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Scientific progress and irreversibility: an economic interpretation of the 'Precautionary Principle'

Christian Gollier[a,*], Bruno Jullien[a], Nicolas Treich[b,c]

[a] *GREMAQ, Université de Toulouse, Place Anatole France, F-31042 Toulouse Cedex, France*
[b] *ERNA-INRA, University of Toulouse, France*
[c] *CIRANO, University of Montréal, Canada*

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Abstract

We consider the problem of the optimal use of a good whose consumption can produce damages in the future. Scientific progress is made over time that provides information on the distribution of the intensity of damages. We show that this progress induces earlier prevention effort only if prudence is larger than twice absolute risk aversion. This paper thus identifies the class of quite restrictive but plausible conditions such that scientific uncertainties justify an immediate reduction of the consumption of a potentially toxic substance. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Recent international events have made it clear that a major source of uncertainty in our Society is our imperfect scientific knowledge. For example, we know that CO_2 emissions can have an impact on our climate in the long run. But scientists are still debating the intensity of potential damages 100 years from now. Current

*Corresponding author.
E-mail address: gollier@cict.fr (C. Gollier)

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estimates vary by a factor of 20. We also have some scientific doubts about whether the so-called 'mad cow' disease can be transmitted to human beings. Thirty years ago, we had doubts about the possibility of asbestos being responsible for some forms of cancer. More recently, in the mid-1980s, physicians had doubts about the possibility of blood products transmitting AIDS. We can also mention uncertainty associated with electromagnetic fields, radon gas exposure and low radiological exposure. There is a clear link among all these examples, namely, the intensity of potential damage depends upon the accumulation of earlier exposure to the risk, or consumption. For example, climate change relates to the stock of CO_2 in the atmosphere, not just to the current flow of CO_2. This hysteresis effect is due to the relatively long lifetime of this gas (approximately 120 years). Also, the risk of developing asbestosis seems to depend upon the duration of the exposure to asbestos. The same observation holds for low radiological exposure and other risks associated with repeated exposure. The hysteresis effect implies that those problems are intrinsically dynamic.

Another obvious link among all these examples is that the uncertainty prevailing at a given time is resolved, at least partially, over time. Learning takes place which induces decision-makers to adapt their strategy. There is ongoing research on the above-mentioned issues that will provide information about the intensity of potential damage. Also, History tells us that scientific progress is made about our understanding of Nature. The expectation that uncertainty will resolve over time, together with the phenomenon of hysteresis, implies that the existing literature of optimal dynamic risk management is not very satisfactory at providing efficient rules to deal with such situations. One challenge of the theory is thus to provide new rules to deal with risks when a flow of information on the actual distribution of potential damage is expected to arrive over time.

Because of the emergence of many such uncertain situations worldwide, new guidelines for decision-makers have been advocated by international organizations. The 'Precautionary Principle' emerged in the mid-1980s as a clause in international treaties such as the Conference of Rio on Environment and Development or the Maastricht Treaty.[1] Article 15 of the Rio Declaration states that *"where there are threats of serious and irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation."*

The Precautionary Principle is the most notable anticipatory principle existing in national[2] and international law with special relevance for human-induced en-

[1]For more information on the Precautionary Principle, see the book edited by O'Riordan and Cameron (1995).

[2]Similar definitions as the one of the conference of Rio have been enacted in the national law of many countries. For instance, in the French law, the Precautionary Principle is defined as a clause of the Loi Barnier (1995) on the reinforcement of the protection of the environment. It states that *"the absence of certainty, given our current scientific knowledge, should not delay the use of measures preventing a risk of large and irreversible damages to the environment, at an acceptable cost".*

vironmental problems. Although devoid of practical content, the principle is formulated in such a manner as to provide the basis for regulatory policies. Its main message to politicians is conceptually clear: scientific progress does not justify the delay of measures preventing environmental degradation.

This paper proposes an economic interpretation of the Precautionary Principle. We investigate the optimality of a rule which consists in developing immediate prevention efforts even if more information is expected to come in the future about the magnitude of the risk. The way we increase the flow of information about the severity of the risk is the most standard one, i.e. it is based upon the expected utility theory and the Bayes rule of revision of beliefs. The decision-maker initially assigns a probability π to future states of the world. This probability π expresses the current scientific knowledge stated in the Precautionary Principle. But the agent acquires scientific information about the risk of damage and so he can revise π through time. We want to examine how the prospect of receiving information affects the current prevention effort.

To do so, we consider two economies that are equivalent except for the flow of information. To simplify, suppose that in economy A, no information is expected in the future which would allow for a Bayesian revision of the distribution of future risks. The risk borne in economy B is the same ex ante, but some scientific information is expected to flow over time which will improve our knowledge about the distribution of the risk. In which of economies A and B is it optimal to invest more in prevention in the short run? The Precautionary Principle would receive some theoretical support if it were socially efficient to make no less prevention effort in economy B than in economy A.

Two contradictory effects are taking place in economy B. The classical argument would be to invest less in prevention in economy B than in economy A, because delaying sacrifices allows for an efficiency gain generated by the better information. Society will adapt its effort to the severity of the risk. This is the 'learn then act' strategy. However, if this strategy is implemented, the risk borne in the future will be larger ex ante, i.e. before information is known. This is due to the sensitivity of the future prevention effort to the information obtained. As stated for instance by the Intergovernmental Panel on Climate Change IPCC (1995), these two effects do not converge but have opposite effects on optimal CO_2 emissions: *''The choice of abatement paths involves balancing the economic risks of rapid abatement now (that premature capital stock retirement will later proved unnecessary) against the corresponding risk of delay (that more rapid reduction will then be required, necessitating premature retirement of future capital stock).''* The second effect mentioned by IPCC is the only one compatible with the precautionary principle, which is to bias decisions in favor of the 'act then learn' strategy. But we cannot a priori predict the impact of the flow of future information on the optimal timing of prevention efforts and need to examine the tension between these two effects in some particular context.

The paper shows that in a simple model of stock externalities the tension between the two effects depends on the shape of the utility function $v(.)$. We show in particular that prudence, a concept introduced by Kimball (1990), plays an important role in the comparative statics analysis. Prudence is equivalent to the positiveness of the third derivative of the utility function. A prudent individual increases her savings in the face of an increase in *risk* associated with future revenues. The link of this concept with our problem is that the second effect mentioned above deals with an increase in early sacrifices to forearm oneself against an increase in future risks. It is thus not so much of a surprise that prudence is at play when considering the effect of an increase in *uncertainty* on the level of early consumption. This is in spite of the lack of a strong relationship between the notions of a Rothschild and Stiglitz (1970) increase in risk and a Blackwell (1951) increase in uncertainty.[3] We exhibit in this paper the necessary and sufficient condition on the utility function to sign the effect of a better information structure on the optimal level of prevention.

There exist different results in the economic literature on the role of uncertainty and learning in dynamic optimization problems. The literature on certainty equivalence established conditions to guarantee that uncertainty does not affect the optimal course of action. Simon (1956) showed that this result holds if the objective function is quadratic. Malinvaud (1969) relaxed this assumption but at the cost of limiting the analysis to small risks. These two results suggest that the direction of the effect of uncertainty on optimal decisions will depend on properties of the third derivative of the utility function.

Other papers have emphasized the role of learning. Spence and Zeckhauser (1972), Drèze and Modigliani (1972) and Kreps and Porteus (1978) pointed out that individuals regard uncertainties resolved at different times as being different. It is thus a widespread belief that the efficient dynamic management of uncertainties should be different if we expect new scientific knowledge to emerge in a near future rather than in a distant one. Epstein (1980) examined the direction in which the optimal decision changes with learning. He developed a method for investigating the comparative statics of a better information structure. In particular, he provided an elegant proof that the prospect of more information increases the incentive to preserve flexibility.[4] In the area of environmental irreversibility, earlier studies showed that an agent should take stronger measures to prevent future irreversible risks if he expects to obtain information (see Arrow and Fischer, 1974, and Henry, 1974). Thus, learning should influence action in favor of the environment.

However, the irreversible nature of our decision today is only one of the elements entering the picture of environmental problems. Most existing models dealing with learning and irreversibility are such that the decision today affects future welfare only through the constraint of the opportunity set available later. To

[3] An increase of uncertainty à la Blackwell can be seen as a mean-preserving spread in the set of distribution functions.

[4] See also Jones and Ostroy (1984).

illustrate, for the problem of global warming, the irreversibility constraint is our inability to emit a negative quantity of greenhouse gas, i.e. to use energy to remove the excess gas. We believe that the problem is not so much that we will not be able to remove the excess CO_2 in the future, but rather that we will have to reduce significantly the emission of CO_2 if we emit too much of it today. The prospect that one will have to reduce emissions so much that one will want to emit a negative volume of gas seems rather unlikely. But we do not deny this possibility in this paper. Our point is that most environmental problems deal with the management of a limited stock of a good. Varying this stock through early consumption generates an environmental externality for future generations.

In our paper, we consider the problem of the consumption of a substance which may be harmful in the future and where the damage may be irreversible. The final payoff for society depends upon the product of the accumulated consumption and a random variable which represents the intensity of the damage. As noticed by Freixas and Laffont (1984), the irreversibility effect can go in the counter-intuitive direction when the welfare depends directly on the first period decision. In particular, Ulph and Ulph (1997) and Kolstad (1996) showed that sufficient conditions established by Epstein (1980) are not directly applicable when taking into account the accumulation phenomenon mentioned above.[5] Because of the difficulty of deriving analytical conditions in this case, recent contributions used numerical simulations, as in Nordhaus (1994), Manne and Richels (1992) and Torvanger (1997).

To our knowledge, we are the first to provide a complete description of the necessary and sufficient condition that is tractable for this problem with both accumulation and irreversibility. This is of course at the cost of considering a specific payoff function. But we believe that it has many potential applications. The model is presented in Section 2. A second contribution of this paper is to provide a useful decomposition of the basic problem examined in the literature into primitive components. The effect of more uncertainty on the current action entails first what we call a 'precautionary effect' that takes place even without any sort of irreversibility. In short, a change in the learning that is anticipated will affect the optimal exposure to risk in the future. According to Kimball (1990), this change in the future risk induces the decision-maker to take precautionary actions today. This effect is examined in Section 3. The existence of an irreversibility constraint adds another effect. The fact that the decision maker knows that he will be constrained in his future actions is expected to affect his current actions if he is not myopic. Moreover, since the intensity of this constraint in the future depends in general upon current actions, more uncertainty should induce the agent to leave

[5] Kolstad (1996) observed that another kind of irreversibility may be involved in the context of stock externalities: ''If one invests in pollution control and then learns that the damage is low one cannot instantly reduce the abatement capital stock''. See also Pindyck (1991) and Dixit (1992) for an analysis of the irreversible nature of some investments.

more options open for the future. This ‘irreversibility effect’ is considered in Section 4. An important result is that the comparative statics analysis is basically the same with or without the irreversibility constraint.

2. Model

2.1. Economic problem

We consider a two-period model where the only source of utility comes from the consumption c_t of a good in each period t. The good is free but its consumption may turn out to be toxic in the future. The damage representing this toxicity is measured in terms of final consumption and is supported only in the second period. This damage is proportional to the quantity of the good already consumed $C=\delta c_1+c_2$ where the parameter δ denotes the fraction of the good consumed in the first period which remains toxic in the second period. The extent of the damage is unknown at the beginning of the first period. Total damage is thus assumed to be $\tilde{x}C$ where $\tilde{x}$ is the random variable representing this uncertainty. But researchers work on refining the estimates on the damage by performing experiments whose outcomes $\tilde{y}$ will allow the consumer to revise the distribution of $\tilde{x}$ according to the Bayes rule.

Assuming time separability of preferences, the decision problem may be written

$$\max_{c_1} u(c_1)+E_{\tilde{y}}\ \left\{\max_{c_2} E_{\tilde{x}|\tilde{y}}\ v(c_2-\tilde{x}(\delta c_1+c_2))\right\} \tag{1}$$

Utility functions u and v are assumed to be three times differentiable, with $u'>0$, $u''<0$, $v'>0$ and $v''<0$. The concavity of v implies that the second-order condition for c_2 is satisfied. We assume that there is always a solution to that problem. This implies that the maximization problem on c_1 is also well-defined. We also assume that this program is concave, with a unique positive solution.[6]

Program (1) is a simple model of stock pollutant. Present consumption accumulates, and this affects future welfare because the remanence parameter δ is assumed to be strictly positive. This represents many situations where the harmful effects of a product depend on its cumulative concentration in some medium (water, soil, air, body...) and not only on its annual rate of deposition.

To simplify the notation, we will hereafter rewrite problem (1) as

$$\max_{c_1} u(c_1)+E_{\tilde{y}} \max E_{\tilde{z}|\tilde{y}}\ v(-\delta c_1+\tilde{z}C) \tag{2}$$

where $C=\delta c_1+c_2$ is the accumulation of the past consumptions and $\tilde{z}=1-\tilde{x}$ is the marginal benefit of consuming the product, net of the loss due to the toxicity it

[6] This is assumed for expositional simplicity. The results are valid for any function $u(c_1)$ provided that c_2 is finite and uniquely defined.

generates. An advantage of this notation is to make it clear that an increase in consumption during the first period must be compensated by an equivalent reduction in consumption (up to factor δ) during the following period in order to maintain the same level of exposure C to risk $\tilde{z}$.

We solve this model recursively, i.e. we begin to solve the last period problem. Note that in period 2 the outcome of the experiment $\tilde{y}$ is known. Let us then denote π_y for the ex post distribution of $\tilde{z}$ after message y. We can then write the value function of the dynamic problem (2) as

$$\begin{aligned} j(c_1, \pi_y) &= \max_C E_{\tilde{z}/y}\, v(-\delta c_1 + \tilde{z}C) \\ &= E_{\tilde{z}/y}\, v(-\delta c_1 + \tilde{z}C(c_1, \pi_y)) \end{aligned} \tag{3}$$

where $C(c_1, \pi_y)$ is the optimal second period decision. The efficient level of consumption in the first period will be the solution of the program (1) that is simply rewritten as

$$\max_{c_1} u(c_1) + E_{\tilde{y}}\, j(c_1, \pi_{\tilde{y}}) \tag{4}$$

2.2. Information structures

We will now compare two information structures $\tilde{y}$ and $\tilde{y}'$. There are a number of ways (often strictly equivalent) that this has been represented in the literature. To do so, it is more convenient to assume that the random variable $\tilde{z}$ is discrete with an arbitrary number m of atoms. We denote by $Z = (z_1, z_2, \ldots, z_m)$ the support of $\tilde{z}$. In the standard terminology of Blackwell (1951), $\tilde{y}$ describes an '*experiment*', that is, a random variable correlated with $\tilde{z}$. We denote $\pi_y(z_i)$ the probability that $\tilde{z} = z_i$ conditional on receiving signal $\tilde{y} = y$, and $\pi_y = (\pi_y(z_1), \ldots, \pi_y(z_m))$ (resp. $\pi'_{y'}$) is the probability distribution conditional on the value y of $\tilde{y}$ (resp. y' of $\tilde{y}'$). Let us also define $\Delta_Z = \{\pi_y \in R^m_+ \mid \Sigma^m_{i=1}\, \pi_y(z_i) = 1\}$ as the set of all probability distributions on the support Z.

Following Blackwell (1951) and Marschak and Miyasawa (1968), we know that experiment $\tilde{y}$ is more informative than experiment $\tilde{y}'$ if and only if:

$$\text{for any } \rho \text{ convex on } \Delta_Z{:}E_{\tilde{y}}\rho(\pi_{\tilde{y}}) \geq E_{\tilde{y}'}\rho(\pi'_{\tilde{y}'}) \tag{5}$$

In short, this definition means that the beliefs are more dispersed in economy $\tilde{y}$ than in economy $\tilde{y}'$ (see, for example, Jones and Ostroy, 1984). Note that at the extreme, if the information given by the observation of $\tilde{y}$ is perfect, each element of π_y is 0 or 1. This represents a maximum dispersion of beliefs. Note also that Eq. (5) implies

$$\pi = E_{\tilde{y}}\pi_{\tilde{y}} = E_{\tilde{y}'}\pi'_{\tilde{y}'} \tag{6}$$

i.e. the two economies have the same prior beliefs on $\tilde{z}$. The risk is the same in each economy; only the experts differ.

As we said in the introduction, the objective of the paper is to compare the optimal current consumption of the good in the two economies $\tilde{y}$ and $\tilde{y}'$. Observe that from the first-order condition of problem (4), the level of consumption in period 1 is lower (resp. larger) in $\tilde{y}$ if and only if the *future marginal* expected utility is larger (resp. lower) in economy $\tilde{y}$. The first lemma of the paper directly follows from this remark and from Eq. (5). This lemma is a particular case of Theorem 1 in Epstein (1980).

Lemma 1 (Epstein, 1980). *The efficient level of consumption in period* 1 *decreases* (*resp. increases*) *with any better information structure if and only if* $j_{c_1}(c_1, \pi)$ *is concave* (*resp. convex*) *in* π.

This lemma gives a general necessary and sufficient condition on the effect of learning on the current decision. However this condition provided by Epstein is not very useful at this stage since we do not know whether future marginal utility is convex or concave in π. Many authors have shown that the models where Epstein's condition can be directly applied are rather special (see, for example, the survey of Graham-Tomasi, 1995). The problem of stock pollutant is typically a problem where this condition leaves it ambiguous, as shown for example in Kolstad (1996) or Ulph and Ulph (1997). In the rest of the paper we provide a set of conditions complementary to Epstein's which allow the ambiguity to be dissipated.

3. Comparative statics analysis

Our objective now is to examine the convexity/concavity of j_{c_1}. By the envelope theorem, we have

$$j_{c_1}(c_1,\pi) = -\delta E v'(-\delta c_1 + \tilde{z}C(c_1,\pi)) \tag{7}$$

where π is the distribution of $\tilde{z}$. Even without some irreversibility constraint, determining whether $j_{c_1}(c_1,\pi)$ is concave in π is technically complex. Indeed π first appears through the expectation operator in a linear way. But it also appears as an argument in C, the solution of the second period problem when $\tilde{z}$ is distributed as π. This link between C and π is implicitly defined by the first-order condition for problem (2):

$$E\tilde{z}v'(-\delta c_1 + \tilde{z}C(c_1, \pi)) = 0 \tag{8}$$

where $\tilde{z}$ has a probability distribution π.

In order to explore the effect of a better information structure on c_1, let us first examine this problem when the utility function belongs to the well-known class of Hyperbolic Absolute Risk Aversion (HARA) functions such that

$$v(x) = \frac{\gamma}{1-\gamma}\left[\eta + \frac{x}{\gamma}\right]^{1-\gamma} \tag{9}$$

Some restrictions on the parameters are required to make $v(.)$ increasing and concave. In particular, we require $\eta + (x/\gamma)$ to be positive. This is the set of functions whose absolute risk tolerance is linear in x. Indeed, we can easily verify that $-v'(x)/v''(x) = \eta + (x/\gamma)$. If $\eta = 0$, we get the standard Constant Relative Risk Aversion (CRRA) functions, with constant relative risk aversion γ. We get Constant Absolute Risk Aversion (CARA) functions if γ tends to infinity.

The following proposition shows that only a sub-class of HARA functions gives support to the Precautionary Principle. This is proven in Appendix A[7].

Proposition 1. *Suppose that v is HARA. A better information structure reduces (resp. increases) the efficient level of consumption in period* 1 *if and only if* $0 < \gamma < 1$ (*resp.* $\gamma > 1$ *or* $\gamma < 0$). *In the case where $\gamma \to 1$, i.e. $v(x) = \ln(\eta + x)$, the structure of the information has no effect on the efficient level of consumption.*

In the special case of $\eta = 0$, we get the standard power utility functions with constant relative risk aversion. Thus, to determine whether a better experiment reduces early consumption in this case, one should check whether relative risk aversion is less than unity. There is much controversy on this, both from the theoretical perspective, and from the empirical one. Econometric estimates of γ vary from 0.1 to 100. In order to solve the equity premium puzzle, one would require γ to be around 30. If this is true, then a better experiment should increase early consumption instead. Because constant absolute risk aversion implies $\gamma = +\infty > 1$, observe that this result also goes in the opposite direction than the one recommended by the Precautionary Principle.

More generally, Proposition 1 shows that the way in which learning affects the current decision depends on the utility function we consider in the model. This result contrasts to what is asserted in the literature since it is generally believed that it is possible to 'separate risk aversion from the irreversibility effect' (see, for example, Kolstad, 1996). It is quite surprising that the role played by the utility function has not been recognized in the literature on the effect of learning, whereas it is central in the literature which examines the effect of uncertainty (see, for example, Rothschild and Stiglitz, 1970 or for a recent systematic analysis Gollier and Kimball, 1996). A reason for this is that the effect of learning on current decisions may be important even under risk neutrality (see, for example, Henry, 1974).

After obtaining Proposition 1, we tried to discover a more general relationship between precaution and the shape of the utility function. We thus developed the

[7] All proofs of the paper are in Appendices A–E.

comparative statics analysis without any restriction on $v(.)$. But we found that such a general relationship does not exist, as shown by the following proposition.

Proposition 2. *If for all discrete supports Z,* $j_{c_1}(c_1,\pi)$ *is concave in* π *on* Δ_Z*, then the utility function v is HARA with* $0<\gamma<1$*; If for all discrete supports Z,* $j_{c_1}(c_1,\pi)$ *is convex in* π *on* Δ_Z*, then the utility function v is HARA with* $\gamma>1$ *or* $\gamma<0$.

The proposition implies that whenever the utility function v is not HARA, the effect of a better information structure is always ambiguous in the following sense: it is possible to find an economic environment (u, δ, and $\tilde{z}$) and two experiments such that the most informative experiment is associated with the smallest consumption in period 1; but it is also possible to find another environment for which the most informative experiment is associated with the largest consumption in period 1.

The proposition identifies the class of utility functions for which it is always possible to find such counterexamples. This is the class of all non-HARA utility functions. This result complements that of Proposition 1, since Proposition 1 characterized the necessary and sufficient conditions for all HARA utility functions. Thus, together with Proposition 1, Proposition 2 provides a full characterization of problem (1).

Yet, apart from the case of HARA preferences (which is quite restrictive), the proposition says that one cannot defend the Precautionary Principle on the basis of the attitude toward risk only, irrespective of the nature of the risk faced. The key feature for the proof of Proposition 2 is that we require the concavity to hold for all probability distributions and all supports. One direction of research is thus to focus on the risk itself, viewing the Principle as applying to particular types of risk.

As will be seen later on, these restrictions on the probability distributions will allow us to identify the shape of preferences that induce a precautionary behavior for such particular risks. We will see that two functions play an important role in the analysis. The first is the well-known Arrow-Pratt coefficient of absolute risk aversion, denoted $A(.)=-v''(.)/v'(.)$. The second is the coefficient of absolute prudence, denoted $P(.)=-v'''(.)/v''(.)$. This was introduced by Kimball (1990) to measure the strength of the precautionary saving motive. The larger P is, the larger is the willingness to save to forearm oneself against future exogenous risks. The necessary and sufficient condition for this to be true is positive prudence, i.e. $P(s)\geq 0$ for any s. A stronger condition than prudence is decreasing absolute risk aversion $A'\leq 0$, which is equivalent to $P\geq A$.

The following proposition shows that for 'small risks'[8], the Precautionary

[8]We use a concept of 'small' risks that has been examined by Samuelson (1970). For more details, see the proof of the following proposition.

Principle is not necessarily justified either. The consumer must be 'sufficiently prudent' [i.e. $P(.)$ must be larger than $2A(.)$] to reduce current consumption with better information.

Proposition 3. *Suppose that in problem* (1) $\tilde{z}$ *is a small risk in the sense of Samuelson* (1970). *A better information structure reduces* (*resp. increases*) *the efficient level of consumption in period* 1 *if and only if absolute prudence is larger* (*resp. smaller*) *than twice the absolute aversion to risk*, $P(s) \geq 2A(s)$ [*resp.* $P(s) \leq 2A(s)$] *for all s*.

A potential intuition for this result is as follows: because $EC\tilde{z}$ is convex in π,[9] a better information structure increases society's expected consumption which follows its optimal signal-dependent strategy. This increase in expected consumption is obtained from the more efficient level of consumption due to better knowledge of the risk. It generates a wealth effect: the increase in consumption generated by the better information structure tends to induce society to worry less about the future. It tends to *increase* the willingness to consume in period 1. The intensity of this effect depends upon the degree of concavity of v, as measured by A. But another effect takes place. More information means that probabilities are more variable between period 1 and period 2. This yields an increase in uncertainty ex ante because risk exposure will be sensitive to more extreme signals. This increase in uncertainty affects the expected marginal utility of future consumption, which in turn tends to *reduce* the optimal level of early consumption if marginal utility is convex. The strength of this effect thus relies on the convexity of v', which is measured by P. This is a 'precautionary effect'. Proposition 1 states that the Precautionary Principle can be justified only if the 'precautionary effect' has a greater impact than the wealth effect. Notice that the condition $P \geq A$, i.e. decreasing absolute risk aversion is weaker than condition $P \geq 2A$. Thus, decreasing absolute risk aversion is necessary, but not sufficient to economically justify the Precautionary Principle.

Notice also that Proposition 3 illustrates the idea that what is true for 'small risks' is not necessary true for all probability distributions (recall Proposition 2).

We now consider another restriction on the probability distribution which is much in line with the risks studied in the paper. Suppose that the posterior distributions of $\tilde{z}$ have a fixed two-point support. Either the severity of the damage to society is 'low'—no damage at all is a possibility—or the severity of the damage is 'large'. We will see in the following proposition that consumer's degree of prudence still controls the comparative statics analysis in this case.

Proposition 4. *Suppose that in problem* (1) *the distribution of* $\tilde{z}$ *has a two-atom support. A better information structure reduces* (*resp. increases*) *the efficient level*

[9] This is true for small risks.

of consumption in period 1 *if and only if absolute prudence is larger* (*resp. smaller*) *than twice the absolute risk aversion*: $P(s) \geq 2A(s)$ [*resp.* $P(s) \leq 2A(s)$] *for all s*.

The two last propositions thus confirm the natural extension of the condition obtained for power utility functions, i.e. relative risk aversion must be less than unity. Indeed since

$$A(x) = \left[\eta + \frac{x}{\gamma}\right]^{-1} \quad \text{and} \quad P(x) = \frac{1+\gamma}{\gamma}\left[\eta + \frac{x}{\gamma}\right]^{-1} \tag{10}$$

we have $P > 2A$ if and only if $\gamma \in]0,1[$.

The limit case of $\gamma = 1$ for the coefficient of relative risk aversion is not new in the theory of finance.[10] Determining whether P is larger or smaller than $2A$ or equivalently $(1/v')$ concave has appeared in different contexts. Drèze and Modigliani (1972) examined the reduction in the level of savings due to the introduction of complete insurance markets. They proved that the level of savings decreases in such a circumstance if and only if $(\mathrm{d}^2(U_1/U_2)/\mathrm{d}c_2^2) \geq 0$ where U_j denotes partial derivatives of a non-time-separable utility function $U(c_1, c_2)$. Assuming $U(c_1, c_2) = u(c_1) + v(c_2)$, this condition becomes $1/v'$ convex.

Caroll and Kimball (1996) showed that the more general condition $P \geq kA$ $(k > 0)$ generally propagates from utility function to value or indirect function by the following two standard operations: first, addition of utility functions across states of nature at a given point in time; and second, intertemporal aggregation, by maximizing the sum of utility in one period and the value from all subsequent periods. The results of Caroll and Kimball (Lemmas 1 and 2) suggest that introducing some other exogenous risks in our model and/or considering more than two periods should not change Propositions 1, 3 and 4.

Condition $P \geq 2A$ has also been used in the economics of asymmetric information. In the principal-agent problem studied by Sinclair-Desgagne and Gabel (1997), for instance, it constitutes a sufficient condition for the optimality of upper-tail audits, that is, audits triggered by good news rather than bad news. The implementation of such audits can create complementarities between environmental tasks and regular business tasks. Another example is provided by Dionne and Fombaron (1996) in their analysis of the efficient allocation of risks in an economy with adverse selection. They find that $P > 2A$ is sufficient for the efficient frontier in the income-states space to be convex.

Finally, maybe one of the best arguments in favor of the restriction $P \geq 2A$ was made by Debreu and Koopmans (1982). These authors argue that a good measure of risk aversion should be the following 'concavity index' $(-v''/(v')^2)$. If we

[10] See for example Grossman and Vila (1992) who showed that a liquidity constraint affects the optimal portfolio of a non-myopic investor one way or another depending upon whether constant relative risk aversion is larger or smaller than 1.

admit the Debreu and Koopmans index, decreasing absolute risk aversion is thus equivalent to $P \geq 2A$. The Debreu and Koopmans index was recently used in political economy theory. For example, Alesina and Tabellini (1990) showed that the uncertainty in the identity of the median tomorrow generates a bias towards deficit if the concavity index is decreasing. Chandler (1998) proved that a decreasing concavity index insures for a variety of objective functions for the principal that the optimal tax schedule is concave and monotonic.

To sum up, we have examined in this section the comparative statics of a better structure of information in problem (2). We have shown that a better information structure reduces the first period consumption only if absolute prudence is larger than twice the absolute aversion to risk. This is sufficient if the utility function is HARA, or if the future risk is 'small' or has a two-point support.

4. Irreversibility effect

In this last section we shall extend the previous analysis by incorporating an irreversibility constraint into model (1). We introduce the irreversibility constraint $c_2 \geq 0$: it is impossible to reduce the stock of toxic product by more than natural rate of decay δ. This is equivalent to $C \geq \delta c_1$ in problem (2). This constraint represents the 'irreversible damage to the environment' mentioned in the Precautionary Principle.

This irreversibility constraint will affect the level of consumption in period 2. It has in turn an effect on the value function j that determines the optimal c_1. As before, we want to determine whether more information leads to less consumption in period 1.

Intuition suggests that the irreversibility constraint should induce a reduction in current consumption c_1. This would be the consequence of an 'irreversibility effect' as initially stressed by Arrow and Fischer (1974) and Henry (1974). The idea underlying this effect is that more information makes more likely the prospect of receiving some 'catastrophic' news and so makes more likely a situation where it would be optimal to reduce the current stock of pollutant. Thus together with learning, the presence of a lower constraint should increase the marginal value of a reduction of current consumption, since this reduction allows the constraint to be relaxed. This value is called a quasi-option value in the standard terminology.

Obviously, since both a 'precautionary effect' and an 'irreversibility effect' now take place in our problem, we need to prove that these effects combine towards precaution. This is not an easy task, as initially observed by Freixas and Laffont (1984). A striking recent illustration of incompatibility of the two effects is found in Ulph and Ulph (1997), who showed that the irreversibility effect does not apply even in their 'textbook model of global warming'. In some situations the irreversibility effect may require larger rather than lower current emissions of greenhouse gases!

The 'irreversibility effect' introduces a 'quasi-option' effect which measures the benefit of relaxing the constraint $C \geq \delta c_1$ by one unit of consumption: reducing current consumption leaves more possibilities open for the future, and thus has an option value. We show in Appendix E that under the conditions identified for precaution to take the form of a reduced consumption, the irreversibility effect goes in the same direction.

Proposition 5. *Consider the problem* (1) *with the irreversibility constraint* $c_2 \geq 0$. *A better information structure reduces the current efficient consumption if* $P(s) \geq 2A(s)$ *for all s, and if either the utility function is HARA or the risk is binary or 'small'.*

The important message of this section is that condition $P \geq 2A$ together with the sufficient conditions for the reversible case are sufficient in the irreversible case. Since $P \geq 2A$ was already necessary in the reversible case, nothing must be added to the sufficient conditions obtained in the previous section, as stated in Proposition 5. Notice finally that the same results would hold under the more general irreversibility constraint $c_2 \geq k(c_1)$, with $k'(c_1) \leq \delta$.

5. Simple numerical example

To illustrate Proposition 5, we developed numerical simulations. We assumed $v(c) = u(c) = ((100 + c)^{1-\gamma}/(1-\gamma))$, $\delta = 0.9$ and a damage scale x of either 1.15 or 0.8. Only two equiprobable messages determine the ex post distribution of $\tilde{x}$. The first message yields $1/2 + \theta$ and $1/2 - \theta$ for the conditional probability of each state of the world, and the other message yields symmetrically $1/2 - \theta$ and $1/2 + \theta$. We considered nine values of θ between 0 and 0.2 for which we computed the optimal solution of c_1. The advantage of this representation is that information is parameterized by θ: the larger θ is, the higher the information is.

The results are shown in Figs. 1–3. In each figure, we plot the numerical solutions both to the model with the constraint $c_2 \geq 0$ and to the model without the irreversibility constraint (respectively represented by dotted lines and solid lines). The figures show that the two solutions coincide as long as the constraint does not bite, i.e. for rates of learning approximately lower than 0.05.

Fig. 1 describes the solution to the problem when the utility function is assumed to be logarithmic, that is when $P = 2A$. In this case learning has no effect on decisions when the problem is unconstrained: the solid line is a horizontal line. This is the result of proposition 1 with $\gamma \to 1$. Note that in many stochastic climate-economy models the utility function is assumed to be logarithmic. It is thus perhaps not surprising that Nordhaus (1994), Manne and Richels (1992) and others found that learning has little or no effect on decisions. A potential explanation would be that considering logarithmic utility functions together with

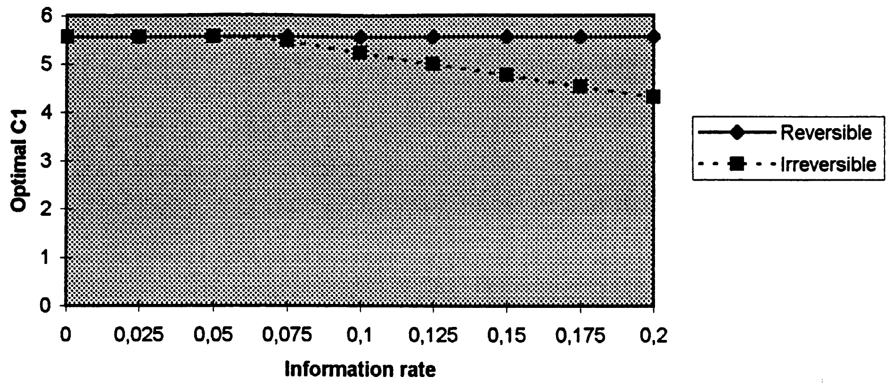

Fig. 1. Numerical simulations with $P = 2A$.

the absence of any irreversibility constraint dilutes the effect of learning on decisions.[11] These results have been used to give support for modest abatement of CO_2 emissions over the next decade.

Fig. 1 also shows that for sufficiently high rates of information, it makes sense to reduce consumption. This is due to the pure ‘irreversibility effect’ which makes precaution optimal. Note that we probably could have reversed this result simply

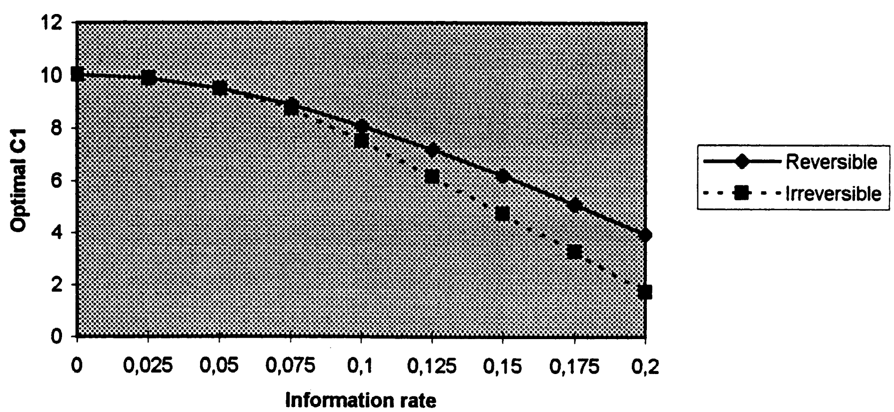

Fig. 2. Numerical simulations with $P > 2A$.

[11] Further research in a model more in line for example with DICE of Nordhaus should say whether it is a good explanation or not.

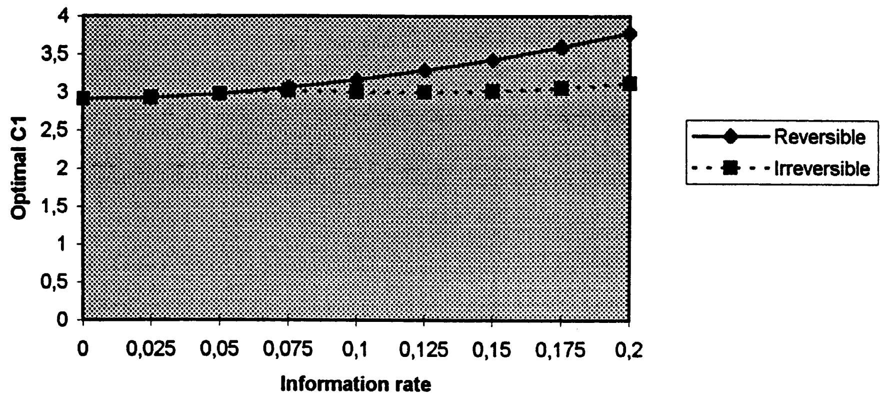

Fig. 3. Numerical simulations with $P < 2A$.

by considering other constraints such as $c_2 \leq kc_1$. Such results are in Kolstad (1996). More precisely, the constraint that Kolstad considers requires that next-period controlled emissions be at least as large as current-period controlled emissions. That new kind of irreversibility makes precautionary policies non-optimal in his model.

Consider now a square-root utility function, i.e. $\gamma = 0.5$. We know from Proposition 1 that precaution is optimal in this case since $\gamma \leq 1$. This is confirmed by our simulations since the solid line is decreasing in Fig. 2. More information leads to reduce consumption even without the constraint. From Proposition 5 we also know that for such a utility function, i.e. for those such that $P \geq 2A$, the 'irreversibility effect' combines with the 'precautionary effect' towards precaution. This means that the reduction of consumption due to learning is higher in the model with the constraint than in the model without the constraint. This is the effect depicted in Fig. 2: for example, going from $\theta = 0$ to $\theta = 0.2$ reduces consumption from 10 to 4 in the model without irreversibility and from 10 to less than 2 in the model with irreversibility.

Finally, suppose that $\gamma = 2$. We know from Proposition 1 that without any constraint precaution is not optimal in this case. In Fig. 3 we can confirm that the solid line is indeed increasing. Now, look at the solution to the constrained problem. For a rate of learning around 0.075, a marginal increase of θ slightly decreases consumption. But around 0.175 such a marginal increase of θ increases consumption. This is a numerical counterexample showing that the global effect of learning is ambiguous at least for some utility function such that $P \leq 2A$. This is because for such utility functions the 'precautionary effect' and the 'irreversibility effect' have opposite directions.

6. Conclusion

This paper has proposed an interpretation of the Precautionary Principle within the standard Bayesian framework. Our interpretation is that more scientific uncertainty as to the distribution of a future risk—that is, a larger variability of beliefs—should induce Society to take stronger prevention measures today.

Three arguments may justify the Precautionary Principle. The first argument is based on the irreversibility of early measures, which entails a 'quasi-option' effect. More uncertainty should induce the decision-maker to favor taking more conservative measures today to value these options more in the future. Arrow and Fischer (1974) and Henry (1974) showed that irreversibility should induce a risk-neutral society to favor current decisions that allow for more flexibility in the future. In a more general model, we isolated this 'quasi-option' effect, and we showed that it always goes in the intuitively appealing direction.

But decisions in the past do not only influence the current opportunity set. They also affect welfare directly: for instance, past emissions of CO_2 increase current exposure to greenhouse risks due to the accumulation of gas in the atmosphere. This accumulation yields two other effects that are not easy to sign, as noticed by Malinvaud (1969), Freixas and Laffont (1984), Kolstad (1996) and Ulph and Ulph (1997). The two additional effects that take place under accumulation are, respectively, a 'precautionary effect' and a 'rigidity effect'. The precautionary effect is that which occurs when the irreversibility constraint is never binding. We showed in this case that more steps should be taken to prevent future risks when the situation is more uncertain—in accordance with the Precautionary Principle—only if absolute prudence is larger than twice the absolute risk aversion. Or, equivalently, if the inverse of marginal utility is concave. Under constant relative risk aversion, this requires relative risk aversion to be less than unity.

We obtained some sufficient conditions guaranteeing that more scientific uncertainty in the future generates a more conservative action today. For example, this is the case if the utility function exhibits linear risk tolerance together with an absolute prudence exceeding twice the absolute aversion to risk. Notice that if absolute risk tolerance is not linear, one can find a prior distribution for the risk and a refinement of the information structure such that the latter induces Society to take less conservative action. If one views the linearity of risk tolerance as very restrictive, our result means that the Precautionary Principle does not follow only from the attitude toward risk. We showed that if we restrict the set of distributions of risk, this conclusion may be relaxed. One line of extension is thus to focus on the nature of the risk faced in situations where the principle is supposed to apply.

Another line of research, which may reflect the motives for the principle, is to view the Precautionary Principle as a safe-guard against the opportunism of decision-makers in situations of asymmetric information or imperfect monitoring by Society.

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Appendix A. Proof of Proposition 1

It is easy to check that if $v(.)$ is defined as in Eq. (9), then the optimal C is proportional to $\eta - \delta(c_1/\gamma)$. More precisely, we obtain $C(c_1,\pi) = c(\pi)(\eta - \delta (c_1/\gamma))$, with $c(\pi)$ defined by $E\tilde{z}(1 + (c\tilde{z}/\gamma))^{-\gamma} = 0$. It yields

$$j(c_1, \pi) = \frac{\gamma}{1-\gamma}\left(\eta - \delta \frac{c_1}{\gamma}\right)^{1-\gamma} g(\pi) \tag{A.1}$$

with $g(\pi) = E(1 + (c(\pi)\tilde{z}/\gamma))^{1-\gamma}$. By the definition of the concept of a better information structure, we know that j is convex in π. Thus, g is convex in π if and only if γ is strictly positive and less than 1. But we have

$$j_{c_1}(c_1, \pi) = -\delta \left(\eta - \delta \frac{c_1}{\gamma}\right)^{-\gamma} g(\pi) \tag{A.2}$$

We conclude that j_{c_1} is concave in π only if γ is strictly positive and less than unity. Lemma 1 concludes the proof. The case of the logarithmic utility function is let to the reader. □

Appendix B. Proof of Proposition 2

Suppose that $j_{c_1}(c_1,\pi)$ is concave in π.

Consider two random variables $\tilde{x}_1$ and $\tilde{x}_2$ such that

$$E\tilde{x}_i v'(-\delta c_1 + \tilde{x}_i) = 0 \tag{B.1}$$

Consider then μ_1 and μ_2 such that

$$\sum_{i=1}^{2} \frac{1}{2} E\mu_i\tilde{x}_i v'(-\delta c_1 + \mu_i\tilde{x}_i) = 0 \tag{B.2}$$

Let $\tilde{z}$ be a risk equal to $\mu_1\tilde{x}_1$ with probability $\frac{1}{2}$, and $\mu_2\tilde{x}_2$ with probability $\frac{1}{2}$. Let π_i denote the distribution of $\mu_i\tilde{x}_i$, $i = 1,2$. Let $\pi = \frac{1}{2}\pi_1 + \frac{1}{2}\pi_2$ be the distribution of $\tilde{z}$. From Eq. (B.1), we know that $C(c_1, \pi_i) = (1/\mu_i)$. From Eq. (B.2), $C(c_1,\pi) = 1$. Then from the concavity assumption:

$$\sum_{i=1}^{2} \frac{1}{2} j_{c_1}(c_1,\pi_i) - j_{c_1}(c_1,\pi) \leq 0$$

For fixed $\tilde{x}_1$ and $\tilde{x}_2$, this is true for all μ_1 and μ_2 satisfying Eq. (B.2). For $\mu_1 = \mu_2 = 1$, $C(c_1,\pi_1) = C(c_1,\pi_2) = C(c_1,\pi) = 1$, so that the LHS vanishes. Thus $\mu_1 = \mu_2 = 1$ is solution of the following program:

$$\max_{\mu_1,\mu_2}\left\{\left\{-\delta\sum_{i=1}^{2}\frac{1}{2}Ev'(-\delta c_1+\tilde{x}_i)\right\}-\left\{-\delta\sum_{i=1}^{2}\frac{1}{2}Ev'(-\delta c_1+\mu_i\tilde{x}_i)\right\}\right\} \tag{B.3}$$

$$\text{subject to constraint (B.2)} \tag{B.4}$$

The first-order condition at $\mu_1 = \mu_2 = 1$ for this problem implies that there exists ν such that:

$$E\tilde{x}_i v''(-\delta c_1 + \tilde{x}_i) = -\nu\{E\tilde{x}_i v'(-\delta c_1 + \tilde{x}_i) + E\tilde{x}_i^2 v''(-\delta c_1 + \tilde{x}_i)\} \tag{B.5}$$

for $i = 1,2$. Using condition (B.1), this condition is equivalent to requiring that

$$E(\tilde{x}_i + \nu\tilde{x}_i^2)v''(-\delta c_1 + \tilde{x}_i) = 0 \tag{B.6}$$

Given that this condition must be satisfied for all pairs of random variables verifying condition (B.1), the value of the coefficient ν must be the same for all such random variables. In short, there exists ν such that:

$$E\tilde{x}v'(-\delta c_1 + \tilde{x}) = 0 \Rightarrow E(\tilde{x} + \nu\tilde{x}^2)v''(-\delta c_1 + \tilde{x}) = 0 \tag{B.7}$$

This implies that exists also β such that for all x: $(x + \nu x^2)v''(-\delta c_1 + x) = \beta x v'(-\delta c_1 + x)$, or, equivalently,

$$-\frac{v'(-\delta c_1 + x)}{v''(-\delta c_1 + x)} = -\frac{1}{\beta} - \frac{\nu}{\beta}x \tag{B.8}$$

Thus absolute risk tolerance is linear in wealth, i.e. if v is HARA.

The proof for the case where $j_{c_1}(c_1,\pi)$ is convex is the same replacing the maximum in program (B.3) by a minimum. □

Appendix C. Proof of Proposition 3

Following Samuelson (1970), let us write $\tilde{z}$ as

$$\tilde{z} = \sigma^2 a + \sigma\tilde{\varepsilon}$$

with $E\tilde{\varepsilon} = 0$. When σ goes to zero, third or higher order moments of $\tilde{z}$ are negligible with respect to the mean and the variance. Thus, when σ approaches zero, the exact Taylor's expansion for $j(c_1, \pi)$ may be truncated before the $E(\tilde{z})^3$:

$$j(c_1,\pi) \cong \max_C v(-\delta c_1) + v'(-\delta c_1)CE\tilde{z} + v''(-\delta c_1)\frac{C^2}{2}E\tilde{z}^2$$

A trivial computation yields the second-order approximation for the solution to this problem:

$$C(c_1, \pi) \cong \frac{-v'(-\delta c_1)}{v''(-\delta c_1)}\frac{E\tilde{z}}{E\tilde{z}^2} \tag{C.1}$$

Note that $\lim_{\sigma\to 0} (E\tilde{z}/E\tilde{z}^2) = (a/E\tilde{\varepsilon}^2)$. This guarantees that the risky position $C(c_1,\pi)$ is not too large.

Similarly, using the truncated Taylor's expansion until $E(\tilde{z})^3$ term for marginal utility yields

$$j_{c_1}(c_1,\pi) \cong -\delta v'(-\delta c_1) - \delta E\big[C\tilde{z}v''(-\delta c_1) + 0.5C^2\tilde{z}^2 v'''(-\delta c_1)\big] \tag{C.2}$$

Replacing C in Eq. (C.2) by its expression in Eq. (C.1), we find:

$$\begin{aligned} j_{c_1}(c_1, \pi) &\cong -\delta v'(-\delta c_1) + \delta\frac{(E\tilde{z})^2}{E\tilde{z}_2}\left[v'(-\delta c_1) - \frac{(v'(-\delta c_1))^2 v'''(-\delta c_1)}{2(v''(-\delta c_1))^2}\right] \\ &= -\delta v'(-\delta c_1) + \delta\frac{(E\tilde{z})^2}{E\tilde{z}^2}\frac{(v'(-\delta c_1))^2}{-2v''(-\delta c_1)}[2A(-\delta c_1) - P(-\delta c_1)] \end{aligned} \tag{C.3}$$

One can easily verify that $((E\tilde{z})^2/E\tilde{z}^2)$ is a convex function of π. Then j_{c_1} is concave in π for compact risks if and only if $P(-\delta c_1)$ is larger than $2A(-\delta c_1)$. Because c_1 and u are arbitrary, this condition should hold for any c_1. □

Appendix D. Proof of Proposition 4

Suppose that $\tilde{z}$ has its support in $\{z_1,z_2\}$, with $z_1 > z_2$. Let w_i denote $-\delta c_1 + C(c_1,\pi)z_i$, together with $p_1 = -z_2/(z_1 - z_2)$ and $p_2 = z_1/(z_1 - z_2)$. The first-order condition (8) is rewritten as:

$$\frac{\pi(z_1)v'(w_1)}{p_1} = \frac{\pi(z_2)v'(w_2)}{p_2} = \lambda(\pi) \tag{D.1}$$

Observe that

$$\frac{j_{c_1}(c_1,\pi)}{-\delta} = \pi(z_1)v'(w_1) + \pi(z_2)v'(w_2) = p_1\lambda(\pi) + p_2\lambda(\pi) = \lambda(\pi) \tag{D.2}$$

Our problem thus simplifies in finding the conditions under which $\lambda(\pi)$ is convex in π. Let us define function ψ such that $\psi(w) = v'^{-1}(1/w)$. Because $p_1 w_1 + p_2 w_2 = -\delta c_1$, condition (D.1) implies that $\lambda(\pi)$ is implicitly defined by

$$\sum_i p_i \psi\left(\frac{\pi(z_i)}{\lambda(\pi)p_i}\right) = -\delta c_1 \tag{D.3}$$

ψ is convex if and only if $(-v'''(w)/v''(w)) \geq 2\ (-v''(w)/v'(w))$, i.e. $P(w) \geq 2A(w)$.

Take any pair of conditional distributions π_1 and π_2 in the support $\{z_1, z_2\}$, and any $t \in [0, 1]$. Suppose that ψ is convex. Then, we obtain

$$\begin{aligned}
&\sum_i p_i \psi\left(\frac{t\pi_1(z_i) + (1-t)\pi_2(z_i)}{t\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\frac{1}{p_i}\right) \\
&= \sum_i p_i \psi\left\{\frac{t\lambda(\pi_1)}{t\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\frac{\pi_1(z_i)}{\lambda(\pi_1)p_i} \right. \\
&\quad \left. + \frac{(1-t)\lambda(\pi_2)}{\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\frac{\pi_2(z_i)}{\lambda(\pi_2)p_i}\right\} \\
&\leq \sum_i p_i \psi\left[\frac{t\lambda(\pi_1)}{t\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\psi\left(\frac{\pi_1(z_i)}{\lambda(\pi_1)p_i}\right) \right. \\
&\quad \left. + \frac{(1-t)\lambda(\pi_2)}{t\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\psi\frac{\pi_2(z_i)}{\lambda(\pi_2)p_i}\right] \\
&= \frac{t\lambda(\pi_1)}{t\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\Sigma_i p_i \psi\left(\frac{\pi_1(z_i)}{\lambda(\pi_1)p_i}\right) \\
&\quad + \frac{(1-t)\lambda(\pi_2)}{t\lambda(\pi_1) + (1-t)\lambda(\pi_2)}\Sigma_i p_i \psi\left(\frac{\pi_2(z_i)}{\lambda(\pi_2)p_i}\right) \\
&= -\delta c_1
\end{aligned}$$

The inequality is the consequence of the convexity of ψ, whereas the last equality is obtained by using condition (D.3) for, respectively, π_1 and π_2. Now compare this result with the definition of $\lambda(t\pi_1 + (1-t)\pi_2)$. Since ψ is increasing, $\lambda(t\pi_1 + (1-t)\pi_2) \leq t\lambda(\pi_1) + (1-t)\lambda(\pi_2)$. This shows that $\lambda(\pi)$ is concave if ψ is convex.

Necessity is obtained by contradiction: if ψ is locally concave, there will exist a support $\{z_1, z_2\}$ and a distribution π on this support such that $\lambda(\pi)$ is locally concave. □

Appendix E. Proof of Proposition 5

Assume $P \geq 2A$ and that $-\delta Ev'(-\delta c_1 + C(c_1,\pi)\tilde{z})$ is concave in π.

Lemma 2. *$Ev'(-\delta c_1 + C(c_1, \pi)\tilde{z}) - E(1-\tilde{z})v'(-\delta c_1 + \delta c_1\tilde{z})$ has the same sign as $C(c_1,\pi) - \delta c_1$.*

Proof. Let $C > \delta c_1$. Then $E\tilde{z}v'(-\delta c_1 + \delta c_1\tilde{z}) > 0$.

Because $(1/v')$ is concave, we have

$$\frac{1}{v'(-\delta c_1 + Cz)} - \frac{1}{v'(-\delta c_1 + \delta c_1 z)} \leq -\frac{v''(-\delta c_1 + \delta c_1 z)}{v'(-\delta c_1 + \delta c_1 z)^2} z(C - \delta c_1)$$

or

$$v'(-\delta c_1 + \delta c_1 z) - v'(-\delta c_1 + Cz) \leq -\frac{v''(-\delta c_1 + \delta c_1 z)}{v'(-\delta c_1 + \delta c_1 z)} z(C - \delta c_1)v'(-\delta c_1 + Cz)$$

Because v is DARA, $-(v''(-\delta c_1 + \delta c_1 z))/(v'(-\delta c_1 + \delta c_1 z))z \leq -(v''(-\delta c_1))/(v'(-\delta c_1))z$, so that

$$Ev'(-\delta c_1 + \delta c_1\tilde{z}) - Ev'(-\delta c_1 + C\tilde{z}) \leq -\frac{v''(-\delta c_1)}{v'(-\delta c_1)}(C - \delta c_1)E\tilde{z}v'(-\delta c_1 + C\tilde{z}) \leq 0$$

Suppose now that $C < \delta c_1$. Then $E\tilde{z}v'(-\delta c_1 + \delta c_1\tilde{z}) < 0$.

$$\frac{1}{v'(-\delta c_1 + \delta c_1 z)} - \frac{1}{v'(-\delta c_1 + Cz)} \leq -\frac{v''(-\delta c_1 + Cz)}{v'(-\delta c_1 + Cz)^2} z(\delta c_1 - C)$$

or

$$v'(-\delta c_1 + Cz) - v'(-\delta c_1 + \delta c_1 z) \leq -\frac{v''(-\delta c_1 + Cz)}{v'(-\delta c_1 + Cz)} z(\delta c_1 - C)v'(-\delta c_1 + \delta c_1 z)$$

$$Ev'(-\delta c_1 + C\tilde{z}) - Ev'(-\delta c_1 + \delta c_1\tilde{z})$$

$$\leq -\frac{v''(-\delta c_1)}{v'(-\delta c_1)}(\delta c_1 - C)E\tilde{z}v'(-\delta c_1 + \delta c_1\tilde{z}) \leq 0$$

□

Define the set

$$\Pi(c_1) = \{\pi \in \Delta_Z | E\tilde{z}v'(-\delta c_1 + \delta c_1\tilde{z}) \geq 0\} \tag{E.1}$$

We want to show that

$$j_{c_1}(c_1,\pi) = \begin{cases} -\delta Ev'(-\delta c_1 + C(c_1,\pi)\tilde{z}), & \text{if } \pi \in \Pi(c_1); \\ -\delta E(1-\tilde{z})v'(-\delta c_1 + \delta c_1\tilde{z}), & \text{if } \pi \notin \Pi(c_1) \end{cases} \tag{E.2}$$

is concave in the distribution π of $\tilde{z}$.

We decompose the problem into four parts.

(i) *π_1 and π_2 are in $\Pi(c_1)$*: Then $\lambda_1\pi_1 + \lambda_2\pi_2$ is in $\Pi(c_1)$. The concavity of j_{c_1} in the region where the constraint is not binding is examined in the previous section.

(ii) *π_1 and π_2 are in $\overline{\Pi}(c_1)$*: In this case, j_{c_1} is linear in π.

(iii) *π_1 is in $\Pi(c_1)$, π_2 is in $\overline{\Pi}(c_1)$, and $\lambda_1\pi_1 + \lambda_2\pi_2$ is in $\overline{\Pi}(c_1)$.*

Concavity reduces here to

$$\lambda_1 E(1-\tilde{z}_1)v'(-\delta c_1 + \delta c_1\tilde{z}_1) + \lambda_2 E(1-\tilde{z}_2)v'(-\delta c_1 + \delta c_1\tilde{z}_2) \tag{E.3}$$

$$\leq \lambda_1 Ev'(-\delta c_1 + C(c_1,\pi_1)\tilde{z}_1) + \lambda_2 E(1-\tilde{z}_2)v'(\delta c_1 + \delta c_1\tilde{z}_2) \tag{E.4}$$

where $\tilde{z}_i$ is distributed as π_i. This is true because $C(c_1,\pi_1) \geq \delta c_1$.

(iv) π_1 is in $\Pi(c_1)$, π_2 is in $\overline{\Pi}(c_1)$, and $\lambda_1\pi_1 + \lambda_2\pi_2$ is in $\Pi(c_1)$.

Concavity writes

$$\lambda_1 Ev'(-\delta c_1 + C\tilde{z}_1) + \lambda_2 Ev'(-\delta c_1 + C\tilde{z}_2) \leq \lambda_1 Ev'(-\delta c_1 + C(c_1,\tilde{z}_1)\tilde{z}_1) + \lambda_2 E(1-\tilde{z}_2)v'(-\delta c_1 + \delta c_1\tilde{z}_2) \tag{E.5}$$

where $C = C(c_1, \lambda_1\pi_1 + \lambda_2\pi_2) \geq \delta c_1$. But

$$\lambda_1 Ev'(-\delta c_1 + \delta c_1\tilde{z}_1) + \lambda_2 Ev'(-\delta c_1 + \delta c_1\tilde{z}_2)$$
$$\leq \lambda_1 Ev'(-\delta c_1 + C(c_1,\pi_1)\tilde{z}_1) + \lambda_2 Ev'(\delta c_1 + C(c_1\pi_2)\tilde{z}_2)$$

and

$$\lambda_2 Ev'(\delta c_1 + C(c_1\pi_2)\tilde{z}_2) \leq \lambda_2 E(1-\tilde{z}_2)v'(\delta c_1 + \delta c_1\tilde{z}_2)$$

because $C(c_1\pi_2) \leq \delta c_1$. Thus, concavity holds. □

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