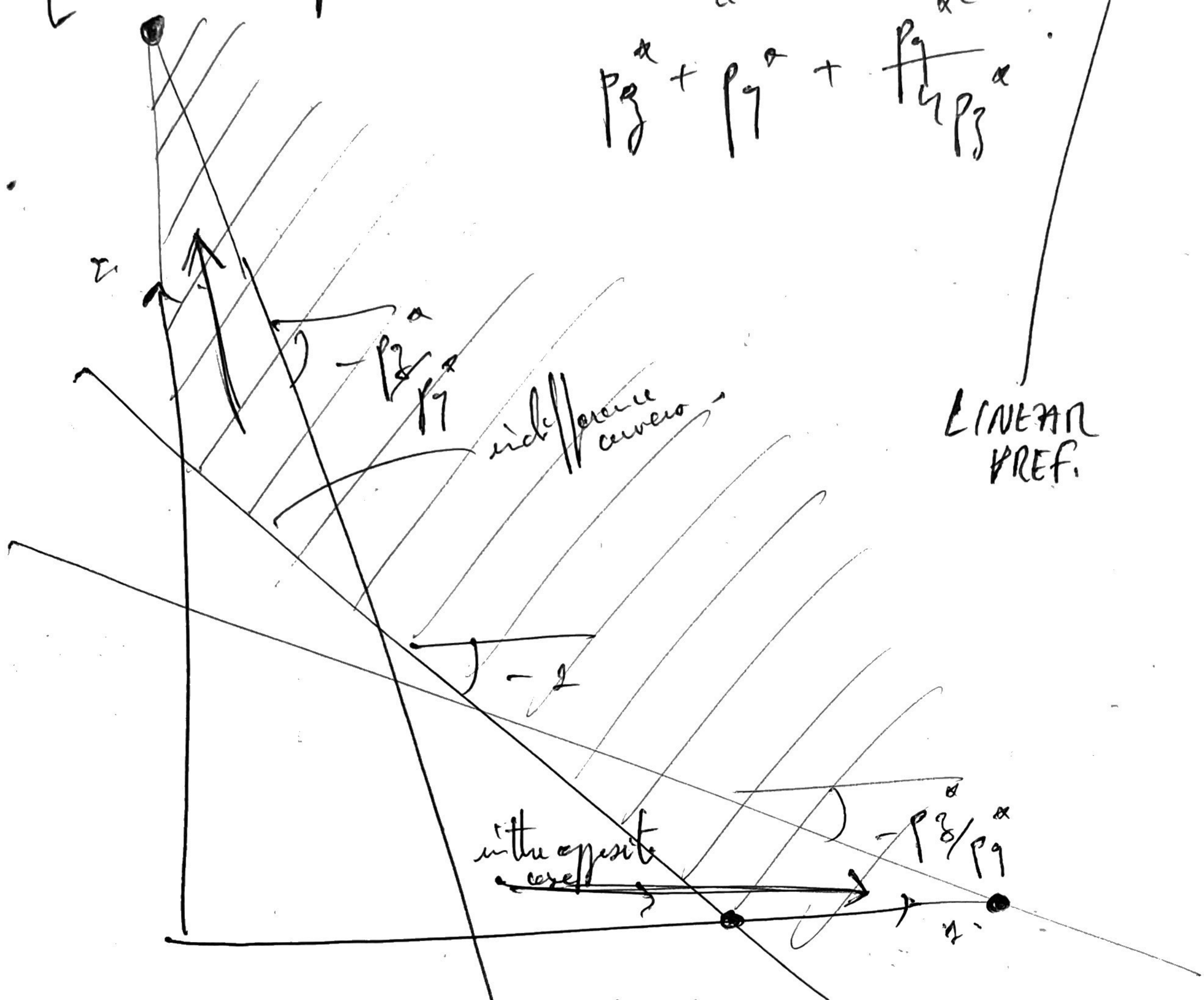
$$\boxed{x_{a2}^* = \frac{1}{2} + 2 \frac{p_j^*}{p_q^*}}$$

* Now for consumer: be the owner of the firm

$$\begin{cases} \max.\ u_1(x_1) = x_{11} + x_{12} \\ \text{s.th}\ \ p^*.x_1 \le \underbrace{p^*.\omega_1 + \pi(p^*)}_{=} \end{cases}$$

$$p_3^* + p_9^* + \frac{p_9^{*2}}{4p_3^*}$$

LINEAR PREF.



$\Longrightarrow$ in any case we have Walras' law.

$$\Longrightarrow\ p^*.x_2^* = p_3^* + p_9^* + \frac{p_9^{*2}}{4p_3^*}.$$

we have.

* if $\boxed{\frac{p_3^*}{p_9^*} < 1}$

$\Longrightarrow$ $x_f^* = (\text{all of } 1, \text{ none of } 2)$.

$$x_{f1}^* = \frac{p_3^* + p_9^* + \pi(p^*)}{p_3^*}.$$

$$\boxed{x_{f1}^* = 1 + \frac{p_9^*}{p_3^*} + \frac{1}{4}\left(\frac{p_9^*}{p_3^*}\right)^2} \text{ and } \boxed{x_{f2}^* = 0.}$$

* if $\boxed{\frac{p_3^*}{p_9^*} > 1.}$ $\Longrightarrow$ $x_f^* = (\text{none of } 2, \text{ all of } 2)$.

$$\Longrightarrow \quad x_{f2}^* = \frac{p_3^* + p_9^* + \pi(p^*)}{p_9^*}.$$

and $x_{f1}^* = 0$

$$\boxed{x_{f2}^* = 1 + \frac{p_3^*}{p_9^*} + \frac{1}{4}\frac{p_9^*}{p_3^*}.}$$

* if $\boxed{\frac{p_9^*}{p_3^*} = 1}$ then any x_f^* on the Budget line fits.

ie any x^* satisfying

$$p_3^* x_{f1}^* + p_9^* x_{f2}^* = p_3^* + p_9^* + \pi(p^*).$$

$$\Longrightarrow \quad x_{f2}^* = \frac{p_3^*}{p_9^*} + 1 + \frac{1}{4}\frac{p_9^*}{p_3^*} - \frac{p_3^*}{p_9^*} x_{f1}^*$$

$$\Longrightarrow \quad x_{b2}^* = 2 + \frac{1}{4} - x_{b1}^*.$$

$$\boxed{x_{b2}^* = \frac{9}{4} - x_{b1}^*}.$$

$\Longrightarrow$ So, the demand of consumer 2 as a fct of the price ratio $\frac{p_g^*}{p_g^*}$ is generally:

$$\boxed{x_b^* = \begin{cases} \left(0,\ 1 + \frac{p_3}{p_1} + \frac{1}{4}\frac{p_9}{p_3}\right) & \text{if } \frac{p_3}{p_1} > 1. \\ \left\{x_b = (x_{b1}, x_{b2}),\ x_{b2} = \frac{9}{4} - x_{b1} \text{ and } x_{b1} \in \left[0, \frac{9}{4}\right].\right\} & \text{if } \frac{p_3}{p_1} = 1. \\ \left(1 + \frac{p_9}{p_3} + \frac{1}{4}\left(\frac{p_9}{p_3}\right)^2,\ 0\right) & \text{if } \frac{p_3}{p_9} < 1. \end{cases} \quad (**)}$$

$\longrightarrow$ Now that we have both y^*, x_a^* and x_b^* as a fct of $\frac{p_3}{p_1}^*$.

We can find the equilibrium ratio by imposing the MARKET CLEARING CONDITION.

i.e. $x_a^* + x_b^* = \omega + y^*$.

$\longrightarrow$ enough to impose it for one of the two goods.

e.g.

$$x_{a1}^* + x_{b1}^* = 5 + (-z^*)$$

* Let's check if $\frac{p_q^*}{p_z^*} = 1$ is compatible with all these requirements?

in this case, $x_{a1}^* = 2 + \frac{1}{2} = \frac{5}{2}$.

$x_{b1}^* \in \left[0, \frac{9}{4}\right]$. and $z^* = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

$\Longrightarrow$ so the market clearing condition imposes $x_{b1}^* = 5 - \frac{1}{4} - \frac{5}{2} = \frac{9}{4}$.

COMPATIBLE.

$\longrightarrow$ we also have $x_{a2}^* = \frac{1}{2} + 2 = \frac{5}{2}$.

, $q^* = \frac{1}{2}$ and $x_{b2}^* = \frac{9}{4} - x_{b1}^* = 0$.

$$\Longrightarrow \boxed{\text{So } \left(\frac{p_q^*}{p_z^*} = 1,\ x^* = \left(x_a^* = \left(\frac{5}{2}, \frac{5}{2}\right), x_b^* = \left(\frac{9}{4}, 0\right)\right), y^* = \left(-\frac{1}{4}, \frac{1}{2}\right)\right) \text{ is a WALRASIAN EQUILIBRIUM.}}$$

* can there be an equilibrium with $\frac{p_g^*}{p_z^*} > 1$?

In this case we have:

$$x_f = \left(1 + \frac{p_g}{p_z} + \frac{1}{4}\left(\frac{p_g}{p_z}\right)^2, 0\right).$$

and $x_a^* = \left(2 + \frac{1}{2}\frac{p_g^*}{p_z^*}, \frac{1}{2} + 2\frac{p_z^*}{p_g^*}\right).$

let's look at the market clearing condition for good 2.

i.e. $x_{a2}^* + x_{f2}^* = 2 + q^*$.

i.e. $\frac{1}{2} + 2\frac{p_z}{p_g} + 0 = 2 + \frac{p_g}{2p_z}$.

$$\Longleftrightarrow 4\frac{p_z}{p_g} - \frac{p_g}{p_z} = 3 \Longleftrightarrow \left(\frac{p_g}{p_z}\right)^2 + 3\frac{p_g}{p_z} - 4 = 0$$

$\Delta = 9 + 4 \times 4 = 25.$

$$\Longrightarrow \frac{p_g}{p_z} = \frac{-3 \pm 5}{2}.$$

we only keep the positive root since $p \gg 0$.

$\Longrightarrow \frac{p_g}{p_z} = 1.$ which CONTRADICTS the initial hyp. that $p_g/p_z > 1$.

$\Longrightarrow$ So there CANNOT exist an eq. with $p_g/p_z > 1$!

* . now for $\frac{p_q^*}{p_z^*} < 1$.

$$\Rightarrow \quad x_f^* = \left(0 \ , \ 1 + \frac{p_z}{p_q} + \frac{1}{4}\frac{p_q}{p_z} \right).$$

the market clearing condit
for good 1 imposes .

$$x_{a1}^* + x_{f1}^* = S - z^* .$$

i.e. $2 + \frac{1}{2}\frac{p_q^*}{p_z^*} + 0 = S - \left(\frac{p_q}{2p_z} \right)^2$.

i.e. $\frac{1}{2}\frac{p_q}{p_z}\left(1 + \frac{1}{2}\frac{p_q}{p_z} \right) = 3$

$$x^2 + x - 3 = 0 . \Rightarrow \quad x_\pm = \frac{-1 \pm \sqrt{13}}{2} = \frac{1}{2}\frac{p_q}{p_z} .$$

we only keep the positive root.
(since $p >> 0$.)

$$\Rightarrow \quad \frac{p_q}{p_z} = -1 + \sqrt{13} .$$

do we have $-1 + \sqrt{13} < 1$.

i.e $\sqrt{13} < 2$.

i.e $13 < 4$. NO . CONTRADICTION.

$\rightarrow$ So, likewise, there cannot exist an equilibrium with $p_q/p_z < 1$ ....

$\Longrightarrow$ So the only eq. that exists is the one found on p. 7/

3) Is the equilibrium PARETO OPTIMAL ?

Yes, the 1st WELFARE THM. applies, because both u_i are strongly mono $\Rightarrow$ mono $\Rightarrow$ LNS.

4) The Pareto optima of this economy satisfy

$$P = \{ x = (x_a, x_b) \, ; \, x_a + x_b = \omega \text{ and } MRS^a_{12}(x_a) = MRS^b_{12}(x_b) \}.$$

$$MRS^a_{12}(x_a) = \frac{x_{a2}}{x_{a1}} \qquad MRS^b_{12}(x_b) = 1.$$

$$\Longrightarrow \boxed{P = \{ x = (x_a, \omega - x_a) \, ; \, x_{a1} = x_{a2} \}.}$$

it was indeed, of course, verified by the Walrasian eq that are found on p. 7/

$\boxed{20}$. Same setup. $L=2$ $I=2$ $J=1$.

Sub. $\omega_1 = (1,0)$ $u_1(x_1) = x_{11}x_{12}$

$\theta_1 = 0$.

and. $\omega_2 = (-,0)$ $u_2(x_2) = x_{21}x_{22}$.

$\theta_2 = 1$.

production set $Y = \{ y_2 \leq 2\sqrt{-y_1} \;;\; y_1 \leq 0 \}$

i.e. production fct. $f(z) = 2\sqrt{z}$.

$= \{(-z, q),\ z \geq 0, \text{ and } q - f(z) \leq 0.\}$.

1). Same as in 17.

2). Likewise, we first solve the PMP.

$$\left\{ \max_{z \geq 0} p_q^* f(z) - p_z^* z \right.$$

$$\Longrightarrow \frac{\partial (p_q^* f(z) - p_z^* z)}{\partial z}\Big|_{z^*} = 0 = \frac{p_q^*}{\sqrt{z^*}} - p_z^*.$$

$$\Longrightarrow \boxed{z^* = \left(\frac{p_q^*}{p_z^*}\right)^2} \text{ and. } \boxed{q^* = 2\frac{p_q^*}{p_z^*}}$$

$y^* = (-z^*, q^*)$

and the profit fct is $\Pi(p^*) = p^* \cdot y^* = p_q^* 2\frac{p_q^*}{p_z^*} - \frac{p_q^{*2}}{p_z^{*}}$

$$\Longrightarrow \boxed{\Pi(p^*) = \frac{p_q^{*2}}{p_z^*}}$$

* solve CMP_1. $\begin{cases} \max . u_1(x_1) = x_{11} x_{12} . \\ \text{s.th.}\ p^* . x_1 \leq p^* . \omega_1 = p_1^* \end{cases}$

$\Longrightarrow$ Walras' law. $\boxed{p^* . x_1^* = p_1^* \underset{(\ast)}{=} p_1^* x_{11}^* + p_2^* x_{12}^*}$

and KT condts for an interior solution.

$$\nabla u_1(x_1^*) = \lambda p^* = \lambda \begin{pmatrix} p_1^* \\ p_2^* \end{pmatrix}.$$

$$\begin{pmatrix} x_{12}^* \\ x_{11}^* \end{pmatrix}$$

$$\Longrightarrow \frac{x_{12}}{p_1} = \frac{x_{11}}{p_2}$$

$$\Longrightarrow \boxed{x_{11} p_2 = x_{12} p_2 .}$$

$\hookrightarrow$ injecting into Walras' law $(\ast)$ yields

$$\Longrightarrow 2 p_1^* x_{11}^* = p_1^* . \Longrightarrow \boxed{x_{11}^* = \frac{1}{2}}$$

and. $x_{12}^* = \dfrac{p_1^*}{p_2^*} x_{11}^*$.

So x_1^* as a fct of the ratio of the prices is. $\boxed{x_1^* = \left(\frac{1}{2} ; \frac{1}{2} \frac{p_1^*}{p_2^*}\right).}$

* for consumer 2, the ~~capitalists~~ we have.

$$\text{UMP}_2 \begin{cases} \max u_2(x_2) = x_{21} x_{22}. \\ \text{s.th.} \; p^* \cdot x_2 \leq \underbrace{0 + 1 \times \Pi(p^*)}_{c}. \end{cases}$$

$\Downarrow$

$$\frac{p_2^{*2}}{p_1^*} = \frac{p_2^{*2}}{p_1^*}.$$

KT yields

$$\boxed{x_{21} p_1 = x_{22} p_2.}$$

While Walras' law yields

$$\boxed{p_1 x_{21} + p_2 x_{22} = p_2^2 / p_1}.$$

injecting KT into Walras' yields

$$2 p_1 x_{21} = p_2^2 / p_1 \longrightarrow \boxed{x_{21}^* = \frac{1}{2}\left(\frac{p_2^*}{p_1^*}\right)^2.}$$

and $\boxed{x_{22}^* = x_{21}^* \frac{p_1^*}{p_2^*} = \frac{1}{2} \frac{p_2^*}{p_1^*}}$.

$$\longrightarrow \boxed{x_2^* = \left(\frac{1}{2}\left(\frac{p_2^*}{p_1^*}\right)^2 ; \frac{1}{2} \frac{p_2^*}{p_1^*}\right).}$$

Now finally imposing Market Clearing will yield the value of $\frac{p_1^*}{p_2^*}$ a at the price equilibrium.

i.e. $x_1^* + x_2^* = \omega + y^*$

Let's consider the clearing of the market associated with good 2.

$$x_{12}^* + x_{22}^* = 0 + q^* = 2\left(\frac{p_2^*}{p_1^*}\right)$$

$$\frac{1}{2}\frac{p_1}{p_2} + \frac{1}{2}\frac{p_2}{p_1}$$

$$\Longrightarrow \left(\frac{p_1}{p_2}\right)^2 = 3 \longrightarrow \boxed{\frac{p_1^*}{p_2^*} = \sqrt{3}}$$

$\rightarrow$ So the Walrasian eq. is unique and given by.

$$\boxed{\left(\frac{p_1^*}{p_2^*} = \sqrt{3},\ x_1^* = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right);\ x_2^* = \left(\frac{1}{6}, \frac{1}{2\sqrt{3}}\right);\ y^* = \left(-\frac{1}{3}, \frac{2}{\sqrt{3}}\right)\right)}$$

2) bis) $\boxed{I = 2\ ,\ J = 1.}$

Two constraints at the same time! inequality

TARETO SET!

$$\begin{cases} \max_{x_1} u_1(x_1) \\ \text{s.th.} \quad u_2(x_2) \geq u^* \\ F(y) \leq 0 \\ x_1 + x_2 = \bar{\omega} + y \end{cases}$$

minimum level of utility for 2.

feasibility of production

choose x_1 and x_2 and y!

$\Downarrow$

monotonicity $\Longrightarrow$ LNS

$\Longrightarrow$ $\begin{cases} u_2(x_2^*) = u^* \\ \text{and} \\ F(y^*) = 0 \Longleftrightarrow \text{or } q^* = f(z^*). \end{cases}$

AND KT cndts.

$$\begin{cases} (1) \quad \nabla_{x_1} u_1(x_1^*) = -\lambda \nabla_{x_1} u_2(x_2^*) \overset{\text{chain rule}}{=} +\lambda \nabla_{x_2} u_2(x_2^*). \\ \text{and} \\ (2) \quad \nabla_{x_2} u_1(x_1^*) = \lambda' \nabla_{x_2} F(y^*) \overset{\text{chain rule}}{=} \lambda' \nabla_y F(y^*) \end{cases}$$

w.r.t 1st constraint

w.r.t 2nd constraint

$\Longrightarrow$ (1) yields:

$$\boxed{MRS^1_{12}(x_1^*) = MRS^2_{12}(x_2^*)}.$$

and (2) yields:

$$\boxed{MRS^2_{12}(x_2^*) = MRT_{12}(y^*)}.$$

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application with $u_1(x_1) = x_{11}x_{12}$,
$u_2(x_2) = x_{21}x_{22}$.

and $f(z) = 2\sqrt{z}$.

$$\frac{1}{\sqrt{-y_1^*}}$$

$$F(y) = y_2 - f(-y_1).$$

$$MRS^1_{12}(x_1^*) = MRS^2_{12}(x_2^*) = MRT_{12}(y^*) = \frac{\frac{\partial F}{\partial y_1}(y^*)}{\frac{\partial F}{\partial y_2}(y^*)}$$

$$\frac{\partial F}{\partial y_2}(y^*) = 1.$$

$$MRT_{12}(y^*) = \frac{1}{\sqrt{-y_1^*}}$$

$\Updownarrow$

$$\boxed{\frac{x_{12}^*}{x_{11}^*} = \frac{x_{22}^*}{x_{21}^*}}\ \boxed{= \frac{1}{\sqrt{z^*}}}$$

... the Pareto set verifies.

$$\boxed{\frac{x_{12}}{x_{11}} = \frac{x_{22}}{x_{21}} = \frac{1}{\sqrt{z}}}$$

$F(y) = 0$ i.e. $\boxed{q = f(z).}$

$u_2(x_2) = u^*$ i.e. $\boxed{x_{21} x_{22} = u^*}$

and.

$x_1 + x_2 = \bar{\omega} + y.$

$$\Longrightarrow \left\{ \boxed{\begin{array}{l} x_{11} + x_{21} = \bar{\omega}_1 - z. \\ \text{and } x_{12} + x_{22} = \bar{\omega}_2 + q = 2\sqrt{z} + \bar{\omega}_2. \end{array}} \right.$$

...

for simplification, like in (20).

$\bar{\omega}_1 = 1 \quad , \bar{\omega}_2 = 0.$

$\Longrightarrow$

$$x_{12} = x_{11} \frac{x_{22}}{x_{21}}.$$

$$= x_{11} \frac{\bar{\omega}_2 + 2\sqrt{3} - x_{12}}{\bar{\omega}_1 - \gamma - x_{11}.} = x_{11} \frac{\left(\omega_2 + 2\frac{x_{11}}{x_{12}} - x_{12}\right)}{\left(\bar{\omega}_1 - \left(\frac{x_{11}}{x_{12}}\right)^2 - x_{11}\right)}$$

$$\Longleftrightarrow x_{12}\bar{\omega}_1 - \frac{x_{11}^2}{x_{12}} - \cancel{x_{11}x_{12}} = x_{11}\bar{\omega}_2 + 2\frac{x_{11}^2}{x_{12}} - \cancel{x_{11}x_{12}}$$

$$\Longleftrightarrow \boxed{x_{12}^2\bar{\omega}_1 - x_{12}\bar{\omega}_2 x_{11} - 3x_{11}^2 = 0}$$

$$\Longrightarrow x_{12}^2 = 3 x_{11}^2 \longrightarrow \boxed{x_{12} = \sqrt{3}\, x_{11}}$$

$$\longrightarrow \frac{x_{12}}{x_{11}} = \sqrt{3} = \frac{1}{\sqrt{\gamma}} \longrightarrow \boxed{\gamma = \frac{1}{3}} \; ; \; \boxed{q = \frac{1}{\sqrt{3}}}$$

$$\frac{x_{22}}{x_{21}} = \frac{x_{12}}{x_{11}} = \sqrt{3} \Longrightarrow x_{22} = \sqrt{3}\, x_{21}$$

and $x_{21} = 1 - \gamma - x_{11} = \boxed{\frac{2}{3} - x_{11} = x_{21}}$

$$x_{22} = \sqrt{3}\, x_{21} = \boxed{\frac{2}{\sqrt{3}} - \sqrt{3}\, x_{11}. = x_{22}}$$

So the Pareto set is:

$$\boxed{P = \left\{ x_1 = (x_{11}, \sqrt{3}\, x_{11}), x_2 = \left(\frac{2}{3} - x_{11}, \frac{2}{\sqrt{3}} - \sqrt{3}\, x_{11}\right) ; \; x_{11} \in \left[0, \frac{2}{3}\right] \right\}}$$

s.t. x_{21} and $x_{22} \geq 0$

3). Discuss the validity of both Welfare thm.

LNS $\Longrightarrow$ 1st Welfare thm applies.

thus the Walrasian eq. found in 2). is Pareto optimal.

$\hookrightarrow$ indeed we can compare with P.: it corresponds to $x_{11} = \frac{d}{2}$.

u_1, u_2, f all strictly concave.

$\Longrightarrow$ 2ND WELFARE THM applies.

$\Rightarrow$ Thus any

$$x, y \in \underset{\shortparallel}{P}$$

$$\left\{ x_1 = (x_{11}, \sqrt{3}x_{11}), x_2 = \left(\frac{2}{3} - x_{11}, \frac{2}{\sqrt{3}} - \sqrt{3}x_{11}\right); \quad y = \left(-\frac{1}{3}, \frac{1}{\sqrt{3}}\right) \right\}.$$

is SUPPORTABLE as a PRICE EQUILIBRIUM WITH TRANSFERS.

4) Find the competitive eq with transfers s.th. both consumers consume the same amount of output.

i.e. we want to find T s.th.

$$\begin{cases} x^*, y^* \in P \\ x^*_{12} = x^*_{22}. \end{cases}$$

$$\Leftrightarrow \quad \sqrt{3}.$$

and w.r.t. p^*, the price system at eq.

$$MRS^1_{12}(x^*_1) = MRS^2_{12}(x^*_2) = MRT_{12}(y^*)$$

$$\shortparallel$$

$$p^*_1 / p^*_2.$$

$$\Longrightarrow \boxed{p_1^* / p_2^* = \sqrt{3}}.$$

and ... th.

$$\begin{cases} p^* \cdot x_1^* = p_1^* + T \\ p^* \cdot x_2^* = p^* \cdot y^* - T. \end{cases}$$

$$\Pi(p^*) = \frac{p_2^{*2}}{p_1}.$$

$x^*, y^* \in P$

and $x_{12}^* = x_{22}^* \Longrightarrow \sqrt{3}\, x_{11}^* = \frac{2}{\sqrt{3}} - \sqrt{3}\, x_{11}^*$.

$$\Longleftrightarrow 2\sqrt{3}\, x_{11}^* = \frac{2}{\sqrt{3}}$$

$$\Longrightarrow \boxed{x_{11}^* = \frac{1}{3}}$$

$$\boxed{x_{12}^* = \frac{1}{\sqrt{3}}}$$

$\Longrightarrow \boxed{x_{22}^* = \frac{1}{\sqrt{3}}}$ and $\boxed{x_{21}^* = \frac{1}{3}}$

$$p^* \cdot x_1^* = p_1^* x_{11}^* + p_2^* x_{12}^*$$

$$= p_1^* + T.$$

$$\longrightarrow \quad x_{11}^* + \frac{p_2^*}{p_1^*} x_{12}^* = 1 + \frac{T}{p_1}.$$

$$\frac{1}{3} + \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}} = \frac{2}{3}.$$

$$\Longrightarrow \quad \boxed{\frac{T}{p_1} = -\frac{1}{3}.}$$

more money from the capitalist lol.

8) bis) Could the transfer be implemented by changing endowments / ownership?

YES, e.g. for ownership. $T = \theta_1' \pi(p^*)$

additional share of the firm given to 1.

$$\theta_1' = \frac{T}{\pi(p^*)} = \frac{p_1 \left(-\frac{1}{3}\right)}{p_2^2 / p_1}$$

$$\theta_1' = -\frac{1}{3}\frac{p_1^2}{p_2^2} = -\frac{1}{3} 3.$$

$\Rightarrow$ $\boxed{\theta_1' = -1.}$

$\longrightarrow$ So after all, No, it would not be possible by transferring ownership of the firm because $0 - 1 = -1 < 0 \ldots$

$\square$.

$\boxed{\begin{array}{l} \textbf{(16)} \text{ Edgeworth box } \quad L=2, \; I=2. \\ \omega_1 = (2,0) \quad u_1(x_1) = x_{11}^{\alpha} + x_{21}^{\alpha} \\ \omega_2 = (1,3) \quad u_2(x_2) = x_{12}x_{22} \end{array}}$

1). $\alpha = \frac{1}{2}$. Find the competitive equilibria and Pareto set.

Pareto set.

$$P = \left\{ x = (x_1, x_2) \; ; \; \begin{array}{l} MRS^1_{12}(x_1) = MRS^2_{12}(x_2) \\ \text{and } x_1 + x_2 = \overline{\omega} \end{array} \right\}$$

$$MRS^{1}_{12}(x_1) = \frac{\cancel{\alpha}\, x_{11}^{\alpha-1}}{\cancel{\alpha}\, x_{12}^{\alpha-1}}.$$

for $\alpha = \frac{1}{2} \longrightarrow$ $\boxed{MRS^{1}_{12}(x_1) = \sqrt{\frac{x_{12}}{x_{11}}}}$

and $\boxed{MRS^{2}_{12}(x_2) = \frac{x_{22}}{x_{21}}}$ //

$x_1 + x_2 = \bar{\omega} = \omega_1 + \omega_2 = (3,3) \Longrightarrow \boxed{\begin{cases} x_{11} + x_{21} = 3 \\ \text{and} \\ x_{12} + x_{22} = 3 \end{cases}}$

$$\frac{x_{22}^2}{x_{21}^2} = \frac{x_{12}}{x_{11}}.$$

$$\Rightarrow x_{12} = x_{11}\frac{x_{22}^2}{x_{21}^2} = x_{11}\frac{(3-x_{12})^2}{(3-x_{11})^2}.$$

$$\Leftarrow \frac{(3-x_{11})^2}{x_{11}} = \frac{(3-x_{12})^2}{x_{12}}.$$

$\Longrightarrow$ solved by : $\boxed{x_{12} = x_{11}}$

$\Rightarrow$ yields $\boxed{x_{22} = x_{21}}$.

$\Rightarrow$ $x_{21} = 3 - x_{11}$

and $x_{22} = 3 - x_{12} = 3 - x_{11}$.

$\Rightarrow$ The Pareto set is:

$$\boxed{P = \left\{ x_1 = (x_{11}, x_{11}),\ x_2 = (3 - x_{11}, 3 - x_{11});\ \text{with } x_{11} \in [0, 3] \right\}.}$$

The competitive equilibrium, in addition, depends on the initial distribution of the total endowment and determines the ratio of the prices.

p^*, x^* is a Walrasian eq.

iff. $x^* \in P$ and $\left\{ \begin{array}{l} p^* \cdot x_1^* = p^* \cdot \omega_1 \\ \text{and } p^* \cdot x_2 = p^* \cdot \omega_2 \end{array} \right.$

$MRS^1_{12}(x_1^*) = MRS^2_{12}(x_2^*)$ and p_1^*/p_2^*

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$$\Longrightarrow \quad MRS^1_{12}(x_1^*) = MRS^2_{12}(x_2^*) = 1.$$

$$\Updownarrow$$

$$p_1^* / p_2^* \Longrightarrow \boxed{\frac{p_1^*}{p_2^*} = 1}$$

$$p^* \cdot x_1^* = p^* \cdot \omega_1 = p_1^* \, 2. \qquad x^* \in P.$$

$$\Updownarrow$$

$$p_1^* x_{11}^* + p_2^* x_{12}^* \longrightarrow x_{11}^* \left(1 + \frac{p_2^*}{p_1^*}\right) = 2.$$

$$\longrightarrow \boxed{x_{11}^* = 1.}$$

$\Longrightarrow$ So the Walrasian eq. satisfies:

$$\boxed{\left\{ \frac{p_1^*}{p_2^*} = 1,\ x_1^* = (1,1),\ x_2^* = (2,2) \right\}}$$

2) Both Welfare thms apply because u_1 and u_2 strongly mono $\Longrightarrow$ LNS for the 1st thm and strictly concave for the 2nd one.

3) We want to find a transfer of the initial endowments of commodity 2 s.th.

$$\boxed{x_{11}^* = x_{21}^*}$$

i.e. we need to solve:

$$\begin{cases} x^* \in P. \\ x_{11}^* = x_{21}^* \\ MRS = \frac{p_1^*}{p_2^*} \\ p^* \cdot x_1^* = p_1^* \omega_1 + p_2 T \\ p^* \cdot x_2^* = p_1^* \omega_2 - p_2 T. \end{cases}$$

transfer of some of good 2 from consumer 2 to consumer 1.

$$\boxed{p_1^*/p_2^* = 1.}$$

$$x_{11} = x_{21}.$$

$$\Downarrow$$

$$x_{11} = 3 - x_{11}.$$

$$\Longrightarrow \boxed{x_{11}^* = \frac{3}{2}} \boxed{= x_{12}^* = x_{21}^* = x_{22}^*.}$$

$$p_1^* x_{11}^* + p_2^* x_{12}^* = p_1^* 2 + p_2^* T.$$

$$\Longleftrightarrow x_{11}^* + \frac{p_2}{p_1} x_{12} = 2 + \frac{p_2}{p_1} T.$$

$$\Longrightarrow T = 3 - 2 = 1.$$

possible!

$$\boxed{T = 1.}$$

we would have new endowments $\omega_1' = (2,1)$ and $\omega_2' = (1,2)$

3) biv). Is it possible using a transfer of initial endowments in good 1?

$\Rightarrow$ we solve.
$$\begin{cases} x^* \in P \\ x_{11}^* = x_{21}^* \\ MRS = p_1/p_2 \\ p \cdot x_1 = p \cdot \omega_1 + p_1 T \\ p \cdot x_2 = p \cdot \omega_2 - p_2 T \end{cases}$$

Otherwise. $x_{11} = \frac{3}{2} = x_{12} = x_{21} = x_{22}$.

$p_1/p_2 = 1$.

but. $p_1 x_{11} + p_2 x_{12} = p_1 (2 + T)$.

$$\Rightarrow \underbrace{x_{11} + \frac{p_2}{p_1} x_{12}}_{= 3} = 2 + T.$$

$\Rightarrow \boxed{T = 1}$ also possible because $\omega_2 = (1, 3)$.

$\Rightarrow$ we would have new endowments. $\omega_1' = (3, 0)$ and $\omega_2' = (0, 3)$

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