FINAL EXAM.

2021

$u(x) = \ln x$. $\quad x_i$ = outcomes.

Exercise 1.

(a). $X = \{ (1, 2, 4, 8) \,;\, p = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}) \}$.

The certainty equivalent is the certain outcome that the decision maker would find equivalent (INDIFFERENT) to playing the lottery: $c(X, u)$

ie

$$u(c(X,u)) = U(X)$$

$$= \mathbb{E}(u(X))$$

$$= \sum_{i=1}^{4} u(x_i)\, p_i$$

$$= \frac{\ln 1 + \ln 2 + \overbrace{\ln 4}^{2\ln 2} + \underbrace{\ln 8}_{3 \ln 2}}{4}$$

$$\Rightarrow \quad u(c(X,u)) = \frac{3}{2} \ln 2.$$

$$\Rightarrow \quad \boxed{c(X,u) = \exp \frac{3}{2} \ln 2 = 2^{3/2}}$$

November 8, 2023

(d). $Y_\pi = \{(1, 8) ; (\pi, 1-\pi)\}$.

$X \sim Y_{\pi^*}$ (ie X indifferent to Y_{π^*}).

iff. $U(X) = U(Y_{\pi^*})$. ~~$3\ln 2$~~.

iff. $u(c(X, u)) = \cancel{\ln(1)}\,\pi^* + (\ln 8)(1-\pi^*)$

$\parallel$

~~$\frac{3}{2}\ln 2$~~

iff. $1-\pi^* = \frac{1}{2}$. iff. $\boxed{\pi^* = \frac{1}{2}}$. $\square$

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defto compute. $U(Y_\pi) - U(Y_{\pi^*})$.

$\parallel$ $U(Y_\pi) - U(X)$ ← should come first.

$=$

$(3\ln 2)(1-\pi) - (3\ln 2)(1-\pi^*)$.

$=$

$\boxed{U(Y_\pi) - U(X) = (3\ln 2)\times(\pi^* - \pi)}$.

So. $\boxed{\pi > \pi^* \Longrightarrow U(X) > U(Y_\pi)}$

(d) $Z = \{(2,4,8) ; (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})\}$.

$$X \succ Y_\pi \text{ iff } \pi > \pi_*.$$

likewise. $\pi > \pi_*$

$$\Rightarrow \frac{1}{2}X + \frac{1}{2}Z \succ \frac{1}{2}Y_{\pi*} + \frac{1}{2}Z.$$

for any lottery Z.

because an expected-utility decision maker.
ie a decision maker whose preferences are representable by a Morgenstern-Von Neumann utility fct of the expected utility form.

satisfy the AXIOM of INDEPENDENCE.

(e).

$$r_A(x) = -\frac{u''(x)}{u'(x)} = \frac{+1/x^2}{1/x}$$

risk averse

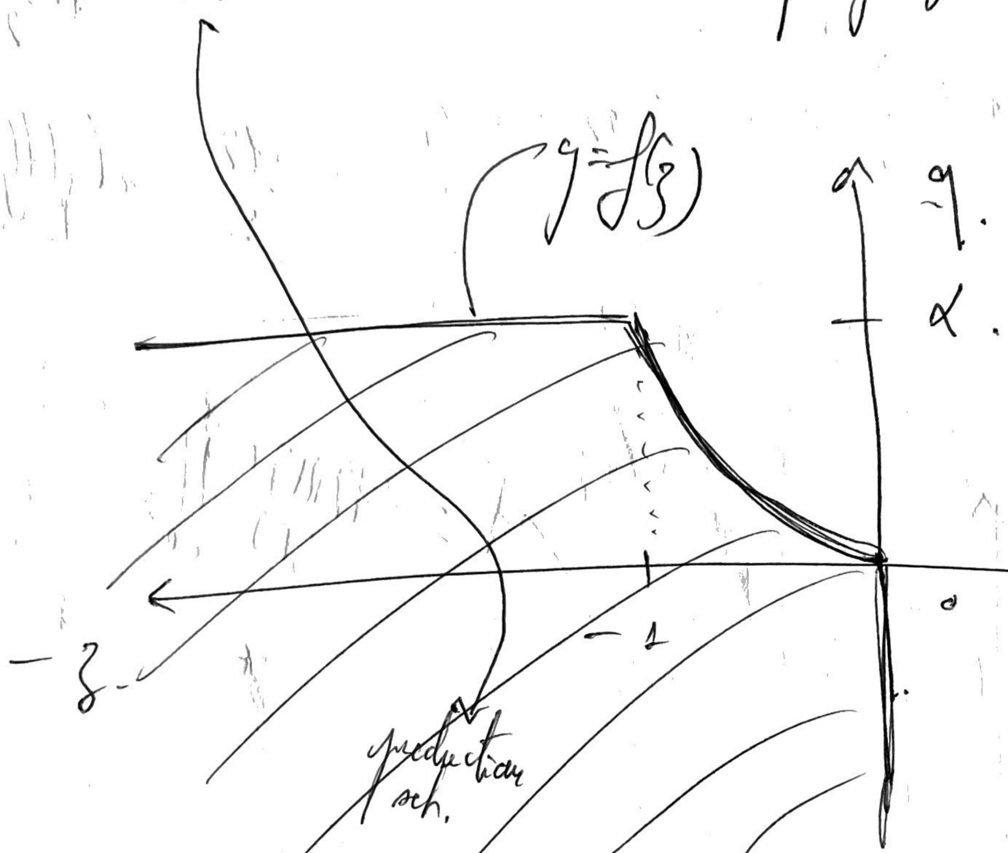
Arrow-Pratt coef of absolute risk aversion

$$\boxed{r_A(x) = \frac{1}{x}}$$

Exercise 2. $L = 2$.

$$f(z) = \begin{cases} \alpha & \text{if } z \geq 1 \\ \alpha z^2 & \text{if } 1 \geq z \geq 0 \end{cases} \qquad \alpha > 0.$$

(a). $Y = \{(-z, q) \; ; \; z \geq 0 \text{ and } q - f(z) \leq 0\}$

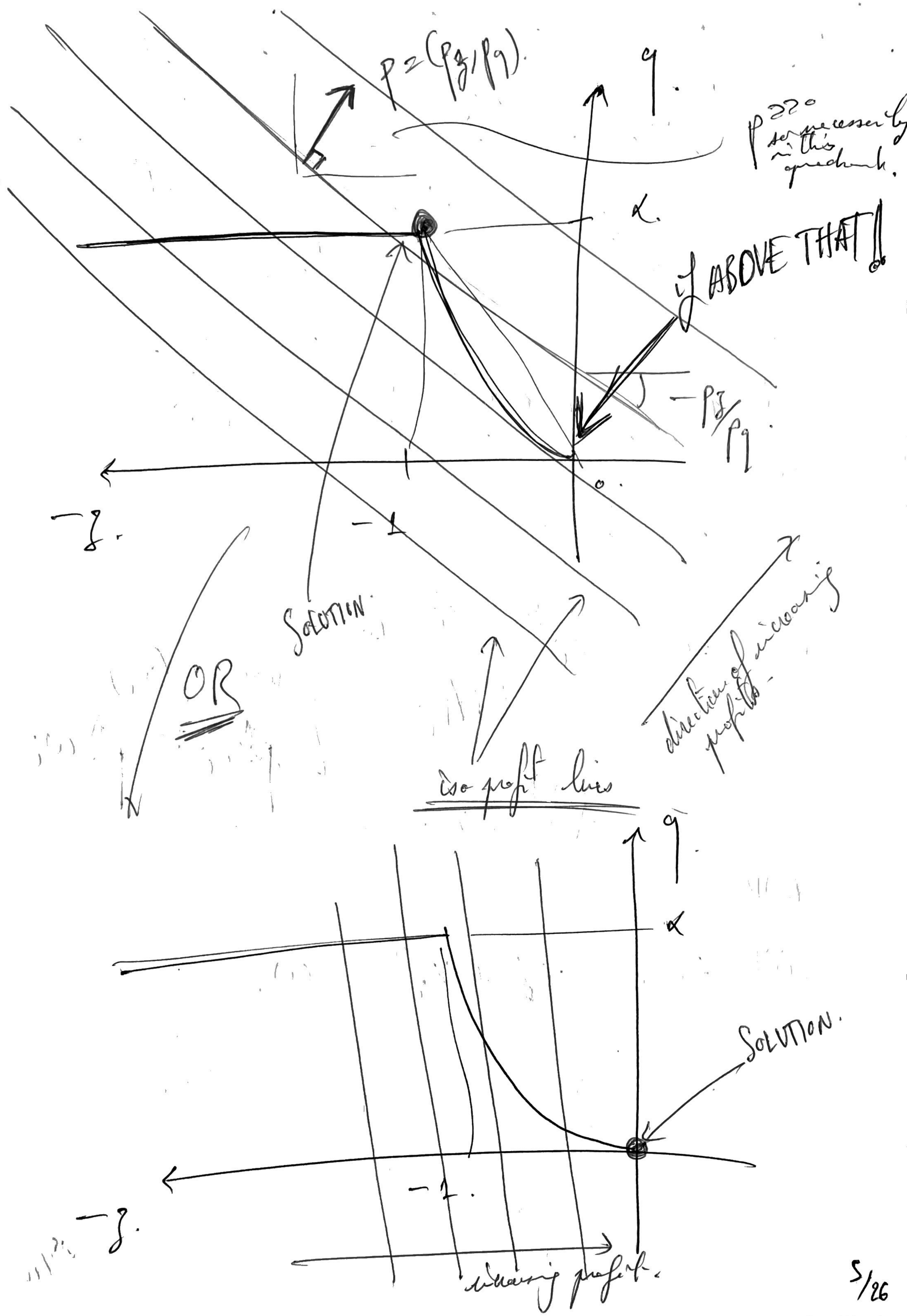


⊕ POSSIBILITY of INACTION.
$\Rightarrow 0 \in Y$??
Yes because $0 - f(0) = 0 \leq 0$. so $0 \in Y$. → it's important to be realistic?...

(b). Let's consider a price system

$$p = (p_z, p_q) \gg 0.$$

The profit of the firm, given a production plan $y \in Y$, is $\underline{p \cdot y}$.



$p=(p_z, p_q)$
q
$p \geq 0$ necessarily so in this quadrant.
α
if ABOVE THAT!!
$-p_z/p_q$
0
$-z$
-1
SOLUTION
OR
iso profit lines
direction of increasing profits
q
α
SOLUTION.
-1
$-z$
increasing profit.

So if $-\frac{p_1}{p_2} < -\alpha$.

$\Longleftrightarrow$ $\boxed{\frac{p_1}{p_2} > \alpha. \quad \text{then} \quad y^* = (0,0).}$

But if.

$\boxed{\frac{p_1}{p_2} < \alpha. \Longrightarrow y^* = (-1, \alpha).}$

So, if

$$\boxed{\frac{p_1}{p_2} > \alpha \longrightarrow \text{the profit is } \pi(p) = p \cdot y^* = 0}$$

and if $\frac{p_1}{p_2} < \alpha \longrightarrow$ the profit is

$$\pi(p) = p \cdot y^* = -p_1 + p_2 \alpha$$

$$= \underbrace{p_2}_{>0} \Big(\underbrace{\alpha - \frac{p_1}{p_2}}_{>0 \text{ by assumption}} \Big).$$

$\longrightarrow$ So

$$\boxed{\frac{p_1}{p_2} < \alpha \longrightarrow \pi(p) \text{ positive and finite}}$$

(c). Let $q \in [0, \alpha]$.

The <u>conditional demand</u> is the solution to the cost minimization problem (CMP)

price of input.

$$\begin{cases} \min \ p_1 \cdot z \\ \text{s.t.h } f(z) \geq q. \end{cases}$$

The <u>cost function</u> is the value fct of the CMP.

$\Rightarrow$ the solution to the CMP is simply $z^* = f^{-1}(q)$. or <u>efficient production</u>

assume $q \in [0, \alpha]$

$\Rightarrow$ then $z^* = \sqrt{\dfrac{q}{\alpha}}$

The cost fct.
for $q \in [0, \alpha]$ is.

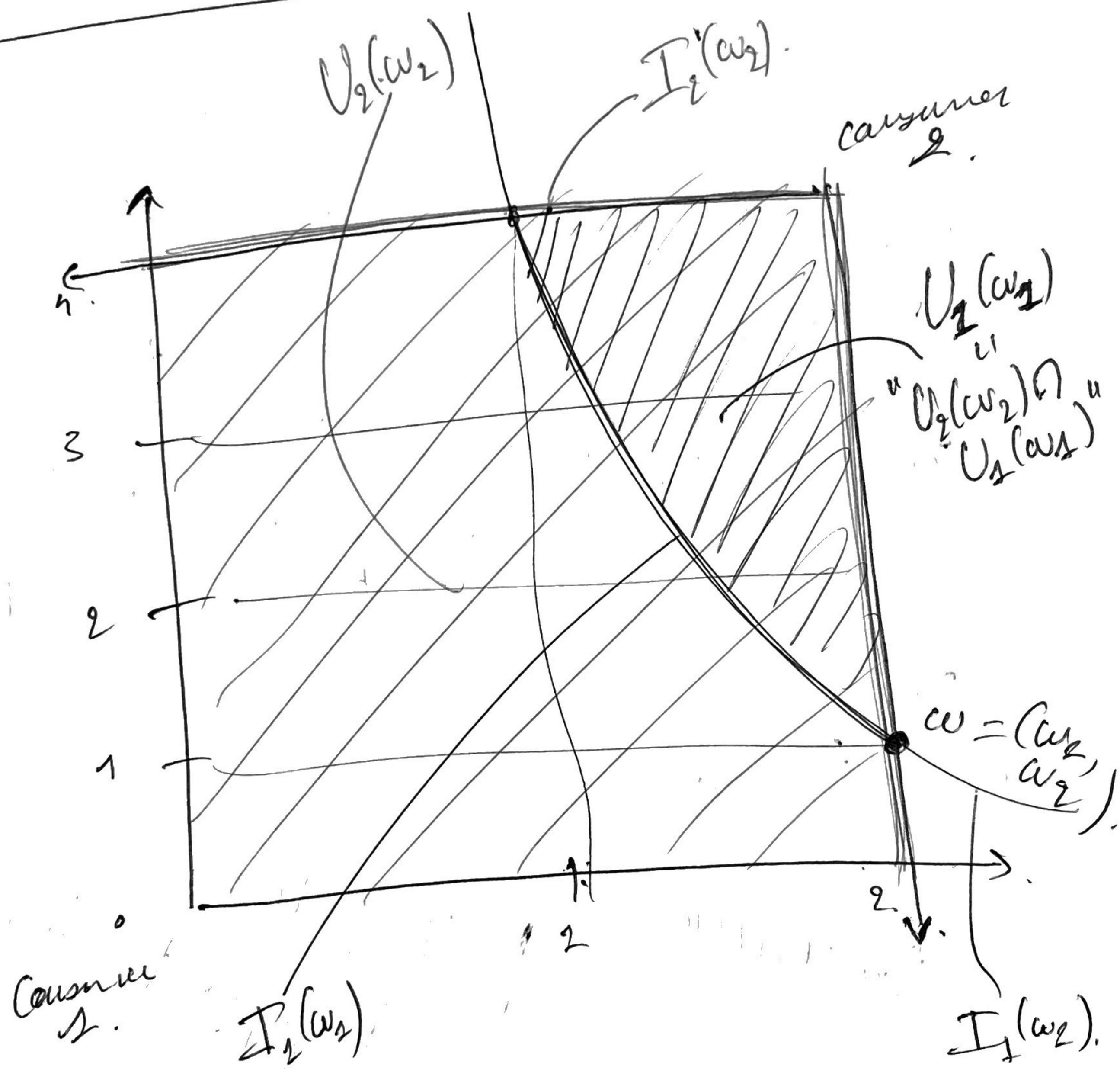
$$\boxed{c(q, p_1) = p_1 z^* = p_1 \frac{q}{\alpha}}$$

EDGEWORTH BOX. ν exercise 6.

Exercise 3. $l = 2$ $\quad I = 2$.

$\omega_1 = (2,1) \qquad u_1(x_1) = x_{11}^{2/3} x_{12}^{1/3}$.

$\omega_2 = (0,3) \qquad u_2(x_2) = x_{21} x_{22}$.



indifference curve for consumer 2 passing through ω_2.

$I_2(\omega_2) = \{ x_2 \,;\, u_2(x_2) = u_2(\omega_2) = 0 \}$.

$= \{ x_2 = (x_{21}, x_{22}) \,;$ either $x_{21} = 0$ or $x_{22} = 0 \}$.

for consumer 1:

$$I_1(\omega_1) = \{ x_1 ; \ u_1(x_1) = u_1(\omega_1) = 2^{2/3} \}.$$

the equation is:

$$x_{12}^{2/3} \, x_{22}^{1/3} = 2^{2/3}.$$

$$\Longleftrightarrow \quad x_{22} = \frac{2^2}{x_{12}^2}$$

$$\boxed{U_1(\omega_1) = \left\{ x_1 = (x_{11}, x_{12}); \ x_{22} = \frac{4}{x_{12}^2} \right\}.}$$

So the whole set $x = (x_1, x_2)$ with $\{ x_1 \in U_1(\omega_1), \ x_2 \in U_2(\omega_2)$ and $x_1 + x_2 = \omega \}$ PARETO DOMINATES ω. $\square$

(h). $\boxed{x^* \text{ is PARETO OPTIMAL iff } \nexists\, x' \text{ s.th. } \left(\forall i \in \{1,2\}.\ x_i' \succsim x_i^* \text{ and } \exists i \in \{1,2\},\ x_i' \succ x_i^*\right)}$.

The set P of pareto optimal allocations satisfy: and $x_1 + x_2 = \omega$

$$P = \left\{ x = (x_1, x_2) \; ; \; MRS^1_{12}(x_1) = MRS^2_{12}(x_2) \right\}.$$

$$MRS^1_{12}(x_1) = \frac{\frac{\partial u_1}{\partial x_{11}}}{\frac{\partial u_1}{\partial x_{12}}}(x_1) = \frac{\cancel{\frac{2}{3}} \frac{1}{x_{11}} \cancel{u_1(x_1)}}{\cancel{\frac{1}{3}} \frac{1}{x_{12}} \cancel{u_1(x_1)}}.$$

$$\boxed{MRS^1_{12}(x_1) = 2\frac{x_{12}}{x_{11}}}.$$

and $\boxed{MRS^2_{12}(x_2) = \frac{\frac{\partial u_2}{\partial x_{21}}}{\frac{\partial u_2}{\partial x_{22}}}(x_2) = \frac{x_{22}}{x_{21}}}.$

$x_1 + x_2 = \omega.$ and

$$\omega = \omega_1 + \omega_2 = (2, 4).$$

$$\Longrightarrow \left\{ \boxed{\begin{aligned} x_{21} &= 2 - x_{11} \\ \text{and } x_{22} &= 4 - x_{12} \end{aligned}} \right.$$

So. $P = \left\{ x = (x_1, \omega - x_1) ; \text{ with } 2\dfrac{x_{12}}{x_{11}} = \dfrac{4 - x_{12}}{2 - x_{11}} \right\}$

$$\Updownarrow$$

$$2\left(\frac{2}{x_{11}} - 1\right) = \frac{4}{x_{12}} - 1.$$

$$\Updownarrow$$

$$\frac{4}{x_{11}} - 1 = \frac{4}{x_{12}} \iff x_{12} = \frac{4}{\frac{4}{x_{11}} - 1}.$$

Let's just evaluate a few pts.

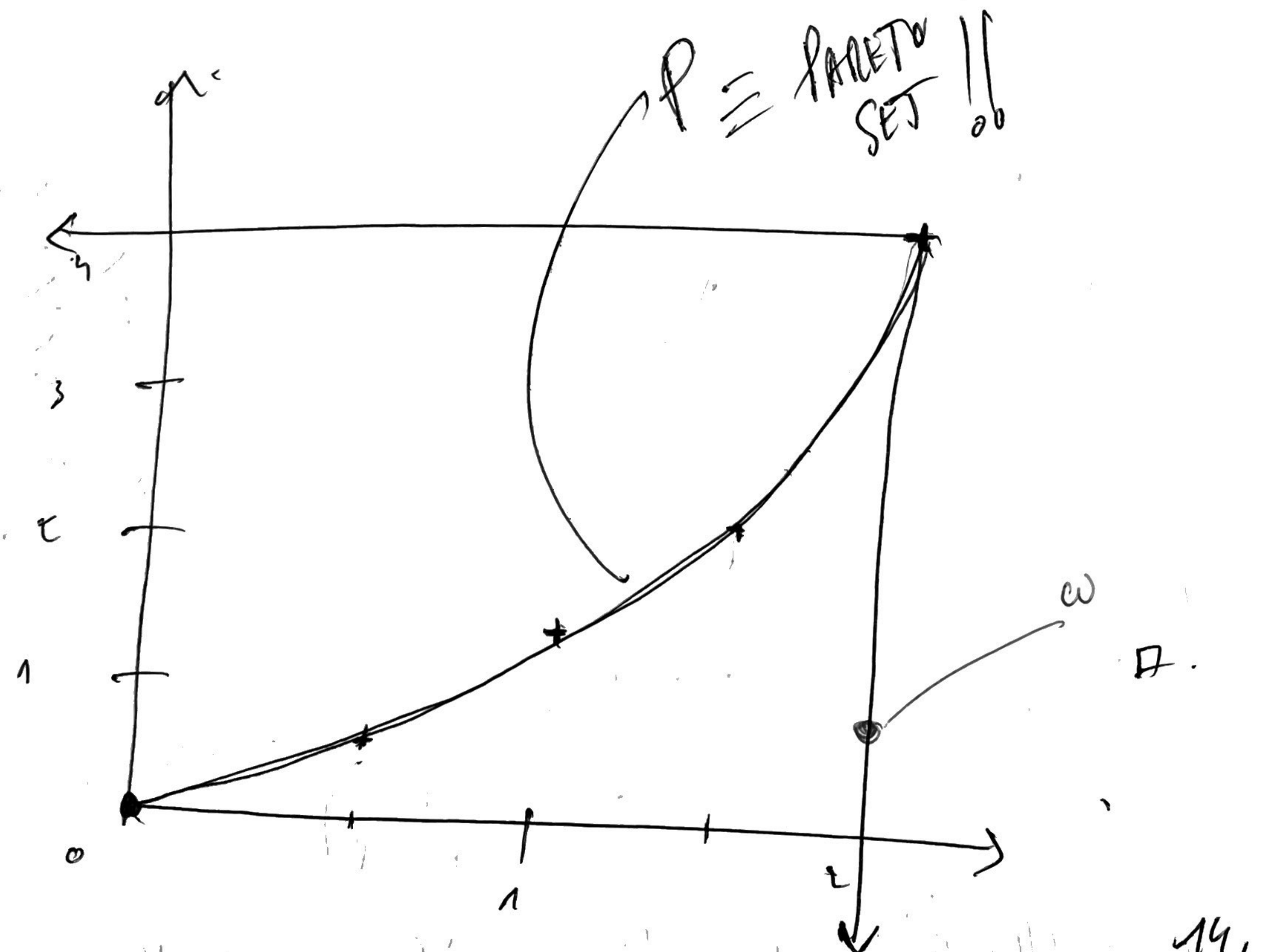
$$x_{11} = 0 \implies x_{12} = 0.$$

$$x_{11} = 1 \implies x_{12} = \frac{4}{3} = 1.5$$

$$x_{12} = 2 \implies x_{12} = 4.$$

$$x_{11} = 1/2 \implies x_{12} = \frac{4}{7} = \text{slightly more than } \frac{1}{2}.$$

$$x_{11} = \frac{3}{2} \implies x_{12} = \frac{4}{\frac{5}{3}} = \frac{12}{5} \text{ slightly more than } 2.$$



(c). A general / Walrasian / competitive / price equilibrium for this economy is a price system p^* and allocation x^* s.th.

(i) * both agents maximize their utility under budget constraints;

(ii) * markets clear.

(i) i.e. x_i^* sol. to.

$$UMP_i \equiv \begin{cases} \max u_i(x_i) \\ p^* \cdot x_i \leq p^* \cdot \omega_i \end{cases}$$

$\forall i = 1, 2.$

AND.

(ii) i.e. $x_1^* + x_2^* = \omega.$

$\Longrightarrow$ Walras' law (from more $\longrightarrow$ LNS) and KT condt yield.

$$\begin{cases} Du_1(x_1^*) = \lambda p^* \\ p^* \cdot x_1^* = p^* \cdot \omega_1 \end{cases} \quad \text{for } UMP_1$$

and

$$\begin{cases} Du_2(x_2^*) = \lambda p^* \\ p^* \cdot x_2^* = p^* \cdot \omega_2 \end{cases} \quad \text{for } UMP_2.$$

$\Downarrow$

$$\begin{pmatrix} \frac{2}{3}\frac{u_1(x_1^*)}{x_{11}^*} \\ \frac{1}{3}\frac{u_1(x_1^*)}{x_{12}^*} \end{pmatrix} = \lambda \begin{pmatrix} p_1^* \\ p_2^* \end{pmatrix}.$$

$$MRS^1_{12}(x_1^*) = \boxed{\frac{p_1^*}{p_2^*} = \frac{2x_{12}^*}{x_{11}^*}}$$

and

$$\boxed{p_1^* x_{11}^* + p_2^* x_{12}^* = 2p_1^* + p_2^*}$$

$$\begin{pmatrix} x_{22}^* \\ x_{21}^* \end{pmatrix} = \lambda \begin{pmatrix} p_1^* \\ p_2^* \end{pmatrix}.$$

$\Downarrow$

$$\frac{x_{21}^*}{p_2^*} = \lambda = \frac{x_{22}^*}{p_1^*}.$$

OR FASTER:

$$MRS^2_{12}(x_2^*) = \boxed{\frac{x_{22}^*}{x_{21}^*} = \frac{p_1^*}{p_2^*}}$$

$\Updownarrow$

$$\boxed{p_1^* x_{21}^* = p_2^* x_{22}^*}$$

and

$$\boxed{p_1^* x_{21}^* + p_2^* x_{22}^* = p_2^* 3}$$

(i). $$\frac{p_1^*}{p_2^*} = \frac{2x_{12}^*}{x_{11}^*} \longrightarrow x_{12}^* = \frac{x_{11}^*}{2}\,\frac{p_1^*}{p_2^*}$$

and $$\frac{p_1^*}{p_2^*}\left(x_{11}^* - 2\right) + \left(x_{12}^* - 1\right) = 0$$

substitute one into the other and get
$x_1^* = (x_{11}^*, x_{12}^*)$ as a fct of $\frac{p_1^*}{p_2^*}$!

$$\frac{p_1^*}{p_2^*}\left(x_{11}^* - 2\right) + \frac{x_{11}^*}{2}\,\frac{p_1^*}{p_2^*} - 1 = 0$$

$$\Longrightarrow \quad x_{11}^* = \frac{1 + \frac{2p_1^*}{p_2^*}}{\frac{p_1^*}{p_2^*} + \frac{p_1^*}{2p_2^*}} \qquad \frac{p_1^*}{p_2^*} + \frac{p_1^*}{2p_2^*} = \frac{3p_1^*}{2p_2^*}$$

$$\Longrightarrow \quad \boxed{x_{11}^* = \frac{2}{3}\frac{p_2^*}{p_1^*} + \frac{4}{3}}$$

and $$\boxed{x_{12}^* = \frac{x_{11}^*}{2}\,\frac{p_1^*}{p_2^*} = \frac{1}{3} + \frac{2}{3}\frac{p_1^*}{p_2^*}}$$

(ii) and

$$x_{22}^* = \frac{p_1^*}{p_2^*} x_{21}^*.$$

and

$$\frac{p_1^*}{p_2^*} x_{21}^* + x_{22}^* = 3.$$

$\Rightarrow$

$$2 \frac{p_1^*}{p_2^*} x_{21}^* = 3.$$

$$\longrightarrow \boxed{x_{21}^* = \frac{3}{2} \frac{p_2^*}{p_1^*}}$$

and $\boxed{x_{22}^* = \frac{p_1^*}{p_2^*} x_{21}^* = \frac{3}{2}.}$

$\Rightarrow$ FINALLY. We find the "crossing" of the offer curves: $x_1^*\left(\frac{p_1^*}{p_2^*}\right)$ and $x_2^*\left(\frac{p_1^*}{p_2^*}\right)$.

i.e. market clearing. $\boxed{x_1^*(p^*) + x_2^*(p^*) = \omega}$

$\longrightarrow$

As we know it's enough to have the market clear for good 1 and it will automatically clear for good 2.

$$x_1^* + x_2^* = \omega \iff \begin{cases} x_{11}^* + x_{21}^* = 2. & (*) \\ \text{and } x_{22}^* + x_{12}^* = 4. \end{cases}$$

from $(*)$, we get.

$$\frac{2}{3}\left.\frac{p_2}{p_1}\right|_* + \frac{4}{3} + \frac{3}{2}\left.\frac{p_2}{p_1}\right|_* = 2.$$

$\times 3 \times 2.$

$$(4+9)\left.\frac{p_2}{p_1}\right|_* + 8 = 12.$$

$$\Rightarrow \boxed{\left.\frac{p_2}{p_1}\right|_* = \frac{4}{13}.}$$

$$\boxed{\frac{p_1^*}{p_2^*} = \frac{13}{4}}$$

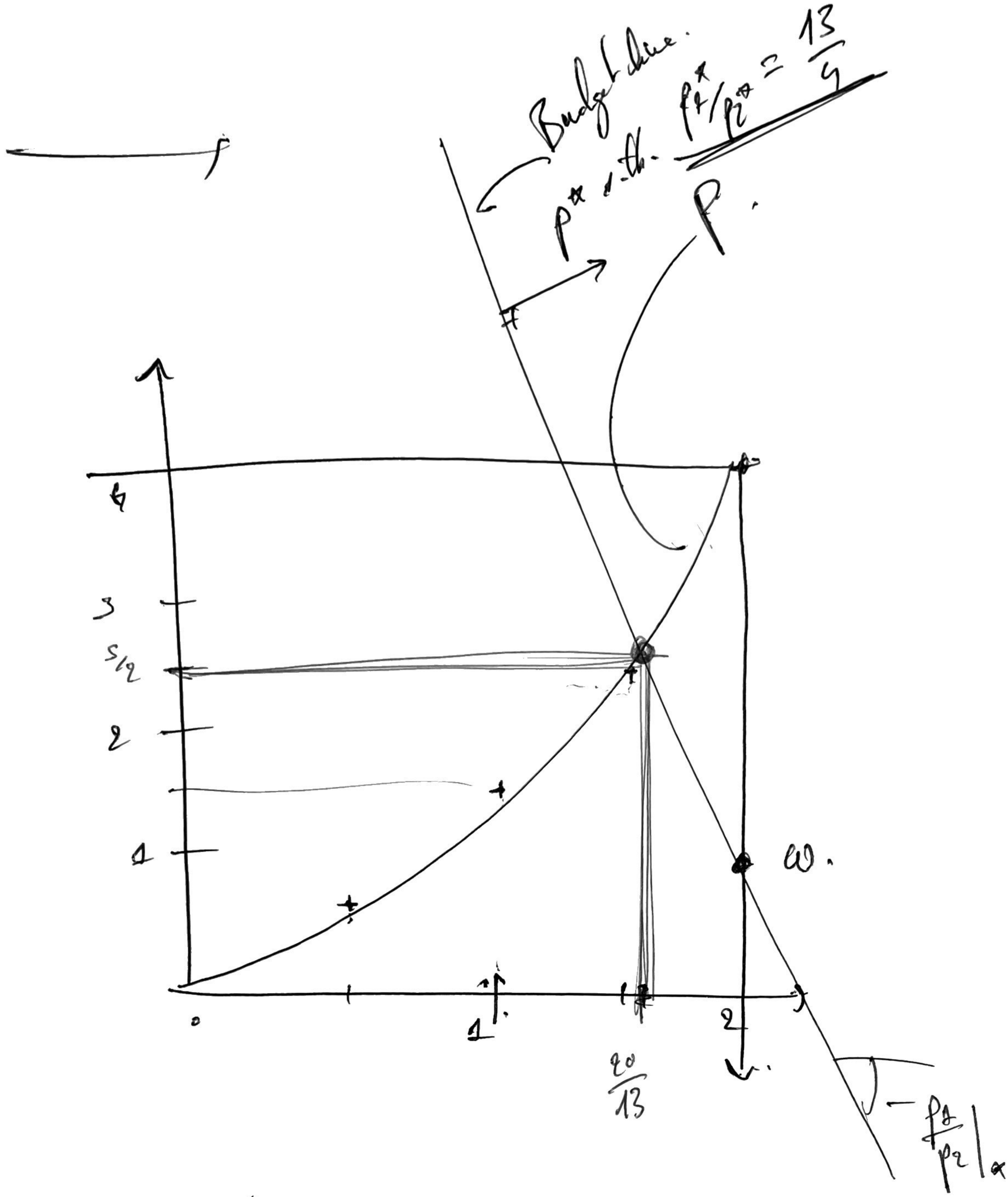
$$\longrightarrow \quad x_{11}^* = 4\left(1 - 2\frac{4}{13}\right).$$

$$= 4\left(\frac{13-8}{13}\right) = \boxed{\frac{20}{13} = x_{11}^*}$$

$$x_{12}^* = 2\left(\frac{13}{4} - 2\right) = \frac{10}{4} = \boxed{\frac{5}{2} = x_{12}^*}$$

$$x_{21}^* = 2 - \frac{20}{13} = \boxed{\frac{6}{13} = x_{21}^*}$$

$$x_{22}^* = 4 - \frac{5}{2} = \boxed{\frac{3}{2} = x_{22}^*}$$



$$\begin{cases} x_{11}^* = \frac{20}{13} \\ x_{12}^* = \frac{5}{2} \end{cases}$$

(d) $x_1 = x_2$. (and $x_1 + x_2 = \omega = (2,4)$).

$\Longrightarrow$

$x_1 = x_2 = (1,2)$.

from LNS, a Walrasian eq is necessarily a Pareto optimum (1st WELFARE THM).

~~so~~ given that. that.

$MRS^1_{12}(x_2) \neq MRS^2_{12}(x_2)$.

we can rule out the possibility that it be a Walrasian eq. $\square$.

(e) Can we find a PARETO OPTIMUM s.th. $x_{11} = x_{21}$?

→ Yes, simply requiring exchange.

$$MRS^1_{12}(x_1) = \frac{2x_{12}}{\cancel{x_{11}}} = \frac{x_{22}}{\cancel{x_{21}}} = MRS^2_{12}(x_2)$$

The simplifying.

$x_{11} = x_{21}$ and Pareto optimum $\Longrightarrow \boxed{2x_{12} = x_{22}}$

→ It can be supported as a PARETO OPTIMUM.

→ So, since u_i are st. concave (verify...)

$\succsim_i$ convex.

→ ACCORDING TO THE 2ND WELFARE THM it can be supported as a price equilibrium with transfers.

$\longrightarrow$ able to find these transfers

$$\begin{cases} x_{11} = x_{21}, \\ 2x_{12} = x_{22}, \end{cases}$$

and

$$\begin{cases} p_1 x_{11} + p_2 x_{12} = p_1 2 + p_2 + T \\ p_1 x_{21} + p_2 x_{22} = p_2 3 - T \end{cases}$$

and market clearing

$$\begin{cases} x_{11} + x_{21} = 2 \\ x_{12} + x_{22} = 4 \end{cases}$$

$$2x_{11} = 2 \longrightarrow \boxed{x_{11} = 1 = x_{21}.}$$

$$3x_{12} = 4 \Longrightarrow \boxed{x_{12} = \frac{4}{3}} \text{ and } \boxed{x_{22} = \frac{8}{3}}$$

price of . LL. will verifie.

$$MRS^1_{12}(x_1) = MRS^2_{12}(x_2) = \frac{p_1}{p_2}.$$

$$\frac{2x_{12}}{x_{11}} = \frac{x_{22}}{x_{21}} = \frac{p_1}{p_2}.$$

$$\boxed{\frac{p_1}{p_2} = \frac{8}{3}}$$

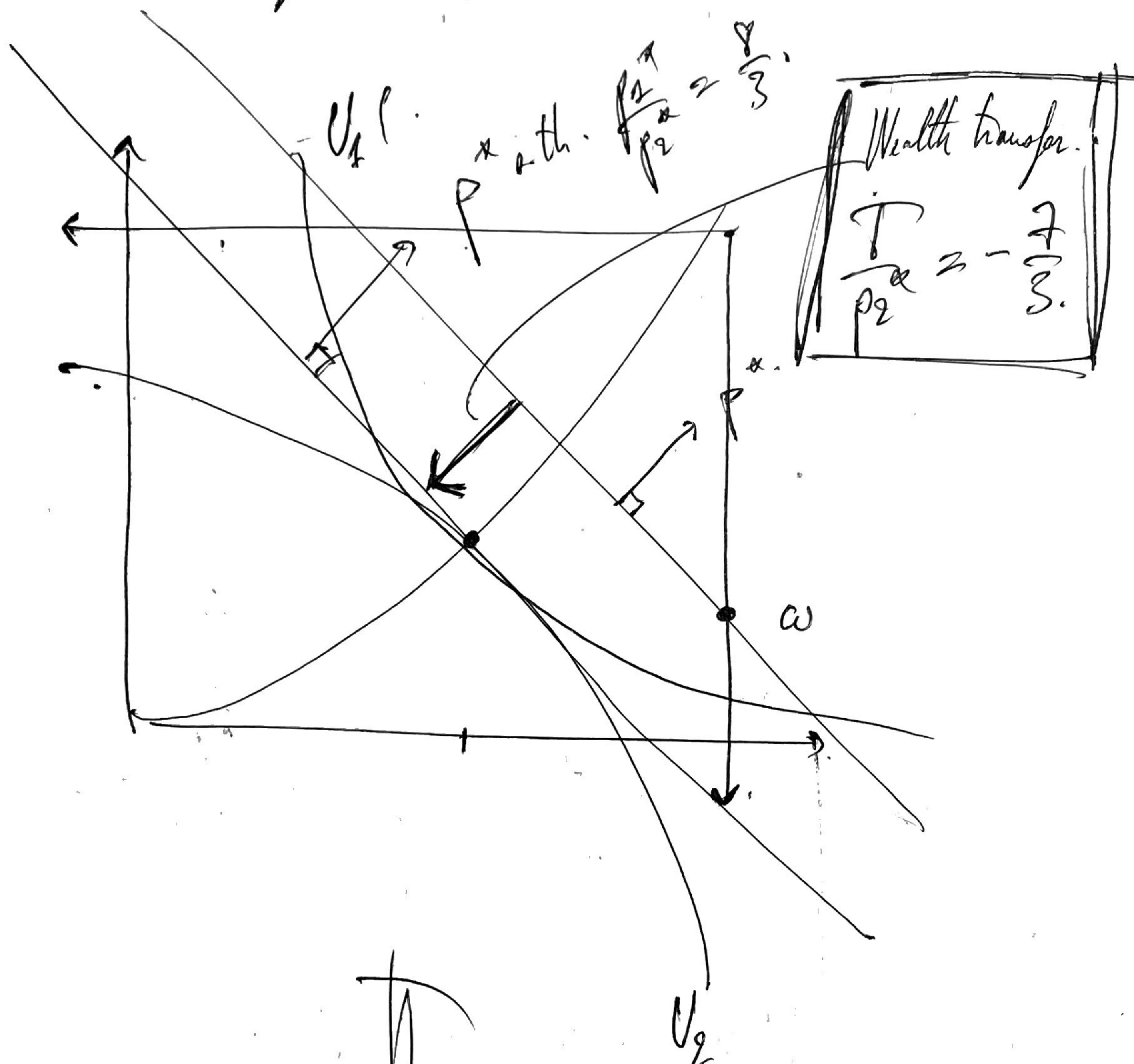
So the lump-sum transfer need to amount to

$$\frac{T}{p_2} = 3 - \frac{p_1}{p_2} x_{21} - x_{22}$$

$$= 3 - \frac{8}{3} - \frac{8}{3} = -\frac{7}{3}.$$

$$\boxed{\frac{T}{p_2} = -\frac{7}{3}}$$

→ Graphical rep.



D.