Optimization – QEM1 (First Year of the QEM program)

First Semester 2025-2026

In University Paris 1 Panthéon-Sorbonne, the course **Optimization (QEM1 – First Year of the QEM program)** consists of two parts:

- **The first part is "Optimization A: Multivariable Calculus".** This part lasts 6 weeks and starts on Monday, September 8, 2025.

- **The second part is "Optimization B: Convex Analysis and Dynamics".** This part lasts 6 weeks and starts on Monday, Octobre 20, 2025.

Both parts are mandatory.

As for the detailed schedule of all the classes (course and tutorials), please contact Mr Guillaume Glade, Paris QEM Schooling: qem-paris@univ-paris1.fr

All the information and the syllabus on the first part "Optimization A: Multivariable Calculus" are available on the pedagogical space of Marcus Pivato.

The final grade of Optimization is a mark between 0 and 100, and it is given by the Attendance (20%), the Midterm Exam on "Optimization A: Multivariable Calculus" (30%), and the Final Exam on "Optimization B: Convex Analysis and Dynamics" (50%).

SYLLABUS of Optimization B: Convex Analysis and Dynamics

This part lasts 6 weeks and it starts on Monday, Octobre 20, 2025.

Lecturer: Mme *Elena del Mercato* – 18 hours of course
TA: Mme *Polina Borisova* – 24 hours of tutorial classes

Final Exam: Thursday, December 11, from 9:00 to 11:00 A.M.

General presentation
This part aims to introduce and develop many of the analytical tools which bear special relevance and are generally used in theoretical and applied Economics. The goal of the course is to provide students with a general understanding of the mathematical objects used in Economics, in order to find efficient methods for solving optimization problems in static and dynamic frameworks.

Prerequisites for “Optimization B: Convex Analysis and Dynamics”
The following topics are assumed to be well-known: supremum, infimum, maximum and minimum in R; calculus in one variable; basic notions in calculus with several variables; Euclidean distance, open balls, open sets, closed sets, bounded sets, compact sets in Rn; continuous functions, continuously differentiable and twice continuously differentiable functions, chain rule, directional derivative; concavity/convexity, quasi-concavity/quasi-convexity, and their characterizations with first and second order conditions; linear algebra: matrix and vector algebra, determinants, systems of linear equations, vector spaces and subspaces. For detailed references on these prerequisites, see "Optimization A: Multivariable Calculus".

Textbooks for “Optimization B: Convex Analysis and Dynamics”

[SHSS]: Sydsaeter K., Hammmond P., Seierstadt A., Strom A. (2005): *Further Mathematics for Economic Analysis*, Prentice Hall.

[S]: Sundaram R.K. (1999): *A First Course in Optimization Theory*, Cambridge University Press, Cambridge.

Week 1

- **Optimization problems.** Basic notions. Optimization in Economics: Examples. Existence result: Extreme Value Theorem (or Weierstrass Theorem). Uniqueness of the solution. [SHSS: 3.1], [S: 2.1, 2.2, some of the examples in 2.3, 2.4, 3.1]
- **Unconstrained optimization.** First order conditions for optimality: Necessary and sufficient conditions. Second order conditions: Necessary and sufficient conditions for local optimality. [SHSS: 3.1, 3.2], [S: 4 up to 4.5]
- **Constrained optimization: Equality constraints.** Optimization problem with equality constraints. Lagrangian function. First order conditions (FOC) for optimality: Necessary and sufficient conditions for FOC. [SHSS: 3.3], [S: 5.2]

Week 2

- **Constrained optimization: Equality constraints (continued).** Second order conditions: Necessary and sufficient conditions for local optimality. Interpretation of the Lagrange multipliers. [SHSS: 3.3, 3.4], [S: 5.3, 5.4]
- **Inequality constrained optimization.** Optimization problem with inequality constraints. Karush-Kuhn-Tucker (KKT) conditions: Necessary and sufficient conditions for KKT. Existence of Lagrange multipliers (Farkas' Lemma). Uniqueness of solutions. [SHSS: 3.5, 3.6, 3.9], [S: 6.1, 6.2, 7.3, 7.4]

Week 3

- **Constrained optimization: Mixed constraints.** Optimization problem with equality and inequality constraints. Karush-Kuhn-Tucker (KKT) conditions: Necessary and sufficient conditions for KKT with mixed constraints. [SHSS: 3.8], [S: 5.1, 5.5.1, 5.5.3, 6.4]
- **Comparative statics.** Value function of parameterized problems. The Envelope Theorems. [SHSS: 3.1, 3.3, 3.7], [S: 2.2, 5.2.3, 6.1.3]

Week 4

- **Finite horizon dynamic programming.** Examples in Economics. Optimization problem with a finite horizon: Basic notions. First order necessary and sufficient conditions for interior solutions. Backward induction method: Bellman equations. The consumption-savings problem: Euler equations. An application. [SHSS: 12.1], [S: 11.1, 11.2, 11.3, 11.4, 11.5, 11.6]

Week 5

- **Infinite horizon dynamic programming.** An example: Ramsey growth model. Stationary optimization problem: Basic notions. Basic assumptions for the existence of a solution. Bellman equation. Concavity and continuity of the value function. Truncated problem at period T. First order conditions: Euler equations. Two applications. Differentiability of the value function. [SHSS: 12.3], [S: 12.1, 12.2, 12.3, 12.4, 12.5]

Week 6

- **Existence of solutions.** Norm, distance, and metric spaces: Basic notions. Weierstrass Theorem in metric spaces. Contraction. The contraction mapping theorem (also called Banach's fixed point theorem). [S: C.3, C.4 up to C.4.3, 12.4.1, 12.4.2]