FINAL EXAM.

2019. **Exercise 1** Done during the tutorials → do it again! = Exercise 32 in the problem sheet.

Exercise 2. — $L=2$. $J=1$. we did it @ you have the solution

two possible technologies:

$$f_1(z)=\sqrt{z} \quad \text{and} \quad f_2(z)=z. \quad z\geq 0.$$

1) $f_1(0.5)=\sqrt{0.5} > 0.5 = f_2(0.5)$.
→ it should choose the first tech. to maximize the quantity of output

2) oppositely, $f_1(2)=\sqrt{2}<2=f_2(2)$.
→ chose the second one.

3) in general, the maximal quantity of output the firm will be able to produce is:

$$f : z \longmapsto \begin{cases} \sqrt{z} & \text{if } 0\leq z\leq 1. \\ z. & \text{if } z\geq 1. \end{cases}$$

4) The production set of the firm is:

$$Y=\{(-z,q) ; q-f(z)\leq 0 \text{ and } z\geq 0\}$$

→ Graphically...

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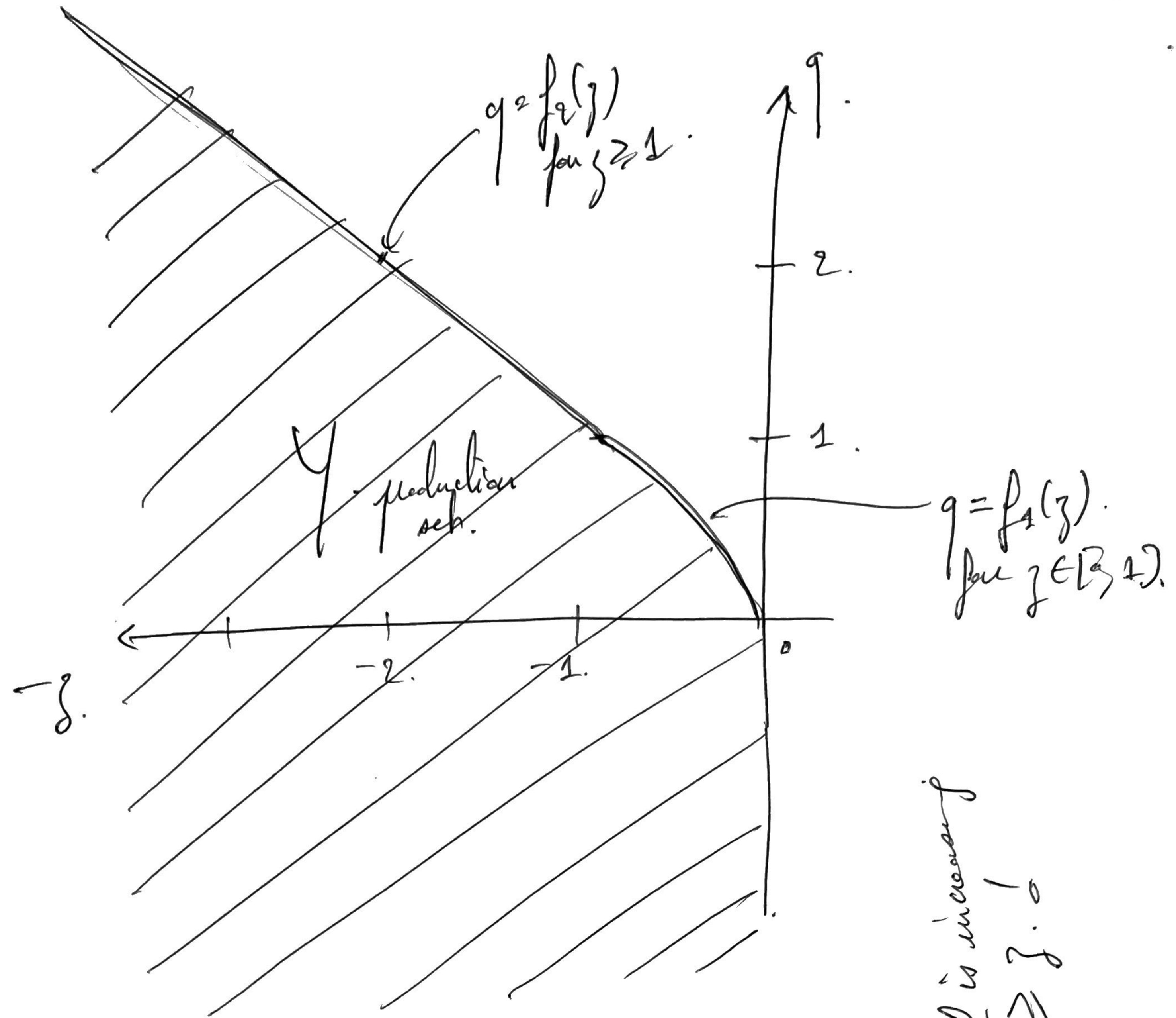


5). Y satisfies free disposal. iff.

$$Y - \mathbb{R}^L_+ \subset Y.$$

i.e.

let $\underset{\shortparallel}{y} \in Y$ and $\underbrace{\delta}_{\geq 0} \in \mathbb{R}^L_+$. Show that $\overset{y'}{\overset{\shortparallel}{y - \delta}} \in Y$.

$(-z, q)$. $\qquad = (\delta_1, \delta_2)$.

$$\vec{y'} = (-z', q') \text{ with } z' = \overbrace{z}^{\geq 0} + \overbrace{\delta_1}^{\geq 0} \geq 0.$$

and $q' = q - \delta_2 = f(z) \; \underbrace{-\delta_2}_{\geq 0} \leq f(z) \leq f(z')$ □.

because f is increasing and $z' \geq z$.

so $y' \in Y$ □.

Exercise 3 : Robinson Crusoe. $L=2$ $I=1$ $J=1$.

$f(z)=\alpha\sqrt{z}$ with $z\geq 0$. $\alpha\geq 0$.

$u(x)=x_1x_2$. $e=(12,0)$ $\theta=1$.

1) Production set is :

$$Y=\{(-z,q)\ ;\ q-f(z)\leq 0 \text{ and } z\geq 0\}.$$

2) The supply correspondence (or fct if single valued) is the correspondence noted y that associate to a price system $p\gg 0$, the set of solutions $y(p)$ to the profit maximization problem (PMP).

$$\begin{cases}\max\limits_{y\in Y} p\cdot y\end{cases} \Longrightarrow y(p)=\operatorname{argmax}_{y\in Y} p\cdot y.$$

The profit fct is the value fct. of the PMP $\longrightarrow$ $\Pi(p)=\max\limits_{y\in Y} p\cdot y$

3). The Walrasian / competitive / price equilibrium of this economy is the price system, allocation and production plan (p^*,x^*,y^*) satisfying :

(i) The consumer is maximizing their utility given their budget constraint.

i.e. $x^*\succsim x$ $\forall x\in B_p{}^{\alpha}(e,y^*)$.

$\iff \quad x^* \in x(p^*, e, y^*)$.

ie x^* is a solution to the UMP.

$$\begin{cases} \max\ u(x) \\ p^*\cdot x \le \underbrace{p^*\cdot e}_{\text{value of initial endowments}} + \underbrace{p^*\cdot y^*}_{\text{profits}} \end{cases}$$

$\theta = 1$!

(ii)* The firm maximizes its profit given its technological constraint.

ie. $p^*\cdot y^* \ge p^*\cdot y \quad \forall y \in Y$.

$\iff$ y^* is a solution to the PMP.

$$\begin{cases} \max\limits_{y\in Y} p^*\cdot y. \end{cases}$$

(iii)* Market clearing conditions. (ie no excess demand nor excess supply for any goods)

$$x^* = e + y^*.$$

$p^* = (s^*, 1)$. ie we normalize the price of the output commodity.

price of the input.

4) PMP. $\iff \begin{cases} \max\limits_{z\ge 0} f(z) - s^*\cdot z \end{cases}$

comes from the monotonicity of f thus F. etc.

$/\ q \le f(z)$

$\oplus$ $q = f(z)$ on prod. frontier...

ie. $\dfrac{\partial}{\partial z}\left(f(z) - s^*\cdot z\right)\Big|_* = 0 = \dfrac{1}{2\sqrt{z^*}} - s^*$.

$\Longrightarrow \quad z^* = \left(\dfrac{\alpha}{2s^*}\right)^2$

$\Longrightarrow \quad q^* = f(z^*) = \dfrac{\alpha^{②}}{2s^*}$ $\quad = \alpha\sqrt{z^*}$

always, i.e. when maximizing profit, there is NO WASTE.

So the supply fct is.

$$\boxed{\begin{aligned} y(\vec{p}^*) &= (-z^*, q^*) \\ &= \left(-\left(\frac{\alpha}{2s^*}\right)^2, \frac{\alpha^{②}}{2s^*}\right) \end{aligned}}$$

and the profit fct is. $\Pi(\vec{p}^*) = \vec{p}^* \cdot y^*$

$$\Pi(p^*) = \frac{\alpha^{②}}{2s^*} - s^*\left(\frac{\alpha}{2s^*}\right)^2 = \boxed{\frac{\alpha^2}{4s^*} = \Pi(p^*)}$$

5). UMP. $\begin{cases} \max u(x) & = x_1 x_2 \\ p^* \cdot x \le p^* \cdot e + \Pi(p^*). \end{cases}$

we maximize w.r.t $\underline{x}$.

$\Longrightarrow \exists\, x^*$ sol^n to UMP satisfies Walras' law (because u strongly mono $\Rightarrow$ mono $\Rightarrow$ LNS). and the Kuhn-Tucker condition.

i.e. (1) $\quad p^* \cdot x^* = p^* \cdot e + p^* \cdot y^*$ $\quad = \Pi(p^*)$.

and (2) KT. $\quad x^* \gg 0 \longrightarrow \boxed{\nabla u(x^*) = \lambda \vec{p}^*.}$

interior sol^n.

$$\nabla u(x^*) = \begin{pmatrix} \frac{\partial u}{\partial x_1}(x^*) \\ \frac{\partial u}{\partial x_2}(x^*) \end{pmatrix} = \begin{pmatrix} x_2^* \\ x_1^* \end{pmatrix} = \lambda \underbrace{\begin{pmatrix} s^* \\ 1 \end{pmatrix}}_{= p^*}.$$

$$\Longrightarrow \lambda = x_1^*.$$

$$\Longrightarrow x_2^* = \lambda s^* \longrightarrow \boxed{x_2^* = x_1^* s^*.}$$

AND. Walras' law.

$$p^* \cdot x^* = s^* x_1^* + x_2^* = 2x_2^*$$

$$= p^* \cdot e + \pi(p^*).$$

$$= s^* 12 + \frac{\alpha^2}{4s^*}$$

$e = (12, 0)$

$$\Longrightarrow \boxed{x_2^* = 6s^* + \frac{\alpha^2}{8s^*}}$$

$$\boxed{x_1^* = \frac{x_2^*}{s^*} = 6 + \frac{\alpha^2}{8{s^*}^2}}$$

$$x(p^*, e, y^*) = \boxed{x^* = (x_1^*, x_2^*)}$$

$\square$.

6) Finally, we impose market clearing w.r.t. each of the ~~two~~ goods.

$$\longrightarrow \quad x^* = e + y^*.$$

$$x_1^* = 12 - z^* = 12 - \left(\frac{\alpha}{2s^*}\right)^2.$$

and $\quad x_2^* = 0 + q^* = \dfrac{\alpha^2}{2s^*}.$

$\longrightarrow$ so we have. $x_1^* = \boxed{6 + \dfrac{\alpha^2}{8s^{*2}} = 12 - \left(\dfrac{\alpha}{2s^*}\right)^2}$

and.

$$x_2^* = \boxed{6s^* + \frac{\alpha^2}{8s^*} = \frac{\alpha^2}{2s^*}}.$$

as expected both yield the same result, i.e.

$$12s^{*2} - \frac{3}{4}\alpha^2 = 0.$$

$$\longrightarrow \boxed{s^* = \frac{\alpha}{4}.}$$

so. $\boxed{x_2^* = 2\alpha = q^*.}$

$\boxed{z^* = 4}$ and. $\boxed{x_1^* = 12 - z^* = 8.}$

7).

→ So $\frac{\partial x_1^*}{\partial \alpha} = 0$, i.e. x_1^* does not depend on α!

→ The utility level at equilibrium is

$$u(x^*) = x_1^{*\alpha} x_2^{*\alpha}$$
$$= 8 \times 2\alpha.$$

→ $\boxed{u^* = u(x^*) = 16\alpha.}$

So $\frac{\partial u^*}{\partial \alpha} = 16 > 0$. i.e. u^* increasing in α.

Exercise 9. $L = 2$ $I = 2$.

$u_1(x_1) = x_{11}$ $u_2(x_2) = x_{22}$.

total resources of the economy. $r = (r_1, r_2) = (3, 4)$.

1) The indifference curves of the 1st consumer are the

$$I_1(y_1) = \{x_1;\ u_1(x_1) = u(y_1)\}.$$
$$= \{x_1;\ x_{11} = \underbrace{y_{11}}_{cst}\}.$$

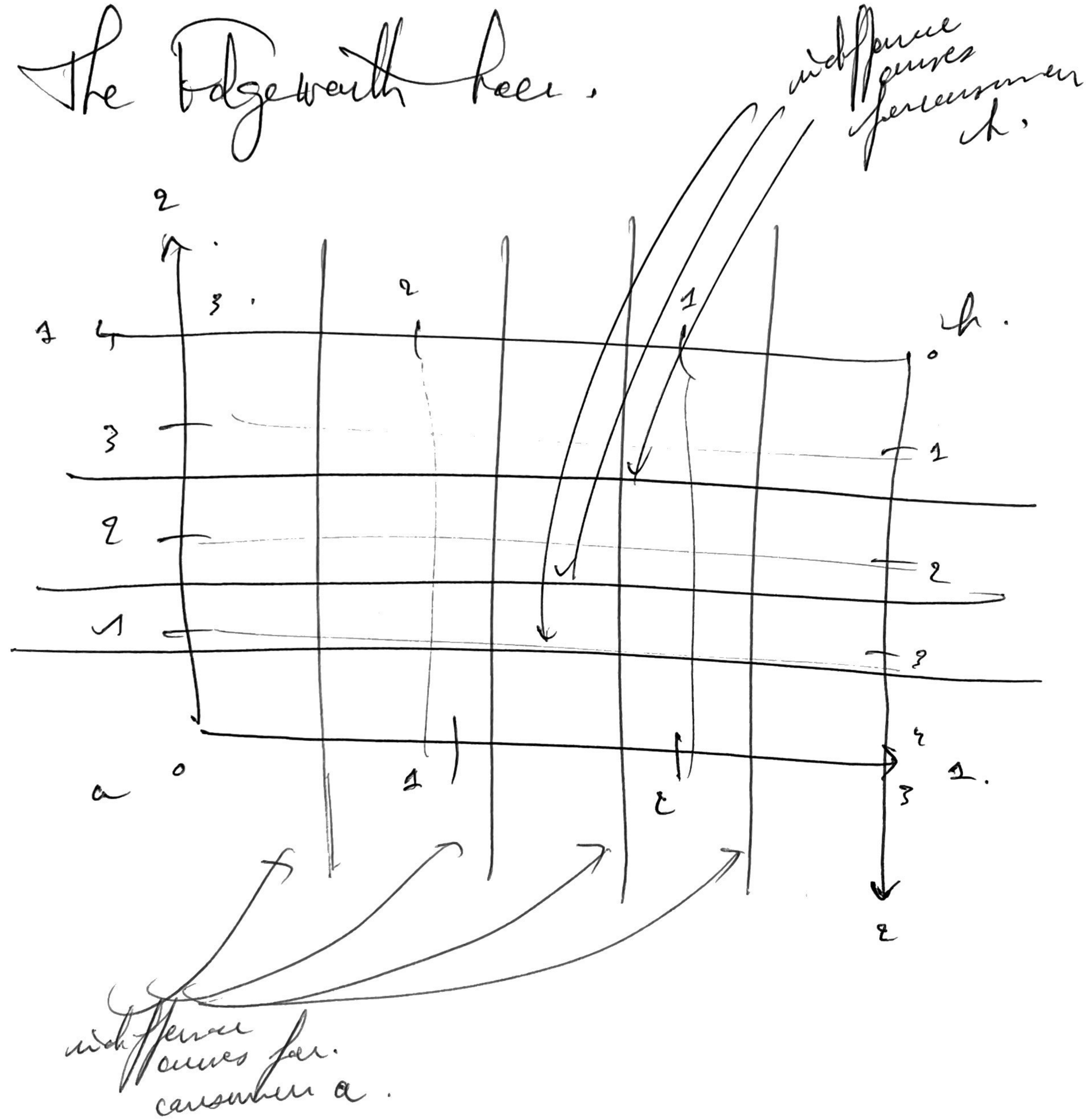
→ i.e. vertical lines.

Likewise $I_2(y_2) = \{ x_2 \; ; \; u_2(x_2) < u_2(y_2) \}$.

$\Rightarrow$ horizontal lines.

be $x_{22} = \underbrace{y_{22}}_{\text{cste}}$.

The Edgeworth box.

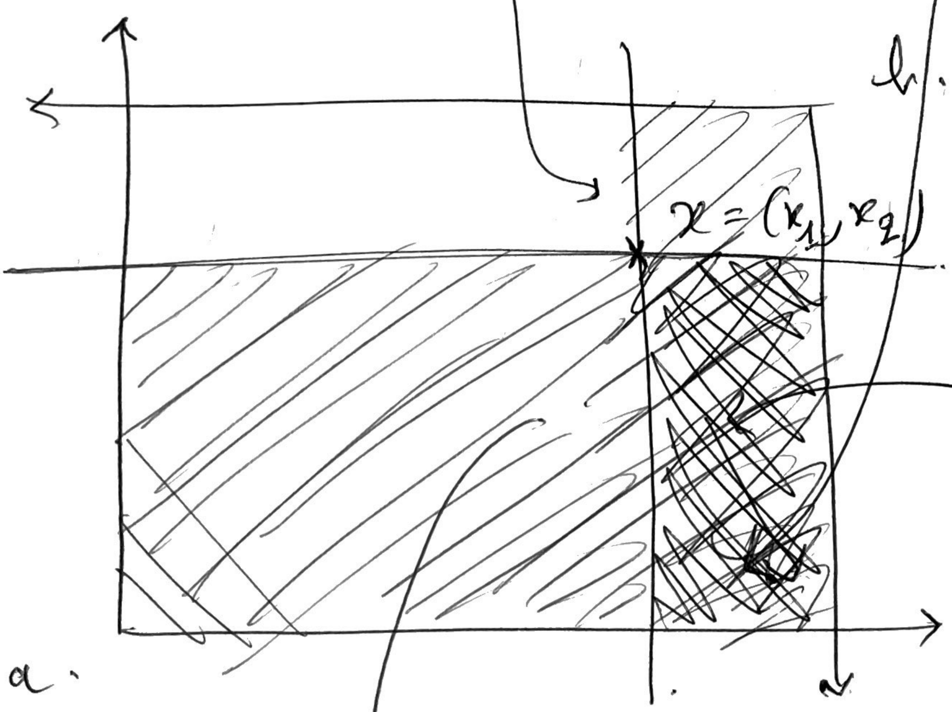


2). A Pareto optimal allocation is an allocation s.th. it is impossible to make one consumer strictly better off without making another one less well off.

For this economy, it writes:

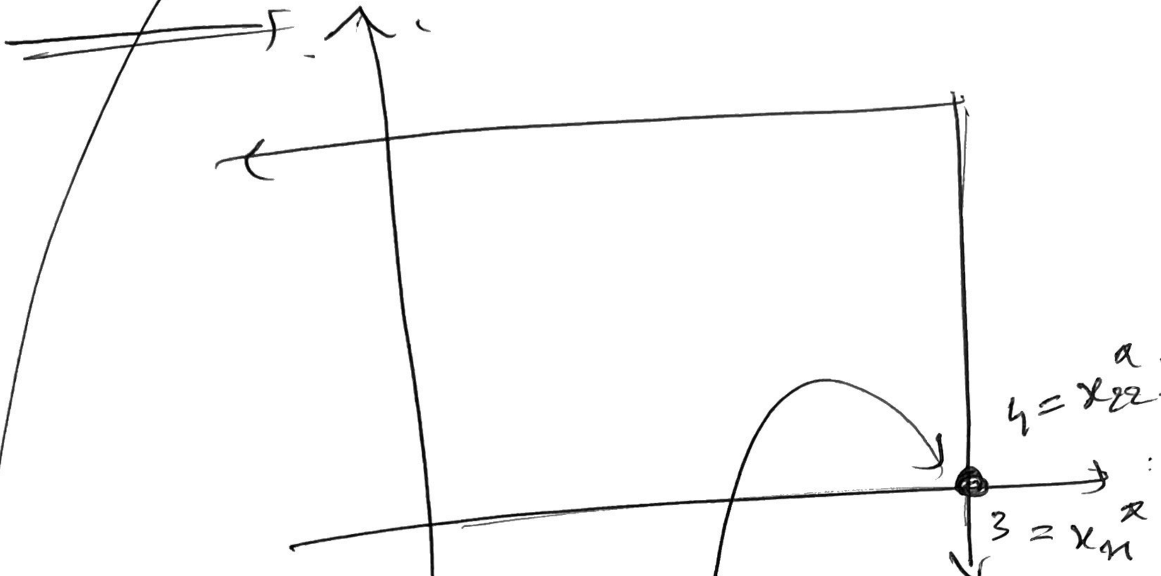
$$\boxed{\begin{array}{l} x^* = (x_1^*, x_2^*) \text{ is PARETO OPTIMAL.} \\ \text{iff } \nexists x' \text{ s.th.} \\ \left(\forall i=1,2,\ x_i' \succsim x_i^* \text{ and } \exists i \in \{1,2\},\ x_i' \succ x_i^*\right). \end{array}}$$

3) Graphically given a pt. this all set, intersection of "$U_1(x_1) \cap U_2(x_2)$". i.e. PARETO DOMINATES x.



x^* is PARETO OPTIMAL.

iff. "$U_1(x_1^*) \cap U_2(x_2^*)$" is a SINGLETON.



$$\boxed{x^* = (x_1^*, x_2^*) \text{ with } x_1^* = (3, 0) \text{ and } x_2^* = (0, 4).}$$

is the PARETO OPTIMUM.

i.e. the first consumer gets all of good 1, and consumer 2 gets all of good 2.

□.

what is meant here as the set.

$\{x = (x_1, x_2)\ ;\ x_1 \in U_1(x_1^*),\ x_2 \in U_2(x_2^*)$ and $x_1 + x_2 = r\}$.