Industrial Organization
M2 APE – Caillaud - Tropeano

Exercises for the course in IO

1 Spatial market segmentation

Consider a horizontally differentiated product market in which two firms are located at the end point of the unit Hotelling interval. Firms produce at marginal costs c. There is a continuum of consumers of mass 1 who are uniformly distributed on the unit interval. They have unit demand, outside utility of $-\infty$ and quadratic transport costs td^2.

1. Suppose that firms simultaneously set a uniform price for all consumers. Characterize the equilibrium of the game and determine equilibrium profits.

2. Suppose now that firms can price discriminate between consumers located on $[0, \frac{1}{2}]$ (segment A) and $(\frac{1}{2}, 1]$ (segment B). Characterize the equilibrium of the game in which firms set simultaneously their pair of prices (p_{iA}, p_{iB}). Compare to the results obtained in question 1 and explain.

3. Suppose now that firm 1 can discriminate between the two consumer segments while firm 2 cannot. Characterize the equilibrium in which firm 1 sets (p_{1A}, p_{1B}) and firm 2 sets p_2 simultaneously. Compare to the results obtained in question 2 and explain.

4. Consider the extended game in which firms may in a first stage invest or not in the possibility of being able to price discriminate the two segments, at cost I, and in the second stage they compete in prices. Characterize the perfect equilibrium and comment on your results.

2 Supply function equilibria

In the electric sector, electricity-generating firms have to submit supply functions before the market actually operates and before the precise state of demand is known. Supply functions indicate how much electricity a firm is willing to inject in the network given the value of the price it will be paid. Firms choosing prices or quantities are polar situations in which they submit an infinite-slope or flat supply function. Therefore, on top of being relevant for some markets, this framework offers a generalization of price-setting or quantity-setting oligopoly games in IO theory.

2.1 No uncertainty.

Two firms face a demand function given by $D(p) = \theta - p$, where θ is a publicly knwon demand parameter. They both have the same decreasing returns to scale technology, with cost function $\frac{c}{2}q^2$ when $q \geq 0$ is produced.

These two firms have to submit twice-continuously differentiable supply functions $S_i(.)$, for $i = 1, 2$, and then, the market-maker finds out the price at which the market clears, i.e. the solution of $D(p) = S_1(p) + S_2(p)$; if this price exists and is unique, both firms are invited to produce $S_i(p)$ and their profits are $pS_i(p) - \frac{c}{2}(S_i(p))^2$; il this price does not exist or is not unique, the market does not operate and profits are null.

Show that for any (p^*, q^*) such that $0 < p^*, q^* < \theta$, $p^* = \theta - 2q^*$ and $p^* < \frac{\theta(1+c)}{3+c}$, both firms bidding

$$S_i(p) = S^L(p) \equiv q^* + \frac{q^*}{p^* - cq^*}(p - p^*)$$

for any $p \in [0, \theta)$ constitutes a pair of symmetric equilibrium strategies.

Moreover, explain why both firms bidding $S_i(p) = \phi(p)$ with $\phi(.)$ strictly increasing, twice-continuously differentiable, such that $\phi(p^*) = q^*$, $\phi'(p^*) = \frac{q^*}{p^* - cq^*}$ and for any p, $\phi(p) \geq S^L(p)$, also constitutes a pair of symmetric equilibrium strategies.

What do you think about the prediction of the supply function model ? Explain.

2.2 Uncertain demand.

Suppose now that demand is random; more precisely θ is a random variable with support $[0, +\infty)$. E.g. it could reflect the fact that demand varies during the day and that electricity-generating firms have to commit the day before on price-quantity schedules. Firms submit supply functions before the realization of θ; once θ is realized, the procedure follows the same steps as under no uncertainty.

1. If firms were perfect competitors, i.e. if they were to produce for a price p so as to equate the price with their marginal cost, what would be the outcome as a function of θ ?

2. If firms were Cournot competitors, i.e. they were to bid a quantity for each realization of θ, what would be the outcome as a function of θ ?

3. Show that both firms bidding

$$S_i(p) = S^U(p) \equiv \left(\sqrt{\frac{4+c}{c}} - 1\right)\frac{p}{2}$$

is a pair of symmetric equilibrium strategies.

4. It is the unique symmetric equilibrium: provide an intuition why the uncertainty framework so drastically reduces the multiplicity of equilibrium outcomes.

5. Compare the equilibrium under the supply function equilibrium with the ones obtained in questions 1 and 2.

3 Advertising and entry

We consider a market with two firms: a branded good (1, firm I) and a generic good (2, firm E). The size of the market is measured by the mass z of consumers. The per consumer advertising expenses of firm 1 are denoted by A. The total expenses of firm 1 are thus equal to zA. The generic producer does not invest in advertising. The consumers are heterogeneous. Each consumer is characterized by its type θ where θ is uniformly distributed on $[0,1]$. The surplus of a consumer of type θ that buys the branded good is equal to $\theta\sqrt{2A} - p_1$ where p_1 is the unit price of firm I. If the same consumer buys the generic good, the surplus is equal to $\frac{1}{2}\theta\sqrt{2A} - p_2$. The generic producer must incur a fixed cost e to enter the market. The cost e is uniformly distributed on $[0, K]$ with $K > 0$.

The timing of the game is the following. At stage 1, firm I invests in advertising. At stage 2, the generic producer observes A and the fixed cost of entry and decides to enter the market or not. At stage 3, both firms compete in price. There are no production costs.

1. Determine the advertising expenses of firm I.

2. What is the impact of the market size z on the advertising decision? Characterize the entry deterrence/accommodation strategy of firm I?

4 Spatial preemption and entry

We study here the brand positioning strategy of an incumbent. For that we consider an incumbent (firm 1) that owns two restaurants: a Chinese restaurant and an Italian restaurant and a new entrant (firm 2) tha owns a Chinese restaurant. There are two consumers.

Surplus of consumer C :

$$\begin{cases} \beta + \frac{\lambda}{2} - p_2^C \text{ if eats at the Chinese restaurant of firm 2} \\ \beta - p_1^C \text{ if eats at the incumbent Chinese restaurant} \\ \beta - \lambda - p^I \text{ if eats at the Italian restaurant} \end{cases}$$

Surplus of consumer I :

$$\begin{cases} \beta - \frac{1}{2}\lambda - p_2^C \text{ if eats at the Chinese restaurant of firm 2} \\ \beta - \lambda - p_1^C \text{ if eats at the incumbent Chinese restaurant} \\ \beta - p^I \text{ if eats at the Italian restaurant} \end{cases}$$

where we let p_i^C denote the price of the Chinese restaurant of firm i ($i = 1, 2$), p^I the price of the Italian restaurant and β and λ are parameters with $\lambda < \beta < \frac{3}{2}\lambda$.

Restaurants compete in prices. The timing of the game is as follows. At stage 1, firm 2 enters or not and at stage 2, firms compete in price. To enter the market, firm 2 must pay a fixed cost $e > 0$. Firms incur no production costs.

1. Determine the entry decision.

2. Is it profitable for the monopoly to shut-down the Chinese restaurant before the entry of the competitor?

5 Predatory behavior and learning by doing

The objective of this exercise is first to build a model of predatory pricing based on learning by doing and then to assess the impact of such a behavior on the economic welfare.

For that purpose we consider a model with two firms (A and B) and two periods of time 1 and 2. At each period t each firm i chooses its output denoted by q_i^t. Both outputs are set simultaneously at each period. Both firms produce the same good for which the inverse demand is denoted by $P = \alpha - Q$ where Q is the total output and P the price. The parameter α is positive. The unit production cost of firm i is equal to β at period 1 and equal to $c_i = \beta - \gamma q_i^1$ at period 2 where parameters β and γ are positive. In addition, at the end of period 1, firm B must incur a fixed cost K to stay on the market at period 2. The level of fixed cost is observed at the end of period 1. At the beginning of the game, both firms know only the distribution of the fixed cost. The cumulative function is $F(x) = C - \frac{D}{\sqrt{x}}$ where C and D are positive parameters. Firm A pays no fixed cost.

1. Consider here the competition at period 2.

(i) Determine the equilibrium at period 2. Deduce the equilibrium profit of firm B at period 2.

(ii) Deduce the probability for firm B to stay on the market at period 2. Show that the probability depends on q_A^1 and q_B^1. Explain the impact of a higher level of output q_A^1 on that probability.

2. We now turn to period 1. Write the whole (on both periods) expected profit of firm A.

3. Assume that firm A ignores the impact of q_A^1 on the probability of firm B to stay on the market. Determine the first order condition that satisfies the optimal level of production q_A^1.

4. We now consider that firm A takes into account the impact of q_A^1 on the probability of firm B to stay on the market. Determine the first order condition that satisfies the optimal level of production q_A^1.

We define a "predatory behavior" as an output q_A^1 set at a higher level than the level obtained in question 3.

5. Compare the optimal level of output q_A^1 obtained in question 3 and 4 (use only the first order condition without calculating explicitly the optimal level of output q_A^1). Explain that result using the typology of Fudenberg and Tirole.

6. (i) Determine the slope of the best response function of firm B at the first period with respect to q_A^1.

(ii) Explain why that slope could be lower or higher than 1.

(iii) Why is a predatory behavior harmful to consumers if the slope is equal to 1?

6 Exclusive dealing to deter entry by a rival

We consider a market with 2 retailers. One producer is on the market (the incumbent, firm I) and faces the threat of entry of a competitor (firm E). The competitor has no variable cost but must incur a fixed cost (f) to enter. The incumbent incurs only a variable cost (constant unit cost $c_I < \frac{1}{4}$). Both producers sell the same product. We study the impact of an exclusive dealing contract between I and one retailer on the entry decision of E. We examine the role of competition between both retailers. For that, we contrast two configurations: one where each retailer operates on a separate market and one where both retailers compete on the same final market. We examine Cournot competition and Betrand competition. Retailers incur no cost except the wholesale price paid to the producers. We assume that $c_I \frac{(1-c_I)}{4} < f < \frac{5}{12} c_I(1-c_I)$.

A. The monopoly case

The demand for each retailer is given by $\frac{1}{2}(1-p)$ where p is the unit price for the final good. The game is as follows:

1. Firm I proposes an exclusive dealing contract to one retailer. That contract constrains that retailer not to deal with the entrant. The retailer accepts or refuses the contract. If the retailer accepts the contract, it receives a monetary transfer T.

2. Firm E decides to enter on the market

3. In case of entry, both producers compete with linear tariff.

Show that there exists a Subgame Perfect Equilibrium (SPE) where the retailer accepts the ED contract and where E does not enter.

B. Cournot competition

We keep the same game with an additional stage (stage 4) where both retailers compete a la Cournot on the final market. The demand is $q = 1 - p$.

(i) Determine the pricing strategy of I and E at stage 3 in case of entry (you will restrict to the case where the wholesale price of E is equal to c_I and you will show that it is the case if c_I is low enough).

(ii) Show that there is no SPE where E does not enter.

C. Betrand competition

We assume here that retailers compete a la Bertrand at stage 4. Using a similar reasoning, show that there is also no SPE where E does not enter.

As a conclusion, explain informally the role of competition between retailers on the inability of the incumbent to impose the exclusive dealing contract to block the entry of E.

7 Targeted advertising

Consider a market a la Varian (1980). There are 2 firms, with zero marginal production cost for their products. The unit mass of consumers with demand of at most one unit of one product with reservation price r consists in: a proportion h of consumers who only consider buying from firm 1 if the price is below r, the same proportion for firm 2, and $s = 1 - 2h$ shoppers who can buy from either firm and will choose the lowest price firm (if the price is below r). h can be viewed as a measure of market differentiation.

Consumers are endowed with these preferences but without advertising they do not know which products exist. Advertising therefore conveys information about the existence of a product and its price. When a firm decides to advertise on a segment of size x, it costs the firm Ax and all consumers within this segment become informed of the product and the price. When a firm does not advertise at all, it therefore gets zero profits. Assume that $r > A$.

Uniform advertising

We first assume that firms are unable to identify the consumers so they decide simultaneously to advertise the whole market or not to advertise at all and which price to charge.

1. Suppose that both firms advertise in equilibrium; explain intuitively why the equilibrium must be in mixed strategies with respect to prices. Prove that if $hr > A$, firms advertise in equilibrium with probability one.

2. Prove that if $A > r(1 - h)$, no firm advertises in equilibrium.

3. Suppose now that $rh < A < r(1-h)$; prove that in a symmetric equilibrium firms necessarily randomize their advertising decisions. Looking for a symmetric mixed strategy (in advertising and price) equilibrium characterized by α, the probability of advertising for a firm, and $F(.)$, the cumulative price distribution function assumed to have no mass and to have support (z, r), prove that equilibrium profits are null. Deduce the equilibrium value of α and then the price cdf and the lower bound of its support.

4. Perform a comparative statics exercise in the mixed strategy case; in particular, explain the relationship between the equilibrium α and the market differentiation parameter h (remember that when h changes, so does $s = 1 - 2h$!).

Targeted advertising

We now assume that each firm can target advertising to any of the three segments of consumers (shoppers, loyal consumers and consumers loyal to the rival), but firms cannot price discriminate and so charge one unique price for all segments. We maintain the assumption: $rh < A < r(1-h)$.

1. Explain intuitively why the equilibrium must be such that (a) firms target their own loyal consumers with probability one (b) do not target their rival's loyal consumers, (c) target the shoppers with some probability $\beta \in (0, 1)$ and (d) choose prices randomly.

2. Assuming that the cdf has no mass and a support (y, r), characterize the equilibrium profits, the equilibrium probability β and the equilibrium price cdf.

3. Propose an economic discussion of the result: in particular, why do firms advertise more on their strong segment, why do firms advertise with probability less than one on the shoppers segment,...

4. Compare the amount of advertising expenditures under uniform advertising and under targeted advertising and explain your result.

8 Vertical exclusion

We consider a vertical relation with two producers and a buyer. One producer has an incumbent position. The objective of the exercise is to study the exclusionary strategy of that incumbent. The incumbent producer is denoted by I and the other producer, the new entrant, is denoted by E. The gross surplus of the buyer if it buys q_i units to producer i $(i = E, I)$ is equal to:

$$v_E q_E + v_I q_I - h(q_I, q_E)$$

with $h(q_I, q_E) = \frac{1}{2}(q_I)^2 + \frac{1}{2}(q_E)^2 + \sigma q_E q_I$, $0 < \sigma < 1$. The unit production cost of the producer i is equal to c_i.

We denote by $w_i = v_i - c_i$. We assume that w_E is distributed on the interval $[\underline{w}, \overline{w}]$ according to the cdf $F(w)$. The density function is equal to $f(w)$. We also assume that $\underline{w} < \sigma w_I < \overline{w} < \frac{w_I}{\sigma}$.

1. We first determine the optimal levels of output (denoted by $q_i^{**}(w_E)$) that maximize the total surplus.

(i) Write the total surplus that will be denoted by $W(q_E, q_I)$.

(ii) Show that $q_E^{**}(w_E) = Max(\frac{w_E - \sigma w_I}{1-\sigma^2}, 0)$ and $q_I^{**}(w_E) = \frac{w_I - \sigma w_E}{1-\sigma^2}$.

2. The timing of the game is the following.

- The incumbent and the buyer agree on a conditional tariff $T(q_I, q_E)$. At that stage, the buyer and the incumbent do not know the true level of w_E.
- The entrant and the buyer oberve w_E. The entrant proposes to the buyer a level of output q_E and a monetary transfer p_E. The buyer accepts or refuses and then orders to the incumbent a level of output q_I.

We denote by $S^{BE}(q_E, q_I)$ the total surplus that is shared by the buyer and the entrant.

(i) Give the expression of $S^{BE}(q_E, q_I)$.

(ii) Write the first order condition satisfied by the level of output q_E that maximizes $S^{BE}(q_E, q_I)$.

3. Show that the expected surplus that is shared by the buyer and the incumbent at stage 1 is equal to:

$$\int_{\underline{w}}^{\overline{w}} \left[W(q_E(w_E), q_I(w_E)) - \Pi^E(w_E)\right] f(w_E) dw_E$$

where $q_E(w_E)$ and $q_I(w_E)$ denote the levels of output decided at stage 2 and where $\Pi^E(w_E)$ is the profit of the entrant earned at stage 2.

4. The levels of output $q_E(w_E)$ and $q_I(w_E)$ depend on the tariff $T(q_E, q_I)$. In order to determine the optimal tariff $T(q_E, q_I)$, we proceed in two steps. First, we determine the levels of output $q_E(w_E)$ and $q_I(w_E)$ that maximizes the expected surplus at stage 1 (under the constraint that $q_E(w_E)$ and $q_I(w_E)$ must also satisfy the maximization constraints of stage 2) and then we show that we can find a tariff $T(q_E, q_I)$ that implements these levels.

(i) Show that:

$$\int_{\underline{w}}^{\overline{w}} \Pi^E(w_E) f(w_E) dw_E = \Pi^E(\underline{w}) + \int_{\underline{w}}^{\overline{w}} q_E(w_E)(1 - F(w_E)) dw_E$$

(ii) Deduce that the expected surplus shared by the buyer and the incumbent is equal to:

$$\int_{\underline{w}}^{\overline{w}} \left[W(q_E(w_E), q_I(w_E)) f(w_E) - q_E(w_E)(1 - F(w_E))\right] dw_E - \Pi^E(\underline{w})$$

The incumbent and the buyer maximize that joint surplus with respect to $q_E(w_E)$ and $q_I(w_E)$. We denote by $q_I^c(w_E)$ and $q_I^c(w_E)$ the solution of that maximization. Explain informally why $q_I^c(w_E) = q_I^*(w_E, q_E)$ where $q_I^*(w_E, q_E)$ denotes the level of q_I that maximizes the total surplus for a given level of q_E while $q_E^c(w_E)$ is distorted downward with respect to $q_E^{**}(w_E)$.

(iii) Show that $q_I^c(w_E) = q_I^{**}(\widetilde{w}_E)$ and $q_E^c(w_E) = q_E^{**}(\widetilde{w}_E)$ with $\widetilde{w}_E = w_E - \frac{1-F(w_E)}{f(w_E)}$.

(iv) Comment the results in terms of exclusion of the entrant.

5. Consider the tariff $T(q_E, q_I) = c_I q_I + \frac{1-F(w_E)}{f(w_E)} q_E + P$ with $P > 0$.

(i) Show that this tariff implements the output levels found at the previous question.

(ii) Compare $\frac{T(0,q_I^*(0,w_E))-T(q_E,q_I^*(q_E,w_E))}{q_I^*(0,w_E)-q_I^*(q_E,w_E)}$ with c_I. Can we say that q_I is sold below cost? For which purpose?

9 Delegated price setting

We consider a market with two producers ($u = 1, 2$) and two distributors ($d = 1, 2$). Both producers' goods are distributed by both distributors. The demand for product u distributed by distributor d is given by:

$$q_d^u = (1-\beta)(1-\gamma) - p_d^u + \beta p_{-d}^u + \gamma p_d^{-u} - \beta\gamma p_{-d}^{-u}$$

where p_j^i denotes the price of product i distributed by j with $j = d, -d$ and $i = u, -u$. Parameters β and γ are such that $0 \leq \gamma < 1$ and $0 \leq \beta < 1$. All costs are equal to 0.

A producer u and a distributor d share equally the revenue they get (the revenue is equal to $p_d^u . q_d^u$).

1.. Both distributors decide to set the prices of both products. Determine the symmetric Nash equilibrium of that pricing game where both distributors set prices simultaneously.

2.. Both distributors adopt the "agency model" in which they both delegate the pricing to both producers. Determine the symmetric Nash equilibrium of that pricing game where both producers set prices simultaneously.

3.. Compare the profitability of both models A and B. Comment and explain the result.

4.. One distributor adopts the agency model and the other sets the prices of both products. Show that the following prices are a Nash equilibrium in which producers and the distributor set prices simultaneously:

- the distributor (distributor 1) that adopts the agency model chooses for $u = 1, 2$:

$$p_1^u = \frac{1-\beta}{(2-\gamma)(2-\beta^2)}(2(\beta-\gamma) - \beta\gamma + 2)$$

- the distributor that sets both prices chooses for $u = 1, 2$:

$$p_2^u = \frac{1-\beta}{(2-\gamma)(2-\beta^2)}(\beta - \gamma - \beta\gamma + 2)$$

Compare prices if $\beta > \gamma$ and explain the result.

5.. Consider a two-stage game where the distributors first decide which model to adopt (agency model or standard model where a distributor sets both products' prices) and then the firms set prices simultaneously. Consider that both distributors decide to adopt the agency model. Using previous results, explain informally why it may be profitable for one distributor to deviate and to decide to set both prices in the case where $\beta > \gamma$.

10 Exclusive territories and inter-brand competition

We study the impact of exclusive territories on inter-brand competition. For that purpose we consider two firms A and B that produce imperfect substitutes. If we denote by p_i the retail price of product i, the demand for that product is equal to $1 - p_i + \beta p_j$ with $0 < \beta < 1$, $j \neq i$ and $i = A, B$. The production cost of each producer is equal to 0. Each producer sells the good to two identical retailers (2 for each producer). The unit cost of each retailer is equal to 0. Both producers propose a two-part tariff to each retailer. Contracts are public. If a retailer refuses the contract, its profit is 0. Retailers compete in price. In a first stage both producers propose tariffs to each retailer (no discrimination between retailers) and in a second stage, retailers compete in price.

1. Determine retail prices and profits at the equilibrium.

2. One producer decides to grant to each of its retailers an exclusive territory. For simplicity each territory corresponds to half of the demand. Show that this decision is profit increasing for the producer. Explain that result.

11 Vertical restraints with intrabrand competition

We consider a vertical structure with a unique brand produced by a firm U and distributed by two retailers D_1 and D_2. The production cost of all firms is normalized to zero. The demand for the good produced by U and distributed by D_i is equal to $v + e_1 + e_2 - p$ where e_i denotes the pre-sale service provided by firm D_i, v is a parameter with $v > 0$ and p is the unit price of the good. The cost of an amount e_i of pre-sale services is equal to $(e_i)^2$. There is no differentiation between both retailers except the level of pre-sale services.

1. We consider in that question the full vertical integration case (case VI) where U, D_1 and D_2 is a unique firm. Determine in that case the level of pre-sale services denoted by e_i^{VI}, the price p^{VI} and the total welfare (consumer surplus and profit).

In case of vertical separation we contrast the performance of two vertical contracts.

2. The vertical contract specifies here only the unit wholesale price denoted by w. The game is as follows:

- firm U sets w
- each firm D_i determines e_i
- firms D_1 and D_2 compete á la Bertrand.

Determine the subgame perfect equilibrium of that game and the total welfare at the equilibrium. Compare the both welfare and explain the result.

3. The producer imposes exclusive territories to each retailer and uses a two-part tariff with a fixed fee F and a unit wholesale price w. Because of exclusive territories, a firm D_i is a monopoly on a demand equal to $\frac{1}{2}(v + e_1 + e_2 - p_i)$ where p_i is the unit price of the product set by firm D_i. The game is the following:

- firm U sets F and w
- each firm D_i determines e_i
- firm D_i determines its price on its territory.

Determine the subgame perfect equilibrium of the game. Show in particular that at the equilibrium $w < 0$. Determine the total welfare. Compare the result with the welfare obtained in case of vertical integration. Explain the result.

4. Show that a resale price maintenance and two-part tariff may be equivalent to vertical integration.

12 Cost-reducing merger

We consider a n-firm Cournot oligopoly. The unit production cost of each firm is equal to c and the inverse demand is equal $P(Q) = A - Q$ where Q denotes the total output. We assume $c < A$. k firms decide to merge. The unit production cost of the merging entity is equal to $c - x$ with $0 < x < c$.

Determine the minimal cost reduction x that makes the merger beneficial for the consumers. Comment the impact of k and n on such a level.

13 Collusion and multi-market contact

We study the impact of multi-market contact on the stability of collusion. We consider two markets that only differ in their number of firms. Two firms A and B are both present in markets 1 and 2 and firm C is present only in market 2. All firms produce the same product. We assume price competition at each period. Production costs are equal to 0 and the demand is equal to $D(p) = 1 - p$ on both markets. Competition is repeated and the discount factor is equal to $\frac{3}{5}$.

1. Suppose first that firms adopt a "grim trigger strategy" on each market separately. More precisely, on each market, a participating firm starts by setting the monopoly price and keeps on choosing that price as long as the other firms choose the same monopoly price; if a firm deviates on a market, the other firms start the punishment phase by returning to the static Nash equilibrium on this market. Show that collusion is sustainable on market 1 but not on market 2.

2. Suppose now that firms consider both markets altogether and adopt the following extended "grim trigger strategy". Each firm starts by setting the monopoly price on all markets to which it participates and firms agree to give firms A and B equal market shares on market 1 and, on market 2, a market share equal to $\frac{2}{5}$ to firm C and $\frac{3}{10}$ each to firms A and B; firms keep on doing so as long as the others do; if a firm deviates on any market from this collusive arrangement, it triggers punishment on all markets simultaneously and firms return to the static Nash equilibrium on both markets. Show that collusion is sustainable on both markets and explain the difference with the previous question.

14 Warranties may signal quality

The objective of this exercise is to develop a simple framework where a full warranty is a signal of quality. For this, we consider M identical consumers and a firm that offers a product that breaks down with probability q. In case of break down, the surplus is equal to $0 - p$ and if the product does not break down, the surplus is equal to $r - p$ where p denotes the price of the product. If the consumer does not buy the good his/her surplus is zero. The firm can be of two types: the probability of break down can be either high (q^H) or low (q^L) with $q^H > q^L$. The consumers do not observe the type of the firm. The type is q^H with probability λ (initial belief of the consumers). The production cost of the good is equal to c whatever the type. We assume that $r(1 - q^H) < c < r(1 - q^L)$. In case of breakdown, if a firm proposes a full warranty, the firm proposes a new product until it does not breakdown.

1. Interpret the condition $r(1 - q^H) < c < r(1 - q^L)$.

2. Show that the following strategies and beliefs constitute a Bayesian Equilibrium:

- The firm of type q^L proposes a full warranty and the price is equal to r.
- The firm of type q^H does not enter.
- If a consumer observes a firm that does not propose a full warranty, he/she believes it is a product of type q^H.

15 Persuasive advertising

Consider a duopoly in the Hotelling setting, where the firms are located at the extreme points of the unit interval. Firms have a unit cost equal to c. There is a unit mass of consumers distributed on the interval $[0, 1]$ (but not necessarily uniformly, see below) with unit demand and valuation for the good given by v large enough so that the market is fully covered. The "transportation cost" is linear in the "distance" between a consumer and the firm from which he buys, with unit cost denoted t.

We formalize the idea that each firm can engage in advertising to mould consumers' preferences so as to convince them that they really want its product and not the rival one. Let a_i denote firm i's advertising intensity; the cost of advertising a is the same for both firms, given by $\frac{\theta}{2}a^2$. . Advertising is assumed to affect the distribution of the taste of consumers: if firms advertising intensities are a_1 and a_2, consumers are distributed according to the density $f(x; a_1, a_2) = (1 + a_1 - a_2) - 2(a_1 - a_2)x$ over the interval $[0, 1]$.

1. Interpret the model. What is the nature of advertising? What is the impact of having $a_1 > a_2$? What is the situation with symmetric advertising decisions $a_1 = a_2$?

2. Determine the demand for each firm given their respective decisions of advertising and price (a_1, p_1) and (a_2, p_2).

3. Consider that firms chooses simultaneously their advertising campaign and their price. Using symmetry, determine the equilibrium price and advertising intensity, as well as profits.

4. Discuss / explain your results. In particular, what would firms do if they could cooperate in their advertising decisions?

16 Variation on Varian's model of sales

Firms 1 and 2 compete in price on a market with a unit mass of consumers. There is a proportion $\theta \in [0, 1]$ of consumers that are captive to firm 1 and value only its product, with a reservation value v. The other consumers are price-sensitive consumers who value both products, with a reservation value also equal to v, and who buy from the cheapest firm. Firms have zero cost of production.

Without any complicated mathematical argument, using mostly economic and/or intuitive arguments, explain the main properties (mixed strategies, common support, mass point,...) of the following equilibrium in this simultaneous price setting game: firm 1 charges $p_1 = v$ with probability θ and randomizes over $(\theta v, v)$ (with density $\frac{\theta v}{p^2}$), firm 2 randomizes over $(\theta v, v)$ (with density $\frac{\theta v}{(1-\theta)p^2}$).

I expect an intuitive discussion and possibly the writing of some equilibrium condition and simple discussion of it, but certainly not any technical analysis.

17 Search for quality with an intermediary

Two firms compete in face of a unit mass of consumers in the presence of an intermediary.

Each firm decides upon a quality variable $r_i \in \mathbb{R}_+$, $i = 1, 2$, that determines the net utility $u(r_i)$ firm i delivers to a consumer buying from it, where $u(.)$ is increasing and r_i denotes the revenue the firm earns per consumer buying from it. Decision r_i costs the firm a fixed cost $K(r_i)$, with $K(0) = K'(0) = 0$ and $K(.)$ increasing, which is spent irrespective of the consumers' purchase decision. There are no price decisions so that competition is purely in quality provision.

There are 2 types of consumers. Informed consumers observe r_1 and r_2 and therefore always buy from the highest quality firm; they constitute a share $1 - \mu$ of all consumers. The μ remaining consumers are uninformed: they do not observe the firms' qualities, but they can listen which firm the intermediary endorses / recommends. In any case, consumers can only visit one firm.

1. Suppose that the intermediary is objective and honest and always recommends the highest quality firm. Prove that there cannot exist a pure strategy Nash equilibrium in qualities.

2. Characterize the mixed strategy equilibrium in quality choice (not spending too much time on technical points!).

3. Assume now that the intermediary is not objective but rather can be hired by a firm. The game is the following: firms choose their qualities (r_1, r_2); then, after observing their respective choices, they bid in a second-price auction for the intermediary's services; the winner wins the privilege of having the intermediary recommends the uninformed consumers to buy from it and it pays the rival's (loosing) bid. It is then well know that firms have a dominant strategy which is to bid exactly the value of their chosen quality: firm i bids r_i. Then, informed consumers buy from the highest quality firm while uninformed consumers follow the intermediary's advice and buy from the firm that has won the auction (hence the highest quality firm). How is your analysis modified ?

18 Knowledge transfer in alliances

We consider an industry with 3 firms that produce an homogeneous good. The inverse demand is given by $P(Q) = 1 - Q$ where Q denotes the industry output. The unit production cost of each firm i is:

$$c_1 = c - x, \quad c_2 = c, \quad c_3 = c$$

with $0 < c < 1$ and $0 < x < c$. Parameter x is the degree of firm 1 cost advantage due to superior knowledge.

Firms 1 and 2 have an option to form an alliance. They negotiate jointly the level of firm's 1 ownership of firm 2. That level of partial ownership is denoted by θ ($0 < \theta \leq 1$). Given θ, firm 1 decides to transfer its superior knowledge to firm 2. In case of knowledge transfer, the cost of firm 2 becomes $c_2 = c - x$. As long as $\theta \leq \frac{1}{2}$, firm 1 does not decide the level of output of firm 2 and chooses only q_1. If $\theta > \frac{1}{2}$, firm 1 has the full control of firm 2 and decides both levels of output q_1 and q_2. We will consider only the case where $\theta \leq \frac{1}{2}$. The objective of the exercise consists in showing that partial ownership facilitates knowledge transfer.

The game is the following:

- firms 1 and 2 decide jointly to form an alliance. Firms jointly choose the level of partial ownership θ;
- firm 1 decides to transfer knowledge or not;
- firms compete à la Cournot.

1. Explain why at stage 3, the objective of each firm is given by the following program:

$$\text{Firm 1: } \underset{q_1}{Max} q_1(P(Q) - c + x) + \theta q_2(P(Q) - c + xk)$$
$$\text{Firm 2: } \underset{q_2}{Max} (1 - \theta) q_2(P(Q) - c + xk)$$
$$\text{Firms 3: } \underset{q_3}{Max} q_3(P(Q) - c)$$

with $k = 0$ if there is no knowledge transfer and $k = 1$ if there is knowledge transfer.

2. We denote by $\pi_i^*(\theta, k)$ the profit of firm i at the equilibrium of stage 3. Show that at the equilibrium, outputs (stage 3) are given by:

$$q_1^* = \frac{(1-\theta)(1-c) + [3 - (1+2\theta)k]\,x}{4-\theta}$$
$$q_2^* = \frac{1 - c - (1-3k)x}{4-\theta}$$
$$q_3^* = \frac{1 - c - (1 + (1-\theta)k)x}{4-\theta}$$

3. Show that firm 1 accepts to transfer knowledge if and only if

$$\frac{x}{(4-\theta)^2}\left[(2\theta-5)\,x-\left(\theta^2-6\theta+2\right)(1-c)\right]\geq 0.$$

4. We define the minimum ownership participation as the minimum level of θ that makes knowledge transfer profitable for firm 1.

(i) Show that there exists such a minimum partial ownership participation (denoted by $\widetilde{\theta}$) if and only if x is lower than a threshold $\overline{x}=\frac{11}{16}(1-c)$.

(ii) Explain why partial ownership makes possible knowledge transfer.

5. Show that $\pi_1^*(\theta,k)+\pi_2^*(\theta,k)$ decreases if θ increases . Explain that result.

6. Deduce that if firms 1 and 2 create an alliance, they choose $\theta=\widetilde{\theta}$.

7. Make explicit the profit comparison done by firms at stage 1 when they decide to form an alliance.

8. Explain intuitively why the alliance is formed if x belongs to an interval $[\underline{x},\overline{x}]$ with $\underline{x}<\overline{x}$.

9. Discuss informally the impact of the alliance of the consumer surplus.

19 Patent races

Consider a model of a symmetric patent race among n firms as presented in the lecture. If a firm spends xdt between time t and $t+dt$, its probability of making a discovery during this interval is $h(x)dt$ (conditional on noone having made the discovery before), where $h(.)$ is increasing concave. The present discounted reward to being first to introduce the new technology is given by V.

We will compare two models of R&D spending, depending on how we formalize the cost of generating the instantaneous probability of discovery $h(x)$: in the first model, firms make a commitment to R&D with an implied present value cost x; in the second model, firms make a commitment to R&D with an implied present value fixed cost F and an implied flow cost x that stops as soon as one firm has made the discovery.

1. Assume here that firms make their R&D spending x_i once and for all at time $t=0$.

- Letting $a_i\equiv\sum_{j\neq i}h(x_j)$, show that firm i's profit is then given by: $\frac{Vh(x_i)}{a_i+h(x_i)+r}-x_i$. Provide an interpretation of this way to formalize R&D costs.

- Show that at an interior optimum, firm i's R&D decision is a decreasing function of a_i, the aggregate innovation rate of its rivals. Comment.

- Show that at a symmetric interior equilibrium, the R&D intensity of each firm is a decreasing function of the number of firms.

2. Assume now that firms spend the flow cost of R&D only when no discovery has been made.

- Letting $a_i \equiv \sum_{j \neq i} h(x_j)$, show that firm i's profit is then given by: $\frac{Vh(x_i)-x_i}{a_i+h(x_i)+r} - F$. Provide an interpretation of this model of R&D costs.
- Show that at an interior optimum, firm i's R&D decision is an increasing function of a_i, the aggregate innovation rate of its rivals. Comment and compare to the previous case.
- Show that in a symmetric interior equilibrium, the R&D intensity of each firm can be an increasing function of the number of firms.

20 R&D spillovers

Consider an industry with 2 firms facing an inverse demand curve $P(Q) = a - Q$. When both firms decide upon research investments x_1 and x_2, firm i's cost of producing q_i is given by: $(c - x_i - \beta x_j)q_i$. Firm i's cost of research, when choosing research investment x_i, is given by $\gamma \frac{x_i^2}{2}$.

1. Provide an interpretation of β.

2. We first analyze the following two-stage game: (1) firms decide simultaneously and non-cooperatively upon their research investments; (2) after observing their rival's research investment, firms decide simultaneously and non-cooperatively upon their production in a Cournot way. Characterize the symmetric equilibrium R&D investments and productions.

3. We now analyze a two-stage situation in which firms cooperate at the R&D stage: (1) firms jointly decide their research investments so as to maximize the sum of their anticipated profits; (2) after observing the research investments, firms decide simultaneously and non-cooperatively upon their production in a Cournot way as before. Characterize the symmetric equilibrium R&D investments and productions in this game of R &D cooperation.

4. Compare the two situations.

21 Licensing with sequential innovations

Two firms, noted 1 and 2, innovate sequentially. In the first period, firm 1 makes a discovery. If it incurs cost c_1, it can turn this discovery into a new product. A mass 1 of consumers has a valuation for this product of r_1. In the second period (no discounting), firm 2 makes a discovery if (and only if) firm 1 has developed the product in period 1. Incurring cost c_2, firm 2 can turn this discovery into a competing product for which consumers have a valuation of r_2. Assume that consumers have unit demands and buy either one unit of product 1, or one unit of product 2, or nothing at all after the second period. Throughout the exercise, assume:

$$
\begin{aligned}
r_2 &> c_2 + r_1 \\
r_1 &> c_1 > 0
\end{aligned}
$$

1. Briefly interpret these assumptions. Which investment decisions are made in the subgame-perfect Nash equilibrium of the game without an allocation of intellectual property rights, i.e., with standard competition in period 2? Explain why this result may be inefficient from a welfare standpoint.

2. For the same parameter constellation, show that a license fee ϕ payable from firm 2 to firm 1 for every unit of the good sold can induce a welfare optimal allocation. (Assume the following timing of the game: first the fee ϕ is set; then firm 1 makes the investment decision; finally firm 2 makes the investment decision.) In which range must ϕ lie to be effective? What happens if it is too high/too low?

3. Now let us further expand the game in the second period. (For this last part of the problem, assume for simplicity that $r_1 = 0$.) Assume that courts only enforce firm 1 's license claims against firm 2 with probability p. Firm 2 can choose two types of monetary investment:

- "Type a" investment: when c_2^a is spent, this investment increases the value of the product for consumers, so that $\frac{\partial r_2}{\partial c_2^a} > 0$, with $r_2(0) > r_1$ and $\frac{\partial^2 r_2}{(\partial c_2^a)^2} < 0$.
- "Type b" investment: when c_2^b is spent, this investment does not affect the value of the product to consumers, but it reduces the probability that firm 2 would be required to pay the license fee by a court, i.e. $\frac{\partial p}{\partial c_2^b} < 0$, with $\lim_{c_2^b \to \infty} p > 0$ and $\frac{\partial^2 p}{(\partial c_2^b)^2} > 0$.

Find a subgame-perfect Nash equilibrium of the following game: in the first stage, firm 1 chooses a fixed ϕ; in the second stage, firm 2 chooses both its investment levels; in the third stage, consumers make their purchase decisions and courts enforce the license fee ϕ with probability $p(c_2^b)$. What changes if firm 1 can set ϕ as a share of r_2 instead of a fixed fee?

22 Essential patents and downstream Cournot competition

Consider the following market structure: there are 2 R&D labs, A and B, that each have a patent on one technology, and there are 2 manufacturers, 1 and 2, that are Cournot duopolists on a market for an homogeneous final good with inverse demand $P(q) = a - bq$, where $q = q_1 + q_2$. Each manufacturer needs a license from both technologies in order to produce the good, that is, both patents are essential, and there are no other costs involved in producing the final good; so, the unit cost of producing the final good consists in the sum of all royalties paid to the labs per unit of output.

1. Solve for the symmetric subgame-perfect Nash equilibrium of the game in which (1) R&D labs first fix their royalty rates r_A and r_B, and (2) manufacturers observe these rates and engage in Cournot competition. Discuss the properties of the equilibrium in economic terms.

2. Suppose that the 2 R&D labs decide to form a patent pool that charges a royalty rate r for the pool of the 2 patents and split the revenues between the two R&D labs. Solve for a subgame-perfect Nash equilibrium of the game in which (1) the pool fixes the pool royalty rate r and (2) manufacturers observe this rate and engage in Cournot competition. Compare the equilibrium outcome with the one obtained in the first question and explain your results.

3. Now assume that the R&D lab A and the manufacturer 1 merge and the new entity ($A1$) maximizes joint profits. No patent pool is formed. Solve for a subgame-perfect Nash equilibrium of the game in which (1) the merged entity $A1$ and R&D lab B first fix their respective royalty rates r_A and r_B, and (2) the merged entity $A1$ and manufacturer 2 observe both rates and engage in Cournot competition. Discuss the properties of the equilibrium and how the merger affects the economic outcome.

23 The patent thicket

Suppose that N firms, $i = 1, ..., N$, each owns a patent that is essential to the production of a given product. For simplicity, we assume that there is a competitive industry that produces this product. Firm i sets a license fee a_i for the use of its patent by the competitive industry and licensing its patent does not involve any cost for firm i. The price of the final product is denoted by p. The competitive industry does not incur any other cost in addition to paying the licensing fees. Demand for the final product is given by $q = d - p$.

1. Suppose that the N patent holders set their license fees independently and non-cooperatively. Characterize the equilibrium prices and the profit of each patent holder at equilibrium.

2. Suppose now that all firms form a patent pool and choose a common license fee $\overline{a}$ to maximize their joint profit. Assuming that the pool's profit is divided equally among the N patent holders, characterize the equilibrium prices and the profit of each patent holder at equilibrium.

3. Discuss the economic consequences of the formation of the pool.

4. Consider now that k firms, with $1 < k < N$, form a patent pool and coordinate their pricing decision, while the other $N-k$ firms continue to set their fees independently. Characterize the equilibrium prices and the profit of each patent holder at the equilibrium of this new situation.

5. Suppose that $N > 3$. If a pool has to be formed through simultaneous individual decisions of the patent holders, which pool would form ? Comment.

6. Compare this result with Lerner-Tirole (2004) and provide an economic discussion.

24 Cross ownership and technology transfer

The objective of the exercise consists in showing that partial ownership facilitates knowledge transfer. For this, we consider an industry with 3 firms that produce an homogeneous good. The inverse demand is given by $P(Q) = 1-Q$ where Q denotes the industry output. The unit production cost of each firm i is:

$$\begin{aligned} c_1 &= c - x \\ c_2 &= c \\ c_3 &= c. \end{aligned}$$

with $0 < c < 1$ and $0 < x < c$. Parameter x is the degree of firm 1 cost advantage due to superior knowledge, as the result of firm 1's R&D past strategy.

Firms 1 and 2 have an option to form an alliance. They negotiate jointly the level of firm's 1 ownership of firm 2. That level of partial ownership is denoted by θ $(0 < \theta \leq 1)$. Given θ, firm 1 may decide to transfer its superior knowledge to firm 2. In case of knowledge transfer, the cost of firm 2 becomes $c_2 = c - x$. We will restrict attention to $\theta \leq \frac{1}{2}$, so that firm 1 has a minority holding in firm 2 and does not decide the level of output of firm 2; firm 1 only chooses q_1.

The game is the following:

- firms 1 and 2 decide jointly to form an alliance and jointly;
- firm 1 decides to transfer knowledge or not;

- firms compete à la Cournot.

1. Letting $k = 0$ if there is no knowledge transfer and $k = 1$ if there is knowledge transfer, characterize the Cournot equilibrium at stage 3 as a function of θ, x and k. Let $\pi_i^*(\theta, k)$ denote firm i's profit at the equilibrium of stage 3.

2. Show that firm 1 accepts to transfer knowledge if and only if:

$$(2\theta - 5)\, x - \left(\theta^2 - 6\theta + 2\right)(1 - c) \geq 0.$$

Then, show that there exists a minimum partial ownership participation (denoted by $\widetilde{\theta}$), i.e. minimum level of θ that makes knowledge transfer profitable for firm 1, if and only if x is lower than a threshold $\overline{x} = \frac{3}{16}(1 - c)$. Explain why partial ownership makes possible knowledge transfer.

3. Show that $\pi_1^*(\theta, k) + \pi_2^*(\theta, k)$ decreases if θ increases. Explain that result and deduce that if firms 1 and 2 create an alliance, they choose $\theta = \widetilde{\theta}$.

4. Make explicit the profit comparison performed by the firms at stage 1 when they decide to form an alliance. Explain intuitively why the alliance is formed if x belongs to an interval $[\underline{x}, \overline{x}]$ with $\underline{x} < \overline{x}$ (do not try to characterize $\underline{x}$).

25 Bundling by a two-sided monopolist

This exercise asks for some creativity on your part. Of course, I do not expect a full-fledge paper in one hour; moreover, Amelio and Jullien already wrote it: "Tying and Freebies in Two-Sided Markets", International Journal of Industrial Organization, vol. 30, 2012, pp. 436-46 ! What I am interested in is the intuition of the phenomenon and how to formalize it in a model that departs only mildly from what has been covered in class.

Many platforms bundle, or tie, the basic service they propose with an additional product. For example, some platforms offer gifts to one segment of users in addition to their basic service, e.g. magazines offering a free DVD in their paper versions. The purpose of this exercise is to make you think about a model that provides an explanation for this type of practice.

Consider a two-sided intermediation monopolist serving users on two sides, $i = 1, 2$. Users on side i benefit from cross-network externalities from the participation by users of the other side. Users on side i, for $i = 1, 2$, are charged a participation / membership fee p_i (and no transaction / usage charge). The platform has also the technological ability to propose an additional product that generates a value θ for all users on side 1, and that costs $c < \theta$ per unit to be supplied (it will be convenient to distinguish the two-sided intermediation "service" from the "product" in your explanation).

Build a model that explains why the platform may find optimal not to propose the service alone, but instead to practice mixed bundling, i.e. to offer, on the one hand, the service and the product as a bundle at a bundle price and, on the other hand, the product alone at a specific price.

The following two assumptions will be central to your explanation:

- Users on side 1 cannot be subsidized: $p_1 \geq 0$.
- Users on side 1 generate very large positive externalities on side 2 users, while the reverse externality is small

26 Taxation of a two-sided platform

A monopolistic media platform faces a population of users (readers), indexed by θ distributed uniformly on $[0,1]$ and a population of advertisers, indexed by π distributed uniformly on $[0,1]$. The media platform collects personal data on each user to provide him an information service of value v and it uses these data to perform targeted advertising for advertisers. So, if the platform publishes an amount a of advertising, a user of type θ enjoys utility from participating in the platform equal to $U = v - \theta - \gamma a - p$ when the platform charges a subscription fee p; θ measures the benefit of privacy for the user and $\gamma > 0$ (or < 0) the disutility (utility) of advertising for the user. When n users participate in the platform, an advertiser of type π enjoys an expected profit from advertising on the platform equal to: $\pi n - r$ when the platform charges sellers a price for advertising r. We assume that advertising has a small effect on readers: $\mid \gamma \mid < v \leq 1$.

Suppose that the platform is subject to a VAT rate t on its readership revenues (advertising revenues are not taxed) so that its after tax profits are given by: $\frac{pn}{1+t} + ra$. In order to protect the social and informative role of media, the central authority decides to reduce the VAT rate t on readership revenues: is this a good idea ? Would your answer change if the platform was instead subject to a unit tax τ on readership revenues so that its after tax profits would be: $n(p - \tau) + ra$ and the central authority was considering reducing this unit tax?

27 Competition between shopping centers

Do not be afraid by how detailed and precise the text of this exercise is. The answers are quite straightforward and quick, except for question 8.

Consider 2 shopping centers, $j = 1, 2$, a unit mass of sellers and a unit mass of buyers (shoppers). Shopping centers charge each seller a lease price p_j for opening a shop at their location and sellers can multihome. Buyers are heterogeneous and they patronize at most one shopping center: from their perspective, the two shopping centers are differentiated *a la Hotelling* on the segment $[0, 1]$ with linear "transportation cost" t, buyers being uniformly distributed on this segment and the market being fully covered.

Formally, an allocation of users $N = (n_s^1, n_s^2, n_b^1, n_b^2) \in [0, 1]^4$ describes, for each shopping center j, the number of sellers n_s^j opening a shop at this center and the number of buyers n_b^j patronizing this center. If n_b^j buyers patronize shopping center j, a seller's shop generates net value: $\beta_s n_b^j - p_j$ for this seller. If n_s^j sellers have opened a shop at shopping center j, a buyer with horizontal characteristics x obtains net utility equal to: $v + \beta_b n_s^1 - tx$ if he visits shopping center $j = 1$ and $v + \beta_b n_s^2 - t(1-x)$ if he visits shopping center $j = 2$. v is assumed large enough so that buyers always want to patronize one shopping center. Being identical, all sellers will be assumed to make the same choice: they either all open shops in both shopping centers (multihome), or they all open a shop in shopping center j only (singlehome at j), for $j = 1$ or $j = 2$, or else they do not open a shop in any shopping center (do not participate). So, we will only consider allocation of users within $\{0, 1\}^2 \times \{(x, y) \in [0, 1]^2, x + y = 1\}$. Finally, assume that: $\beta_b < t$.

We analyze the following situation: (1) shopping centers choose simultaneously their lease prices $p_j \in [0, \beta_s]$, and then (2) buyers and sellers observe these prices and make simultaneously their decisions, given their anticipations about the other side's strategy.

Definition 1 *: Given a pair of lease prices $P = (p_1, p_2) \in [0, \beta_s]^2$, an allocation of users N is said to be a* ***demand configuration*** *if the following holds:*

- *if $n_s^j = 1$ then, anticipating that n_b^j buyers patronize center j, each seller weakly prefers to open a shop at center j; if $n_s^j = 0$ then, anticipating that n_b^j buyers patronize center j, each seller weakly prefers not to open a shop at center j;*
- *anticipating that (n_s^1, n_s^2) sellers open a shop at shopping center $j = 1, 2$, there are n_b^j buyers patronizing shopping center j when each buyers individually decides optimally which center to patronize.*

Definition 2 *: An* ***equilibrium*** *$(P^*, N(.))$ is defined as a pair of lease prices, $P^* = (p_1^*, p_2^*)$ and a "demand mapping" $N(.)$ from the set of possible prices $[0, \beta_s]^2$ into the set of possible demand configurations, such that P^* constitutes a Nash equilibrium of the price game in which each shopping center anticipates that it will face a demand from buyers and sellers given by $N(P)$ when prices are equal P.*

1. Characterize the set of prices M for which there exists demand configurations in which all sellers multihome. Provide a full characterization of such demand configuration.

2. Characterize the set of prices S_j for which there exists demand configurations in which all sellers singlehome at shopping center j. Provide a full characterization of such demand configuration.

3. Characterize the set of prices O for which there exists demand configurations in which no seller participates. Provide a full characterization of such demand configuration.

4. Does a given vector of lease prices give rise to a unique possible demand configuration? Explain carefully your answer in economic terms. How does it impact the characterization of an equilibrium and why is the above text so careful about the definition of an equilibrium?

5. Show that if $(P^*, N(.))$ is an equilibrium, $(P^*, \hat{N}(.))$ is also an equilibrium when $\hat{N}(P) = N(P)$ for any $P = (p_1^*, p_2)$ and for any $P = (p_1, p_2^*)$ and $\hat{N}(P)$ takes any value in the other cases. So, when characterizing an equilibrium at price P^*, the demand mapping needs only be determined on the two segments $\{(p_1^*, p_2), p_2 \in [0, \beta_s]\}$ and $\{(p_1, p_2^*), p_1 \in [0, \beta_s]\}$.

6. Show that there exists an equilibrium with lease prices equal to $\frac{\beta_s}{2}$ and characterize an associated demand mapping. How is this equilibrium related to what we saw in the course?

7. For any pair of prices $P^* \in M \cap S_1 \cap S_2$, exhibit a demand mapping such that P^* can be supported as an equilibrium with this demand mapping (you may restrict, if you want, to symmetric prices $p_1^* = p_2^*$). Compare this situation with the one characterized at the previous question with respect to the welfare of each category of agents and discuss.

8. Show that there cannot exist an equilibrium P^* such that $P^* \notin M \cap S_1 \cap S_2$. Conclude. *The complete and fully correct answer to this question is rather long, as several cases must be considered; but the argument is always of the same nature. A informative sketch of the proof will be sufficient.*

28 Multihoming and singlehoming

Two platforms, $k = A, B$, provide access to the content published by content providers to a population of mass 1 of viewers; the platforms bear no cost (neither on the viewers' side nor on the content providers' side); viewers may "multi-home", i.e. use the services of both platforms, while content providers are assumed to "single-home", i.e. can publish their content on one platform only.

For each platform, there exists a unit mass of content providers who can publish their content on this platform. When a content provider publishes her content, her benefit is proportional to the total number of viewers who view it, the benefit per viewer being π. So, a content provider publishing her content on platform k at price r_k gets benefits equal to $\pi(n_k + n_M) - r_k$, where n_k denotes the number of viewers exclusively using platform k and n_M the number of viewers multihoming and using both platforms. Whenever indifferent, a content providers chooses to publish her content.

From the viewers' perspective, platforms are differentiated a la Hotelling on a unit segment with uniform distribution and linear transportation cost, and we allow viewers to multi-home. When using the service of platform k only, at price p_k, a viewer located at $x \in [0, 1]$ enjoys a net utility equal to $bm_k - p_k - td(x, x_k)$, where $m_k \in \{0, 1\}$ denotes the number of content providers who publish their content on platform k and $d(x, x_k)$ measures the distance between this viewer at x and platform k, platform A being located at $x_A = 0$ and platform 2 at $x_B = 1$. When using the service of both platforms, the same viewer at x enjoys a net utility equal to $b(m_A + m_B) - p_A - p_B - t$. Note that $n_A + n_B + n_M \leq 1$ and a priori there may be a segment of viewers not using any platform (if the inequality is strict).

Part I: Symmetric equilibrium in a sequential price setting framework

In this part, we consider the following sequential game form under perfect observability: (1) platforms first choose simultaneously viewers' prices (p_A, p_B), (2) viewers all simultaneously decide which platform to use, (3) platforms then choose simultaneously content providers' prices (r_A, r_B), and (4) content providers all simultaneously decide whether to publish their content, all previous moves being observed by players at any stage of the game.

We are going to characterize a symmetric subgame perfect equilibrium, and the condition under which it exists, such that the equilibrium outcome has a segment of viewers multihoming and all content providers publish their content.

I.1. Explain *intuitively* why there may be viewers' multihoming in this framework, what form it would take, and why such an equilibrium can exist only if the differentiation between both platforms is not too large.

I.2. Suppose that after stage 2, the allocation of viewers is given by (n_A, n_B, n_M), all non-negative; show that the continuation equilibrium in this subgame is in dominant strategies and characterize it.

I.3. At stage 2, suppose platforms have chosen prices (p_A, p_B). Given the players' expectation about the continuation game, show first that one can restrict attention to prices for viewers such that $p_k \leq b$. Then determine the equilibrium allocation of viewers for any pair of prices in this compact square of prices. In particular, characterize the region of prices for which there is viewers' multi-homing and the region of prices for which there is viewers' single-homing.

I.4. Let assume in this question that $b + \pi > t$ (H1). Show that under assumption (H1), there exists a unique symmetric equilibrium and it is such that some viewers multi-home; characterize this equilibrium.

I.5. Assuming (H1), comment on this equilibrium outcome in economic terms. In particular: compare it with the outcome in case there is a unique platform and explain; discuss it according to whether $b > \pi$ or $b < \pi$ and explain.

I.6. Assuming that (H1) does not hold, characterize the unique symmetric equilibrium and comment on it.

Part II: A simultaneous price setting framework

In this part, we consider the following sequential game form under perfect observability: (1) platforms first choose simultaneously viewers' prices (p_A, p_B) and content providers' prices (r_A, r_B), (2) Simultaneously, all viewers decide which platform to use and all content providers decide whether to publish their content or not, after observing all prices.

Does the equilibrium outcome characterized in Part I constitute an equilibrium outcome, or even the unique equilibrium outcome, in this simultaneous price setting game ? Explain the difference with Part I.

Part III: Sequential setting with compatible content providers

In this last part, we modify the model of Part I (sequential price setting) by assuming that the group of content providers who can publish their content only on platform k, $k = A, B$, is of size s and that there is a third group of content providers of mass $1-s$ who can publish their content on both platforms, if they find it profitable to do so: that is, we allow some content providers to multihome. We assume that all these compatible content providers behave identically. We also assume that viewers get benefit from viewing a content once, but no additional benefit from viewing it twice, e.g. on both platforms when they multihome.

III.1. Suppose that $s = 0$ so that there is only a group of fully compatible content providers who can multihome. Discuss *intuitively* the possibility of viewers' multihoming.

III.2. Suppose that $0 < s < 1$; how is your discussion of question III.1. modified?

Bonus question. Try to follow the analysis of Part I in the framework of Part III with $0 < s < 1$.