

DO MERGER EFFICIENCIES ALWAYS MITIGATE PRICE  
INCREASES?\*ZHIQI CHEN<sup>†</sup>GANG LI<sup>‡</sup>

In a Cournot model with differentiated products, we demonstrate that merger efficiencies in the form of lower marginal costs for the merging firms (the insiders) lead to higher post-merger prices under certain conditions. Specifically, when the degree of substitutability between the two insiders is not too high relative to that between an insider and an outsider, increased efficiencies may exert upward rather than downward pressure on the prices of the merging firms. Our results suggest that in cases where firms engage in quantity competition, antitrust authorities should not presume that efficiencies will necessarily mitigate the anticompetitive effects of the merger.

## I. INTRODUCTION

MERGERS CAN CREATE EFFICIENCIES, arising from sources such as scale economy, rationalized production schedule between plants, and acquisition of complementary technologies. It is widely accepted by economists that lower marginal costs brought about by merger efficiencies will encourage firms to compete more aggressively, thus mitigating the loss of competition that may be caused by a merger. As Werden [1996 p. 409] has noted, ‘If a merger caused a reduction in marginal cost for the merging firms, the cost reduction would offset the anticompetitive effect of the merger on prices. Indeed, if the merger reduced the marginal costs of the merging firms by a sufficient amount, it would cause all prices in the industry to fall.’

Consistent with this conventional wisdom among economists, antitrust authorities typically associate the amount of price reduction with the magnitude of merger efficiencies. For example, the U.S. *Horizontal Merger Guidelines* [2010 pp. 30–31] states, ‘the Agencies consider whether cognizable efficiencies likely would be sufficient to reverse the merger’s potential to harm customers in the relevant market, *e.g.*, by preventing price increases

\*We would like to thank Yongmin Chen, Gamal Atallah, Ping Lin, Chengzhong Qin, Aggey Semenov, Adam Wong, Wen Zhou, the Editor and three anonymous referees as well as seminar participants at University of Manitoba and Shanghai University of Finance and Economics for their comments.

<sup>†</sup>Authors’ affiliations: School of Economics, Nanjing University, Nanjing, China and Department of Economics, Carleton University, Ottawa, Ontario, Canada.

*e-mail:* zhiqi.chen@carleton.ca

<sup>‡</sup>School of Economics, Nanjing University, Nanjing, China.

*e-mail:* gangli@nju.edu.cn

in that market.’ The EC *Guidelines on the Assessment of Horizontal Mergers* [2004, paragraph 79] indicates that ‘the relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger. For that purpose, efficiencies should be substantial...’ In Canada, the *Competition Act* contains a provision that instructs the Competition Tribunal not to block a merger that ‘has brought about or is likely to bring about gains in efficiency that will be greater than, and will offset, the effects of any prevention or lessening of competition that will result or is likely to result from the merger...’ Indeed, in discussions of merger enforcement policies and practices it is often held as self-evident that merger efficiencies mitigate anticompetitive effects and exert downward pricing pressure (see, e.g., Fisher *et al.* [1989] and Salop [1987]).

In this paper, we challenge the conventional wisdom that merger efficiencies in the form of lower marginal costs will always counteract the price increases arising from the loss of competition. We do so by examining the effects of a merger in a model where firms produce differentiated goods and compete in quantities. The merger generates efficiencies so that the marginal costs of the merging firms (the insiders) are lower after the merger. We show that an increase in efficiencies may either raise or reduce the post-merger prices of the insiders, depending on the substitutability pattern among the products offered by the insiders and their rivals (the outsiders) as well as the number of competitors. Specifically, increased efficiencies exert upward – rather than downward – pressure on prices if the degree of substitutability between the two insiders is not too high relative to that between the insiders and outsiders, and the number of competitors is not too small.

The intuition behind this result rests on a strategic effect of efficiencies on the insiders’ prices. As the marginal costs of the insiders fall, they expand their output. This direct effect of efficiencies exerts downward pressure on the insiders’ prices. The outsiders, on the other hand, reduce their quantities in response to the output expansion by the insiders. This strategic effect of efficiencies exerts upward pressure on the insiders’ prices. Our analysis shows that the strategic effect dominates the direct effect under the conditions mentioned in the preceding paragraph.

Note, however, this result does not hold in a model where firms produce a homogeneous good. This is because under Cournot competition the slope of a firm’s best-response function is between  $-1$  and  $0$ ; in other words, a firm would reduce its quantity by less than 1 unit in response to a 1-unit increase in output by its rivals. This implies that in terms of its impact on the *aggregate output* produced by all firms, the direct effect of efficiencies is greater than the strategic effect. In other words, a reduction in the marginal costs of the insiders leads to a larger aggregate output. When firms produce a homogeneous good, a larger aggregate output causes the price to fall independent of which firms produce the incremental

output. Therefore, efficiencies always lead to a lower price in a model where firms produce a homogeneous good.

In our model of differentiated goods, on the other hand, a larger aggregate output does not necessarily cause the price of a good to fall. When the degrees of substitutability vary across goods, the magnitude of the price impact of an additional unit of output depends on which firm produces the incremental unit; the impact on a firm's price is smaller if the incremental unit is produced by a rival with a low degree of substitutability than if it is manufactured by a competitor with a high degree of substitutability. This observation suggests that even if efficiencies have a larger impact on the output of insiders than on that of outsiders, they may still cause the prices of insiders to rise if the degree of substitutability is low between the two insiders relative to that between insiders and outsiders.

As the preceding discussions show, our result depends critically on the assumption of differentiated goods. In the literature, in contrast, theoretical analyses of merger efficiencies are typically conducted in the framework of homogeneous Cournot oligopoly.<sup>1</sup> They include Farrell and Shapiro [1990], Levin [1990], Cheung [1992], Froeb and Werden [1998], Motta and Vasconcelos [2005], Banal-Estañol *et al.* [2008], Amir *et al.* [2009], and Jovanovic and Wey [2012]. Since firms produce a homogeneous good in these models, a reduction in marginal costs after a merger always leads to a lower price.<sup>2</sup>

This paper is organized as follows. The model is presented in Section II, and the price effects of merger efficiencies are analyzed in Section III. Sections IV and V examine the overall effects of the merger on profits and prices, while Section VI considers the impact of efficiencies on consumer welfare. In Section VII, we discuss the implications of our findings for merger enforcement policy, and we conclude in Section VIII.

## II. THE MODEL

Consider an industry where firms produce differentiated goods and compete in quantities. Initially, there are  $n (\geq 3)$  firms. Each firm produces one good at constant marginal cost, denoted by  $c$ . Then two of these firms, firm 1 and firm 2, decide to merge.

We are interested in a situation where the merger generates efficiency gains. Specifically, suppose the merger generates efficiency gains that reduce the marginal costs of the merging firms to  $(1-e)c$ , where  $e \in (0, 1)$

<sup>1</sup> One exception is Werden [1996], which studies merger efficiencies in a differentiated Bertrand model.

<sup>2</sup> In particular, Froeb and Werden [1998] derive a formula for the amount of reduction in marginal costs needed to prevent a merger from increasing price in a Cournot industry.

measures the magnitude of merger-induced efficiencies.<sup>3</sup> The marginal costs of the other firms, on the other hand, are not affected by the merger. The focus of our analysis will be on how the price impact of the merger is influenced by the size of  $e$ .<sup>4</sup>

Buyers in this industry could be either consumers or other firms. In the latter case, the firms could be manufacturers that use these goods as inputs, distributors that resell the goods to other firms, or retailers that resell the goods to final consumers.

To keep the analysis simple and tractable, we suppose that the demand functions are symmetric for the insiders and, respectively, for the outsiders. This enables us to analyze the equilibrium behavior of the insiders and outsiders as two groups. Specifically, suppose that firm  $i$  ( $= 1, 2, \dots, n$ ) faces the following inverse demand function:  $p_i = P^i(\mathbf{q}_I, \mathbf{q}_O)$ , where  $p_i$  is the price of product  $i$ ,  $\mathbf{q}_I \equiv (q_1, q_2)$  the vector of quantities produced by the insiders (i.e., firm 1 and firm 2), and  $\mathbf{q}_O \equiv (q_3, \dots, q_n)$  the vector of quantities produced by the outsiders (i.e., firms 3 through  $n$ ). We will use subscript  $j$  to denote the partial derivative of  $P^i(\mathbf{q}_I, \mathbf{q}_O)$  with respect to  $q_j$ . Thus,  $P_j^i \equiv \partial P^i / \partial q_j$  indicates the responsiveness of price  $i$  to a small change in the quantity of good  $j$ . For  $i \neq j$ ,  $P_j^i$  measures the degree of substitutability between good  $i$  and good  $j$ .

To be more specific, we assume that the demand structure is symmetric in the sense that firms within each group have the same degree of substitutability when they produce the same quantity. Moreover, we assume each inverse demand function is separable in its arguments.<sup>5</sup> These assumptions imply that the system of demand functions can be written in the following form:

$$(1) \quad P^i(\mathbf{q}_I, \mathbf{q}_O) = \Psi_I(q_i) + \Phi_{II}(q_j) + \sum_{k=3}^n \Phi_{IO}(q_k), \quad i, j = 1, 2, \quad i \neq j$$

$$P^i(\mathbf{q}_I, \mathbf{q}_O) = \Psi_O(q_i) + \sum_{j=1}^2 \Phi_{IO}(q_j) + \sum_{j \neq 1, 2, i} \Phi_{OO}(q_j), \quad i = 3, 4, \dots, n$$

(2)

where  $\Psi_I(q_i)$ ,  $\Psi_O(q_i)$ ,  $\Phi_{II}(q_i)$ ,  $\Phi_{IO}(q_i)$  and  $\Phi_{OO}(q_i)$  are twice continuously differentiable functions of  $q_i$ . Furthermore, we assume that these functions satisfy the following assumptions.

<sup>3</sup> In this analysis, we do not consider merger efficiencies in the form of a reduction in fixed costs because its implication for post-merger prices is obvious. Provided that the number of active outsiders remains the same after the merger, the reduction in the insiders' fixed costs will have no impact on post-merger prices of all firms.

<sup>4</sup> Hence, the merger and its efficiencies are exogenous in this model. Banal-Estañol *et al.* [2008] and Jovanovic and Wey [2012] consider endogenous mergers and endogenous efficiencies in the homogeneous Cournot framework. It would be interesting to extend their analyses to a Cournot model with differentiated goods.

<sup>5</sup> This assumption serves to eliminate the second-order cross partial derivatives of the inverse demand functions (i.e.,  $P_{jk}^i = 0$  for  $j \neq k$ ) from the comparative static analysis. This makes it much easier to interpret our results, in particular, the conditions in Propositions 1 and 3.

*Assumption 1.* The goods are imperfect substitutes, that is,  $\Psi'_I(q) < \Phi'_{II}(q) < 0$ ,  $\Psi'_O(q) < \Phi'_{OO}(q) < 0$ ,  $\Psi'_I(q) < \Phi'_{IO}(q) < 0$  and  $\Psi'_O(q) < \Phi'_{IO}(q) < 0$ .

*Assumption 2.* The marginal revenue of each firm is a decreasing function of its own quantity both before and after merger, that is,  $2\Psi'_I(q) + q\Psi''_I(q) < 0$ ,  $2\Psi'_I(q) + q\Psi''_I(q) + q\Phi''_{II}(q) < 0$  and  $2\Psi'_O(q) + q\Psi''_O(q) < 0$ .

To understand Assumption 1, observe that  $P^i_i = \Psi'_I$  and  $P^i_j = \Phi'_{II}$  for the insiders (i.e.,  $i, j=1, 2, i \neq j$ ), and  $P^i_i = \Psi'_O$  and  $P^i_j = \Phi'_{OO}$  for the outsiders (i.e.,  $i, j=3, \dots, n, i \neq j$ ). This suggests that  $\Phi'_{II}$  (respectively,  $\Phi'_{OO}$ ) measures the degree of substitutability between the insiders (respectively, between any pair of outsiders). Moreover, note that  $P^i_j = \Phi'_{IO}$  for  $i=1, 2$  and  $j=3, \dots, n$ . In other words,  $\Phi'_{IO}$  represents the degree of substitutability between an insider and an outsider.

Regarding Assumption 2, note that the marginal revenue of the merged entity with respect to  $q_i$  ( $i=1, 2$ ) is  $\Psi_I(q_i) + \Phi_{II}(q_j) + \sum_{k=3}^n \Phi_{IO}(q_k) + q_i\Psi'_I(q_i) + q_j\Phi'_{II}(q_j)$ , where  $j=1, 2$  and  $j \neq i$ , and the marginal revenue of other firms takes the form  $P^i(q_I, q_O) + q_i\Psi'_K(q_i)$ , where  $K=I$  if  $i=1, 2$  (before the merger) and  $K=O$  if  $i=3, \dots, n$  (before and after the merger). Since the marginal cost is constant for every firm, diminishing marginal revenue ensures that the second-order condition of a firm's profit-maximization problem is satisfied.

Since  $\Phi_{II}(q)$ ,  $\Phi_{IO}(q)$  and  $\Phi_{OO}(q)$  can be different functions, this demand system admits the possibility that the degree of substitutability between the two insiders is different from that between an insider and an outsider, which, in turn may be different from that between any pair of outsiders. One advantage of this demand specification is that it enables us to investigate how the price impact of efficiencies is influenced by the degree of substitutability between the two insiders relative to that between an insider and an outsider.<sup>6</sup>

Given the symmetry in the demand system, we will focus our analysis on symmetric equilibria in which the two insiders produce the same quantity and all outsiders choose the same output. We will continue to use subscript  $I$  to denote the variables of an insider and subscript  $O$  those of an outsider.

Next, we present the firms' profit-maximization problems and the resulting first-order conditions before and after the merger. Recall that

<sup>6</sup> In practice, one factor considered by antitrust authorities in their assessment of the unilateral effects of a merger is whether the products of the merging parties are close substitutes relative to those offered by other competitors. See, for example, Section 6.1 of the *U.S. Horizontal Merger Guidelines* [2010].

Assumption 2 ensures that the second-order conditions of these optimization problems are satisfied.

Before the merger, each firm solves the following profit-maximization problem:

$$(3) \quad \max_{q_i} [P^i(\mathbf{q}_I, \mathbf{q}_O) - c]q_i.$$

This yields the standard first-order condition:

$$(4) \quad P^i + q_i P_i^i = c.$$

We use superscript  $C$  to indicate the pre-merger equilibrium. Accordingly,  $q_I^C$  and  $q_O^C$  denote the pre-merger quantity of an insider and an outsider, respectively. Similarly,  $\pi_I^C$  and  $\pi_O^C$  denote the pre-merger equilibrium profit of an insider and an outsider, respectively.

After the merger, the insiders' profit-maximization problem becomes

$$(5) \quad \max_{q_1, q_2} [P^1(\mathbf{q}_I, \mathbf{q}_O) - (1-e)c]q_1 + [P^2(\mathbf{q}_I, \mathbf{q}_O) - (1-e)c]q_2.$$

Note in (5) that the post-merger marginal costs of the insiders are  $(1-e)c$ . The first-order conditions can be written in the form:

$$(6) \quad P^i + q_i P_i^i + q_j P_j^i = (1-e)c \quad (i, j = 1, 2, i \neq j).$$

The optimization problem and the first-order condition of an outsider are the same as (3) and (4). The post-merger equilibrium quantities are then determined by (4) and (6).

Let  $q_I^M$  and  $q_O^M$  denote the post-merger quantity of an insider and an outsider, respectively. Then (4) and (6), along with (1)-(2), imply that  $q_I^M$  and  $q_O^M$  are determined by the following two equations:

$$(7) \quad \Psi_I(q_I^M) + \Phi_{II}(q_I^M) + (n-2)\Phi_{IO}(q_O^M) + q_I^M \Psi'_I(q_I^M) + q_I^M \Phi'_{II}(q_I^M) = (1-e)c;$$

$$(8) \quad \Psi_O(q_O^M) + 2\Phi_{IO}(q_I^M) + (n-3)\Phi_{OO}(q_O^M) + q_O^M \Psi'_O(q_O^M) = c.$$

We use  $\pi_I^M$  and  $\pi_O^M$  to denote the post-merger equilibrium profit of an insider and an outsider, respectively.

### III. PRICE EFFECTS OF EFFICIENCIES

The objective of this paper is to challenge the conventional wisdom that efficiencies in the form of lower marginal costs will always mitigate the upward pricing pressure arising from a horizontal merger. Hence, we start

by examining the impact of efficiencies on the post-merger prices of the insiders and outsiders. We do so in this section under the assumption that the merger is profitable for the insiders. We will address the issue of merger profitability in Section IV, and we will combine the findings from Sections III and IV to analyze the overall effects of the merger on prices in Section V.

We first consider the impact of efficiencies on equilibrium quantities. Conducting comparative statics on (7) and (8), we obtain:

$$(9) \quad \frac{\partial q_I^M}{\partial e} = - \frac{c[2\Psi'_O + q_o^M \Psi''_O + (n-3)\Phi'_{OO}]}{J},$$

$$(10) \quad \frac{\partial q_O^M}{\partial e} = \frac{2c\Phi'_{IO}(q_I^M)}{J},$$

where  $J \equiv [2(\Psi'_I + \Phi'_II) + q_I^M(\Psi''_I + \Phi''_II)][2\Psi'_O + (n-3)\Phi'_{OO} + q_o^M \Psi''_O] - 2(n-2)\Phi'_{IO}(q_I^M)\Phi'_{IO}(q_O^M)$  is the Jacobian determinant associated with (7)-(8).

For simplicity, we have omitted the arguments in the functions  $\Psi_I(q_I)$ ,  $\Psi_O(q_O)$ ,  $\Phi_{II}(q_I)$  and  $\Phi_{OO}(q_O)$ . But we have retained the argument in the function  $\Phi_{IO}(q_i)$  to avoid ambiguity.

From (9) and (10), we observe:

*Lemma 1.*  $\partial q_I^M / \partial e > 0$  and  $\partial q_O^M / \partial e < 0$  if and only if  $J > 0$ .

*Proof.* The numerators of (9) and (10) are negative because  $2\Psi'_O + q_o \Psi''_O < 0$ ,  $\Phi'_{OO} < 0$  and  $\Phi'_{IO} < 0$ . Hence, the signs of (9) and (10) are determined by the sign of  $J$ , as described in Lemma 1. ■

Lemma 1 states that larger efficiencies increase the quantity of each insider but reduce the quantity of each outsider as long as  $J > 0$ . If  $J < 0$ , on the other hand, we would have a perverse situation where a reduction in the marginal cost of the merged entity causes the insiders to contract output and the outsiders to expand output.<sup>7</sup> Recall that the main objective of this paper is to demonstrate that efficiencies can lead to higher prices. This result would not be very convincing if it were obtained from a perverse situation where a lower marginal cost reduces the insiders' quantities and increases the outsiders' quantities. Therefore, we make the following assumption to rule out this situation:

<sup>7</sup> This is considered a perverse situation because we normally expect that a firm with a lower marginal cost should be able to produce more rather than less output in equilibrium. Moreover, the equilibrium under  $J < 0$  is 'unstable' in the sense that if the equilibrium is perturbed, adjustments of quantities by the insiders and outsiders in accordance with (7) and (8) would move  $(q_I, q_O)$  further away from the equilibrium point.

*Assumption 3.*  $J > 0$ .

We are now able to present our core result that efficiencies can cause the insiders' prices to rise. Using (1), we calculate how the post-merger price of an insider changes with the size of efficiencies:

$$(11) \quad \frac{\partial p_I^M}{\partial e} = [\Psi'_I(q_I^M) + \Phi'_{II}(q_I^M)] \frac{\partial q_I^M}{\partial e} + \Phi'_{IO}(q_O^M)(n-2) \frac{\partial q_O^M}{\partial e}.$$

Further analysis of (11) reveals:

*Proposition 1.* A larger  $e$  raises the post-merger prices of the insiders if and only if

$$(12) \quad \frac{1 + \Phi'_{II}(q_I^M)/\Psi'_I(q_I^M)}{\Phi'_{IO}(q_O^M)/\Psi'_I(q_I^M)} < \frac{2(n-2)\Phi'_{IO}(q_I^M)}{[2\Psi'_O + q_O^M\Psi''_O + (n-3)\Phi'_{OO}(q_O^M)]}$$

holds in the post-merger equilibrium.

*Proof.* Substituting (9) and (10) into (11), we find,

$$(13) \quad \frac{\partial p_I^M}{\partial e} = \frac{c[2(n-2)\Phi'_{IO}(q_I^M)\Phi'_{IO}(q_O^M) - (\Psi'_I + \Phi'_{II})(2\Psi'_O + q_O^M\Psi''_O + (n-3)\Phi'_{OO})]}{J}.$$

Since  $J > 0$ , the sign of (13) is determined by the sign of its numerator. The latter is positive if and only if (12) holds. ■

Proposition 1 suggests that efficiencies can indeed exert upward pressure on the insiders' post-merger prices under some circumstances.<sup>8</sup> This would arise if the left-hand side of (12) is relatively small, i.e., if the degree of substitutability between the products of the two insiders (normalized by the size of an insider's own price effect),  $\Phi'_{II}(q_I^M)/\Psi'_I(q_I^M)$ , is low relative to that between the products of an insider and an outsider,  $\Phi'_{IO}(q_O^M)/\Psi'_I(q_I^M)$ .

To see the intuition behind condition (12), note from (11) that efficiencies affect the insiders' prices through two channels. The first channel is through each insider's output ( $\partial q_I^M/\partial e$ ). Lemma 1 states that lower marginal costs induce the insiders to expand output after the merger, and the

<sup>8</sup> To be clear, the focus of this section is on the impact of efficiencies (i.e., lower marginal costs) on the post-merger prices alone. Here we do not consider whether the post-merger prices are higher or lower than the pre-merger prices. The comparison of pre- and post-merger prices is done later in Section V.

output expansion tends to reduce the insiders' prices. This is the direct effect of efficiencies on the post-merger prices of the insiders. The second channel is through every outsider's output ( $\partial q_O^M / \partial e$ ). In Lemma 1, we see that larger efficiencies cause each outsider to reduce its output. This is the strategic effect arising from the fact that outputs are strategic substitutes. As the insiders expand outputs in response to their lower marginal costs, the outsiders react by contracting their outputs. The latter tend to push up the insiders' prices.

As we can see from (11), the size of the direct effect decreases with the magnitude of  $|\Psi'_I(q_I^M) + \Phi'_{II}(q_I^M)|$ , while the size of strategic effect increases with the magnitude of  $|\Phi'_{IO}(q_O^M)|$ .<sup>9</sup> Accordingly, the direct effect is dominated by the strategic effect if  $|\Psi'_I(q_I^M) + \Phi'_{II}(q_I^M)|$  is sufficiently small relative to  $|\Phi'_{IO}(q_O^M)|$ . Alternatively, we can normalize the degrees of substitutability,  $|\Phi'_{II}(q_I^M)|$  and  $|\Phi'_{IO}(q_O^M)|$ , by the size of the own price effect  $|\Psi'_I(q_I^M)|$ , and say that the direct effect is dominated by the strategic effect if  $\Phi'_{II}(q_I^M) / \Psi'_I(q_I^M)$  is not too large relative to  $\Phi'_{IO}(q_O^M) / \Psi'_I(q_I^M)$ . In a nutshell, that is how we interpret condition (12).<sup>10</sup>

Note that (12) depends also on other factors such as the degree of substitutability among outsiders ( $\Phi'_{OO}$ ) and the number of firms ( $n$ ). By inspection we see that, *ceteris paribus*, the right-hand side of (12) decreases with the value of  $|\Phi'_{OO}|$  but has an ambiguous relationship with  $n$ .

To flesh out fully the implications of condition (12), we consider the special case of a linear demand system. Specifically, suppose the inverse demand functions take the following form:

$$(14) \quad p_i = \alpha - q_i - \gamma q_j - \delta \sum_{k=3}^n q_k \quad (i, j = 1, 2, i \neq j),$$

$$(15) \quad p_i = \alpha - q_i - \delta \sum_{j=1}^2 q_j - \varphi \sum_{j \neq 1, 2, i} q_j \quad (i = 3, 4, \dots, n).$$

In (14)-(15),  $\alpha > c$  and the values of  $\gamma$ ,  $\delta$  and  $\varphi$  are strictly between 0 and 1.

Relating to the general demand functions in (1)-(2), we now have  $\Psi'_I = \Psi'_O = -1$ ,  $\Phi'_{II} = -\gamma$ ,  $\Phi'_{IO} = -\delta$ , and  $\Phi'_{OO} = -\varphi$ . Accordingly,  $\gamma$  measures the degree of substitutability between the two insiders (firms 1 and 2),  $\delta$  indicates the degree of substitutability between an insider and an outsider, and  $\varphi$  captures the degree of substitutability between any pair of outsiders. A larger  $\gamma$  (respectively,  $\delta$ ) means that the goods produced by the two insiders (respectively, by an insider and an outsider) are closer substitutes.

<sup>9</sup> Recall that  $\Psi'_I(q_I^M)$ ,  $\Phi'_{II}(q_I^M)$  and  $\Phi'_{IO}(q_O^M)$  are all negative.

<sup>10</sup> We will provide more intuition below using the case of linear demand functions.

Using the linear demand functions, we can obtain the closed-form solutions to the equilibrium quantities and prices before and after the merger. We present these and other details of this special case in Appendix A.

Regarding our key condition (12), we use the linear demand functions (14)-(15) to find the following conditions.

*Proposition 2.* Suppose the market demand functions are linear, as represented by (14)-(15). A larger  $e$  raises the post-merger prices of the insiders if and only if

$$(16) \quad \gamma < \frac{2(n-2)\delta^2}{(n-3)\varphi+2} - 1.$$

To satisfy (16), it is necessary that  $\varphi < 2\delta^2$  and

$$(17) \quad n > \frac{4\delta^2+2-3\varphi}{2\delta^2-\varphi}.$$

*Proof.* Condition (16) is obtained by rewriting (12) using the linear demand system (14)-(15). Since  $\gamma > 0$ , the right-hand side of (16) has to be positive. To satisfy the latter, we need  $2(n-2)\delta^2 > (n-3)\varphi+2$ . Rearranging terms, we find

$$(18) \quad (2\delta^2-\varphi)n > 4\delta^2+2-3\varphi.$$

Suppose that the right-hand side of (18) is positive. Then to satisfy (18), it is necessary that  $\varphi < 2\delta^2$  (to ensure that the left-hand side is positive). Solving (18) for  $n$ , we obtain (17).

As the last step of the proof, we show that the right-hand side (18) indeed has to be positive. Suppose not. That is, suppose  $4\delta^2+2-3\varphi \leq 0$ . This would imply that  $\varphi \geq (4\delta^2+2)/3 > 2\delta^2$  because  $\delta < 1$ . In other words, the left-hand side of (18) would be negative as well. Then to satisfy (18), we would need

$$(19) \quad n < \frac{3\varphi-(4\delta^2+2)}{\varphi-2\delta^2}.$$

Note that the right-hand side of (19) is smaller than 3 for  $\delta < 1$ . In other words, (19) contradicts the assumption  $n \geq 3$ . Therefore, it is impossible to satisfy (18) if its right-hand side is not positive. ■

Note that the condition  $\varphi < 2\delta^2$  in Proposition 2 is not binding for some values of  $\delta$ . Specifically, it is slack if  $2\delta^2 \geq 1$ , or equivalently, if

$\delta \geq \sqrt{2}/2$ . Moreover, as shown in Appendix A, the right-hand side of (17) is greater than 3 for  $\delta \in (0, 1)$ . This implies that (16) holds only if the number of firms is four or greater.

Proposition 2 provides more clarity to the finding in Proposition 1. It clearly shows that efficiencies exert upward pressure on the prices of the merging firms if the degree of substitutability between the two insiders (as measured by  $\gamma$ ) is not too high relative to that between an insider and an outsider (as measured by  $\delta$ ). For this to occur, moreover, the number of competitors ( $n$ ) cannot be too small and (in the case where  $\delta < \sqrt{2}/2$ ) the degree of substitutability between any pair of outsiders ( $\varphi$ ) cannot be too high.

To illustrate the intuition behind the above observations, we rewrite (11) using the linear demand functions:

$$(20) \quad \frac{\partial p_I^M}{\partial e} = -(1+\gamma) \frac{\partial q_I^M}{\partial e} - \delta(n-2) \frac{\partial q_O^M}{\partial e}.$$

The two terms on the right-hand side of (20) represent the direct effect and strategic effect of efficiencies. From (20), we see that the strategic effect may dominate the direct effect if  $\gamma$  is small relative to  $\delta(n-2)$ . The number of firms play a role here because the aggregate size of the strategic effect depends on the number of outsiders ( $n-2$ ).

To be more specific,  $\gamma$  influences the magnitude of the direct effect because it determines the size of the reduction in an insider's price when the other insider expands output (in response to its lower marginal cost). On the other hand,  $\delta$  affects the magnitude of the strategic effect because it determines the amount of increase in the insider's price when an outsider contracts output. For the direct effect to be dominated by the strategic effect, therefore,  $\gamma$  cannot be too large relative to  $\delta$ .

Furthermore, the linear demand system enables us to gain a better understanding about the role played by the degree of substitutability between any pair of outsiders ( $\varphi$ ). While  $\varphi$  does not appear in (20) explicitly, its impact on  $p_I^M$  manifests itself primarily through the strategic effect term  $\partial q_O^M / \partial e$ .<sup>11</sup> Recall that (17) implies  $n > 3$ , which means that there is more than one outsider and hence each outsider competes with at least one other outsider (as well as with the two insiders). As observed earlier, when insiders expand their quantities due to their lower marginal costs, outsiders will reduce their quantities. But the magnitude of output reduction by each

<sup>11</sup> To be more precise,  $\varphi$  also affects the magnitude of the direct effect term  $\partial q_I^M / \partial e$ . This arises because when the outsiders reduce their quantities in response to the expansion of output by the insiders, this reduction by outsiders induces the insiders to expand output further. As shown in Appendix A, however, at the aggregate level the magnitude of this 'echo' effect is smaller than that through  $\partial q_O^M / \partial e$ . Hence, our intuition here focuses on the effect of  $\varphi$  on  $\partial q_O^M / \partial e$ .

outsider depends on  $\phi$  because of the competition among the outsiders. *Ceteris paribus*, a reduction in the output of one outsider induces other outsiders to expand their quantities. This factor mitigates the initial reduction in an outsider's output. The smaller is the value of  $\phi$  (i.e., the lower is the degree of substitutability among the outsiders), the smaller is the influence of this mitigating factor and hence the larger is the size of the strategic effect. This explains why a relatively small  $\phi$  may be needed in order to satisfy (16).

Two additional observations about the conditions in Proposition 2 are worth noting. First, condition (16) permits but does not require that  $\gamma < \delta$ . In particular, it is shown in Appendix A that in the case where  $\phi = \delta$ , condition (16) implies that  $\gamma < \delta$ . In this case, larger efficiencies lead to higher prices only if the insiders' products are more distant substitutes for each other than for those of the outsiders. This case is worth noting because it is the type of situations where a merger would normally be considered as unlikely to cause price increases.<sup>12</sup>

In the case where  $\phi < \delta$ , on the other hand,  $\gamma < \delta$  is not necessary for (16) to hold. As shown in Appendix A, both (16) and  $\gamma > \delta$  may hold simultaneously if  $n > 4$  and  $\phi < (n-4)/(n-3)$ . Therefore, we should interpret the restrictions on the sizes of  $\gamma$  and  $\delta$  imposed by (16) in relative terms, rather than in absolute terms. Efficiencies may lead to higher prices for the insiders even if  $\gamma$  is greater than  $\delta$  in absolute terms.

Second, the positive relationship between post-merger prices and efficiencies does not arise if the firms produce a homogeneous good. As shown in Appendix A, the conditions in Proposition 2 cannot be satisfied by  $\gamma = \delta = \phi$ . Intuitively, when firms produce a homogeneous good, the price of the good depends on the aggregate output but not on the distribution of output among different firms. Since the slope of a firm's best response function under Cournot competition is between  $-1$  and  $0$ , a firm would reduce its quantity by less than 1 unit in response to a 1-unit increase in output by its rivals. This implies that in terms of its impact on the *aggregate output* produced by all firms, the direct effect of efficiencies is greater than the strategic effect. In other words, a reduction in the marginal costs of the insiders leads to a larger aggregate output. Therefore, efficiencies always lead to a lower price in a model where firms produce a homogeneous good.<sup>13</sup>

<sup>12</sup> A merger is unlikely to generate substantial unilateral price increases if non-merging parties offer very close substitutes for the products offered by the merging firms' (*U.S. Horizontal Merger Guidelines* [2010 Section 6.1]).

<sup>13</sup> While the discussion in this paragraph is based on the case of linear demand functions, this result remains valid if we consider a general demand function  $P(\sum_{i=1}^n q_i)$  that satisfies the standard assumptions  $P' < 0$  and  $P' + q_i P'' < 0$ . Details are available upon request.

In the case of differentiated goods, on the other hand, a larger aggregate output does not necessarily cause the price of an individual good to fall. When the degrees of substitutability vary across goods, the magnitude of the price impact of an additional unit of output depends on which firm produces the incremental unit; the impact on a firm's price is smaller if the incremental unit is produced by a rival with a low degree of substitutability than if it is manufactured by a competitor with a high degree of substitutability. This observation suggests that even if efficiencies have a larger impact on the output of insiders than on the output of outsiders, they may still cause the prices of insiders to rise if the degree of substitutability is low between the two insiders relative to that between an insider and an outsider.

While our main interest is in the effects of efficiencies on the prices of the insiders, the analysis would not be complete without a consideration of the outsiders' prices. Going back to the general model, we use (9) and (10) to derive the following result.

*Proposition 3.* A larger  $e$  leads to higher prices for each outsider if and only if

$$(21) \quad \Psi'_O + q_O^M \Psi''_O > 0$$

holds in the post-merger equilibrium.

*Proof.* Differentiating (2) and substituting (9) and (10) for  $\partial q_I^M / \partial e$  and  $\partial q_O^M / \partial e$ , we find:

$$(22) \quad \frac{\partial p_O^M}{\partial e} = - \frac{2c(\Psi'_O + q_O^M \Psi''_O) \Phi'_{IO}(q_I^M)}{J}.$$

Since  $J > 0$  and  $\Phi'_{IO} < 0$ , (22) is positive if and only if  $\Psi'_O + q_O^M \Psi''_O > 0$ . ■

Note that a necessary condition for (21) to hold is that  $\Psi''_O > 0$ . If  $\Psi''_O \leq 0$ , on the other hand, a larger  $e$  would lead to lower prices for the outsiders. This implies that in the case of linear demand functions (where  $\Psi''_O = 0$ ), larger efficiencies lead to lower post-merger prices for the outsiders.

Intuitively, the effects of efficiencies on the outsiders' prices can go either way because of two opposing forces implied by Lemma 1. On the one hand, a larger  $e$  reduces the post-merger quantity of each outsider, which tends to raise the price of an outsider. On the other hand, larger efficiencies cause the insiders to expand output, which tends to push down the price of an outsider. The net effect is, in general, ambiguous.

Finally, if we combine the results in Propositions 1 and 3 and consider the impact of larger efficiencies on the (weighted) average price of insiders and outsiders, it is clear that the average price may either increase or decrease depending on whether (12) and (21) are satisfied. To be more specific, the average price will increase if both (12) and (21) are met (and hence both the insider prices and outsider prices rise with  $e$ ).<sup>14</sup> If both (12) and (21) are violated, on the other hand, the average price will fall with  $e$ . The impact of efficiencies on the average price will be ambiguous if one, and only one, of (12) and (21) is satisfied.

#### IV. MERGER PROFITABILITY

In this section, we investigate the effects of the merger on the profits of the insiders and outsiders. Since we have used a Cournot model of oligopoly, our analysis would not be complete without addressing the merger paradox. It is well-known in the literature that a merger is usually not profitable (for the merging firms) in a homogeneous Cournot model if the merger generates no efficiency gains.<sup>15</sup> In the present model, however, the merger can generally be profitable because of product differentiation and efficiencies. Below we characterize the conditions under which the merger is indeed profitable.

Differentiating an insider's post-merger profit, we can easily confirm that  $\partial\pi_I^M/\partial e > 0$  (see eq. (24) below for detail); that is, larger efficiencies improve the insiders' profitability. Therefore, it is reasonable to expect that the merger should be profitable for the insiders if efficiencies are sufficiently large.

To establish formally the above result in the general model, we need an additional assumption. Recall from Lemma 1 that the quantity of an insider rises, and the quantity of an outsider falls, with the size of efficiencies  $e$ . It is then possible that if  $e$  is sufficiently large, the post-merger quantity of an insider could be larger than its pre-merger quantity. In this and the next section, we assume that this is indeed the case at  $e=1$ , that is,

*Assumption 4.*  $q_I^M > q_I^C$  if  $e=1$ ,

Intuitively,  $e=1$  means that each insider's marginal cost falls to 0 after the merger. Therefore, if  $c$  is sufficiently large to start with, a merger with  $e=1$  would confer the insiders such a large cost advantage that they would produce larger quantities than before the merger. Indeed, in the special case of linear demand functions, it is straightforward to show that Assumption 4 is satisfied if  $c$  exceeds a certain threshold.<sup>16</sup>

<sup>14</sup> In Section VI, we will present a demand system under which this can indeed occur.

<sup>15</sup> See, for example, Salant, *et al.* [1983] and Lommerud and Sörgard [1997].

<sup>16</sup> See condition (A11) in Appendix A.

Recall that  $\pi_I^C$  and  $\pi_I^M$  denote an insider's profit before and after the merger, respectively. Hence, the merger is profitable for the insiders if  $\pi_I^M > \pi_I^C$ .

*Proposition 4.* There exists an  $e_\pi \in [0, 1)$  such that  $\pi_I^M > \pi_I^C$  if  $e > e_\pi$ .

*Proof.* Note that an outsider's best-response function is the same before and after the merger. Accordingly, (8) represents a relationship between an outsider's quantity and an insider's quantity both before and after the merger. We solve (8) to express  $q_O$  as a function of  $q_I$ , denoted by  $q_O = \tilde{q}_O(q_I)$ . Then  $q_O^C = \tilde{q}_O(q_I^C)$  and  $q_O^M = \tilde{q}_O(q_I^M)$ . Differentiating (8), we find:

$$(23) \quad \tilde{q}'_O(q_I) = -\frac{2\Phi'_{IO}(q_I)}{2\Psi'_O + q_O\Psi''_O + (n-3)\Phi'_{OO}} < 0.$$

Assumption 4 and (23) imply that  $q_O^M < q_O^C$  at  $e=1$ .

To compare  $\pi_I^M$  at  $e=1$  with  $\pi_I^C$ , note that  $\pi_I^M = P^1(\mathbf{q}_I^M, \mathbf{q}_O^M)q_I^M > P^1(\mathbf{q}_I^C, \mathbf{q}_O^M)q_I^C$  because  $\mathbf{q}_I^M$  is the insiders' best response to  $\mathbf{q}_O^M$ . Then  $q_O^M < q_O^C$  implies  $P^1(\mathbf{q}_I^C, \mathbf{q}_O^M)q_I^C > P^1(\mathbf{q}_I^C, \mathbf{q}_O^C)q_I^C > [P^1(\mathbf{q}_I^C, \mathbf{q}_O^C) - c]q_I^C = \pi_I^C$ . Hence,  $\pi_I^M > \pi_I^C$  at  $e=1$ .

Differentiating  $\pi_I^M$  and applying the envelope theorem, we find

$$(24) \quad \frac{\partial \pi_I^M}{\partial e} = cq_I^M + (n-2)\Phi'_{IO}(q_O^M)\tilde{q}'_O \frac{\partial q_I^M}{\partial e} > 0.$$

If  $\pi_I^M < \pi_I^C$  at  $e=0$ , there exists a unique  $e_\pi \in (0, 1)$  such that  $\pi_I^M = \pi_I^C$  at  $e = e_\pi$  and  $\pi_I^M > \pi_I^C$  for  $e > e_\pi$ . On the other hand, if  $\pi_I^M > \pi_I^C$  at  $e=0$ , we set  $e_\pi=0$ . ■

Proposition 4 confirms the conjecture that the merger is profitable for the insiders when efficiencies are sufficiently large. This result complements the finding in Perry and Porter [1985] that merger in a Cournot model is profitable if the slope of each firm's (rising) marginal cost curve is sufficiently steep. In their model, merger generates efficiencies because it flattens the marginal cost curve, and the efficiencies are larger when the pre-merger marginal cost curve is steeper. Hence, the Perry and Porter result essentially means that merger is profitable if efficiencies are sufficiently large. Proposition 4 suggests that the same result holds if efficiencies reduce the vertical intercept rather than the slope of marginal cost curve.

Next, we move on to consider the effects of the merger on the outsiders' profits. Recall from Lemma 1 that larger efficiencies cause the outsiders to reduce output. Hence, we expect the post-merger profit of each outsider to fall below its pre-merger level if efficiencies are sufficiently large. This brings us to another critical value of  $e$ .

*Lemma 2.* There exists an  $e_p \in (e_\pi, 1)$  such that  $q_I^M = q_I^C$ ,  $q_O^M = q_O^C$ ,  $p_I^M = p_I^C$  and  $p_O^M = p_O^C$  at  $e = e_p$ .

*Proof.* Note that the left-hand side of (6) is less than that of (4) for  $q_i = q_j = q_I$ . This implies that if there are no merger efficiencies (*i.e.*, if  $e = 0$ ), an insider's marginal revenue after the merger is lower than that before the merger. Since the best-response function of each outsider is the same before and after the merger, this implies that  $q_I^M < q_I^C$  and  $q_O^M > q_O^C$  at  $e = 0$ . On the other hand, Assumption 4 and (23) imply that  $q_I^M > q_I^C$  and  $q_O^M < q_O^C$  at  $e = 1$ . By the continuity of  $q_I^M$  in  $e$ , there exists  $e_p \in (0, 1)$  such that  $q_I^M = q_I^C$ . Then (23) implies  $q_O^M = q_O^C$  at  $e = e_p$ . From the inverse demand functions (1)-(2), we infer that  $p_I^M = p_I^C$  and  $p_O^M = p_O^C$  at  $e = e_p$ .

In the case where  $e_\pi = 0$ ,  $e_p > 0$  implies that  $e_p > e_\pi$ . Now suppose  $e_\pi > 0$ . Note that  $[P^i(\mathbf{q}_I, \mathbf{q}_O) - (1-e)c]q_i > [P^i(\mathbf{q}_I, \mathbf{q}_O) - c]q_i$  for  $e > 0$ . Hence,  $\pi_I^M > \pi_I^C$  at  $e = e_p$ .

Since  $\pi_I^M = \pi_I^C$  at  $e = e_\pi$ , (24) implies that  $e_p > e_\pi$ . ■

Lemma 2 defines a critical value of  $e$  such that the equilibria before and after the merger involve exactly the same quantities and prices. Note that  $e_p > e_\pi$ , implying that the merger is profitable for the insiders even before their quantities reach their pre-merger levels. For the outsiders, on the other hand,  $e_p$  represents a turning point for their post-merger profits *vis-à-vis* their pre-merger levels.

*Proposition 5.*  $\pi_O^M < \pi_O^C$  if and only if  $e > e_p$ .

*Proof.* Since (3) represents an outsider's profit both before and after the merger, Lemma 2 implies that  $\pi_O^M = \pi_O^C$  at  $e = e_p$ . Differentiating an outsider's post-merger profit with respect to  $e$  and applying the envelope theorem, we obtain:

$$(25) \quad \frac{\partial \pi_O^M}{\partial e} = 2q_O^M \Phi'_{IO}(q_I^M) \frac{\partial q_I^M}{\partial e} < 0,$$

in which  $\partial q_I^M / \partial e > 0$  by Lemma 1. Then (25) implies  $\pi_O^M < \pi_O^C$  if and only if  $e > e_p$ . ■

Proposition 5 indicates that the impact of merger on the outsiders' profits depends on the size of efficiencies. The outsiders benefit from the merger if efficiencies are small but are harmed by it if efficiencies are large. This observation suggests that the expected impact of a merger on the outsiders' profits is not a good indicator of the presence or absence of efficiencies.<sup>17</sup>

Furthermore, Proposition 5 suggests that efficiencies may eliminate the 'free-riding problem' of merger. As early as Stigler [1950], economists have recognized that firms which do not participate in a merger may benefit more from the merger than the participants. This is particularly the case in the standard Cournot model with a homogeneous good where, in the absence of efficiencies, a merger always benefits the outsiders but may be unprofitable for the insiders [Salant, *et al.* 1983]. Proposition 5 implies that when a merger generates efficiencies, the outsiders can no longer be assured of benefiting more from the merger than the insiders.<sup>18</sup> In fact, they may be harmed by the merger.

#### V. PRICE EFFECTS OF MERGER

Recall from Proposition 1 that the prices of the insiders rise with the size of efficiencies under condition (12). This result suggests an intriguing possibility that a merger could lead to lower prices when efficiencies are small but higher prices when efficiencies are large. We investigate this possibility in this section.

Lemma 2 identifies a critical level of efficiencies,  $e_p$ , around which the merger leads to higher insider prices if the level of efficiencies is on one side of  $e_p$  and lower insider prices on the other side of  $e_p$ . To be more specific, if  $\partial p_I^M / \partial e < 0$  so that the insider prices decrease with efficiencies for  $e$  over the entire interval  $[0, 1]$ , we have the familiar situation where efficiencies mitigate the upward pricing pressure caused by the merger. In that case, the insiders' post-merger prices will fall below their pre-merger level if the size of merger efficiencies exceeds  $e_p$ . If  $\partial p_I^M / \partial e > 0$  at  $e = e_p$ , on the other hand, we have the intriguing situation where the merger leads to lower prices when efficiencies are small but higher prices when efficiencies are large.

<sup>17</sup> There is a literature, pioneered by Eckbo [1983], that uses the abnormal stock returns of outsiders to ascertain whether a merger is motivated by efficiency gains. Schumann [1993] criticizes this approach by arguing that 'practically any pattern of rivals' abnormal returns can be consistent with some story of predominately procompetitive or anticompetitive mergers.' Proposition 5 lends theoretical support to Schumann's argument because it implies that merger efficiencies can cause the abnormal stock returns of outsiders to go in either direction.

<sup>18</sup> The same observation has been made by Heywood and McGinty [2008] and Cunha and Vasconcelos [2015] in the context of Stackelberg models. On the other hand, the free-riding problem exists in Perry and Porter's [1985] model (Heywood and McGinty [2007]).

*Proposition 6.* If condition (12) holds at  $e=e_p$ , there exists an  $e_l \in (e_\pi, e_p)$  such that  $p_I^M < p_I^C$  for  $e \in (e_l, e_p)$ , and an  $e_h \in (e_p, 1)$  such that  $p_I^M > p_I^C$  for  $e \in (e_p, e_h)$ .

*Proof.* Recall that  $p_I^M = p_I^C$  and  $p_O^M = p_O^C$  at  $e=e_p$ . By Proposition 1, condition (12) implies that  $\partial p_I^M / \partial e > 0$  at  $e=e_p$ . Then the result follows from the continuity of  $p_I^M$  in  $e$ . ■

To illustrate the implications of Proposition 6, consider the extreme case where the demands for the products of the two insiders are independent of each other, i.e.,  $\Phi'_{II}(q)=0$  for all  $q \geq 0$ .<sup>19</sup> In this case, the merger would not have any effect on quantities and prices in the absence of efficiency gains. By the conventional wisdom, such a merger should not raise any competition concerns because the only motivation for this merger is to reap efficiency gains. Yet in this model, such a merger may actually cause the prices of the merging firms to rise if condition (12) holds at  $e=e_p$ . Here, the merger may lead to higher prices *because of*, rather than *despite*, efficiencies.

Finally, we end this section by noting that  $e_p$  is a critical value for the outsiders' prices as well. That is, the post-merger prices of the outsiders are higher than their pre-merger prices if the level of efficiencies is on one side of  $e_p$  and lower than their pre-merger counterpart on the other side of  $e_p$ . To be more specific, if condition (21) in Proposition 3 is satisfied, the merger leads to lower prices for the outsiders if  $e < e_p$  and higher prices if  $e > e_p$ . The opposite is true if (21) does not hold.

## VI. CONSUMER WELFARE

In this section, we suppose that the goods are sold to final consumers and investigate the impact of efficiencies on consumer welfare. The purpose is to demonstrate that efficiencies may be detrimental to consumer welfare.

Propositions 1 and 3 imply that larger efficiencies would cause the prices of both insiders and outsiders to rise if conditions (12) and (21) hold in the post-merger equilibrium. In that scenario, larger efficiencies unambiguously reduce consumer welfare. However, as noted in Section III, condition (21) does not hold in the case of linear demand functions. This raises the question: can conditions (12) and (21) both be satisfied in a post-merger equilibrium?

To answer this question, we consider a utility function that generates a system of non-linear demand functions and demonstrate that, under the

<sup>19</sup> We thank an anonymous referee for the suggestion that we consider this special case.

conditions specified below, efficiencies indeed lead to higher prices for all goods and hence reduce consumer welfare.

Specifically, suppose that the market demand functions are generated from the following utility function of a representative consumer:

$$(26) \quad u(\mathbf{q}_I, \mathbf{q}_O) + y = \sum_{i=1}^2 \alpha_I q_i - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \gamma q_1 q_2 - \delta q_1 \sum_{i=3}^n q_i - \delta q_2 \sum_{i=3}^n q_i + \sum_{i=3}^n \alpha_O q_i + \frac{\beta}{\tau} \sum_{i=3}^n q_i^\tau - \frac{\phi}{2} \left[ \sum_{i=3}^n q_i \sum_{j=3}^n q_j - \sum_{i=3}^n q_i^2 \right] + y,$$

where  $y$  denotes the quantity of a numeraire good,  $\alpha_I$  and  $\alpha_O$  are both greater than marginal cost  $c$ , and each element in  $(\gamma, \delta, \phi, \tau)$  is strictly between 0 and 1.<sup>20</sup> It is easy to verify that  $u(\mathbf{q}_I, \mathbf{q}_O)$  displays diminishing marginal utility. As shown in Appendix B, moreover,  $u(\mathbf{q}_I, \mathbf{q}_O)$  is concave in  $(\mathbf{q}_I, \mathbf{q}_O)$  for quantities in the relevant range.

Solving the utility-maximization problem associated with (26), we obtain the following system of inverse demand functions:

$$(27) \quad p_i = \alpha_I - q_i - \gamma q_j - \delta \sum_{k=3}^n q_k, \quad (i, j = 1, 2; i \neq j);$$

$$(28) \quad p_i = \alpha_O + \beta q_i^{\tau-1} - \delta \sum_{j=1}^2 q_j - \phi \sum_{j \neq 1, 2, i} q_j, \quad (i = 3, 4, \dots, n).$$

The main difference between this demand system and the linear demand system (14)-(15) is that the demand function for each outsider is non-linear in its own quantity. But the demand function for an insider remains linear, with  $\gamma$  measuring the degree of substitutability between the insiders and  $\delta$  indicating the degree of substitutability between an insider and an outsider.

In a symmetric equilibrium, (26) can be written in the form:

$$(29) \quad V = 2\alpha_I q_I - (1 + \gamma)q_I^2 - 2(n-2)\delta q_I q_O + \alpha_O(n-2)q_O + \frac{\beta}{\tau}(n-2)q_O^\tau - \frac{\phi}{2}(n-2)(n-3)q_O^2 - 2p_I q_I - (n-2)p_O q_O.$$

We use (29) to measure consumer welfare before and after the merger.

*Proposition 7.* Suppose the market demand functions are represented by (27)-(28) and  $e$  is sufficiently large that the merger is profitable for the insiders. A larger  $e$  leads to higher prices for the insiders if and only if

<sup>20</sup> Additional restrictions on these parameters are needed to ensure that the equilibrium quantities of all firms are positive. See Appendix B for details.

$$(30) \quad \gamma < \frac{2(n-2)\delta^2}{(n-3)\varphi + \tau(1-\tau)\beta(q_O^M)^{\tau-2}} - 1,$$

holds in the post-merger equilibrium. A larger  $e$  unambiguously raises the prices of the outsiders. Consequently, a larger  $e$  reduces consumer welfare if (30) is satisfied.

*Proof.* Noting that  $\Psi'_I = -1$ ,  $\Phi'_{II} = -\gamma$ ,  $\Psi'_O = -(1-\tau)\beta(q_O^M)^{\tau-2}$ ,  $\Phi'_{IO} = -\delta$ ,  $\Psi''_O = (2-\tau)(1-\tau)\beta(q_O^M)^{\tau-3}$  and  $\Phi'_{OO} = -\varphi$ , we rewrite (12) to find (30). Condition (21) is satisfied because  $\Psi'_O + q_O^M \Psi''_O = (1-\tau)^2 \beta(q_O^M)^{\tau-2} > 0$ . Proposition 3 then implies that a larger  $e$  raises the prices of the outsiders. Consumer welfare falls with a larger  $e$  because the indirect utility function associated with (26) is a decreasing function in prices  $p_I$  and  $p_O$ . ■

Therefore, larger efficiencies will harm consumers if condition (30) is satisfied.<sup>21</sup> As shown in Appendix B, a necessary condition for (30) to hold is that

$$(31) \quad \gamma < \frac{2(n-2)\delta^2}{(n-3)\varphi} - 1.$$

This is consistent with our earlier observation that efficiencies lead to higher prices for the insiders if the degree of substitutability between the insiders is not too high relative to that between an insider and an outsider.

One drawback of (30) is that it contains an endogenous variable,  $q_O^M$ , which itself depends on parameters such as  $\gamma$ ,  $\delta$  and  $\varphi$ . To demonstrate clearly that there indeed exist parameter values for which the condition in Proposition 7 is met, we use a set of numerical examples.

Specifically, we present here numerical examples for different values of  $e$  with the values of other parameters fixed at  $n=12$ ,  $\beta=0.14$ ,  $\gamma=0.3$ ,  $\delta=\varphi=0.8$ ,  $\tau=0.2$ ,  $\alpha_I=70$ ,  $\alpha_O=44$ , and  $c=1.8$ . Additional technical details about the numerical simulations can be found in Appendix B. Below we display two diagrams to demonstrate our main results.

Figure 1 illustrates the post-merger increase in the insider prices for  $e \in [0.15, 0.30]$ . We choose this range of  $e$  because, on the one hand, the merger is not profitable for  $e < 0.15$ , and, on the other hand, condition (30) is violated and hence the insider prices fall with efficiencies for  $e > 0.27$ . On the vertical axis of Figure 1 is the change in the prices of the insiders,  $(p_I^M - p_I^C)/p_I^C$ . From Figure 1 we see that, for  $e$  in the stated range, the merger causes the insider prices to rise by more than 12%, and larger efficiencies push the prices to move even higher.

<sup>21</sup> Note that (30) is a sufficient condition for consumer welfare to decrease with efficiencies. Consumer welfare could fall even if (30) is not satisfied. This would arise if the impact of higher outsider prices outweigh that of lower insider prices.

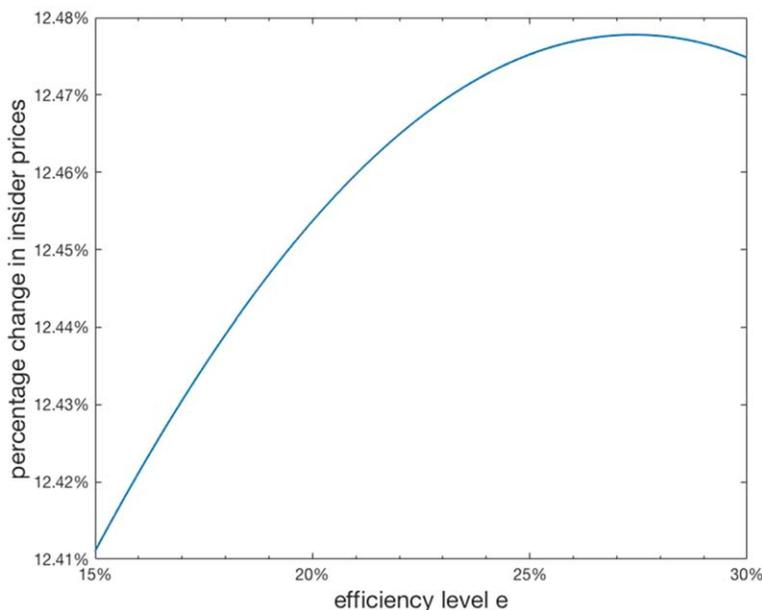


Figure 1  
Impact of Merger on Insider Prices  
[Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

Figure 2 demonstrates the impact of efficiencies on consumer welfare. On the vertical axis is the change in consumer welfare,  $(V^M - V^C)/V^C$ , where  $V^C$  and  $V^M$  denote the value of (29) in pre-merger and post-merger equilibrium, respectively. Figure 2 shows that for  $e \in [0.15, 0.30]$  the merger reduces consumer welfare by more than 19%, and larger efficiencies exacerbate the reduction in consumer welfare.

Note that among the parameter values used in the numerical examples,  $\gamma=0.3$  and  $\delta=0.8$ . That is, the degree of substitutability between the insiders is substantially smaller than that between an insider and an outsider. Therefore, these examples represent a situation where the products offered by an insider and an outsider are very close substitutes while those offered by the two insiders are distant substitutes. Moreover, the merger generates significant efficiencies in excess of 15%. Yet the merger leads to substantial increase in prices and causes significant harm to consumers.

## VII. IMPLICATIONS FOR MERGER ENFORCEMENT POLICY

In this section, we discuss the implications of our findings for merger enforcement policy. The fundamental contribution of this paper is the identification of an effect whereby efficiencies can impact prices; specifically, a post-merger reduction in marginal costs can cause prices to rise depending

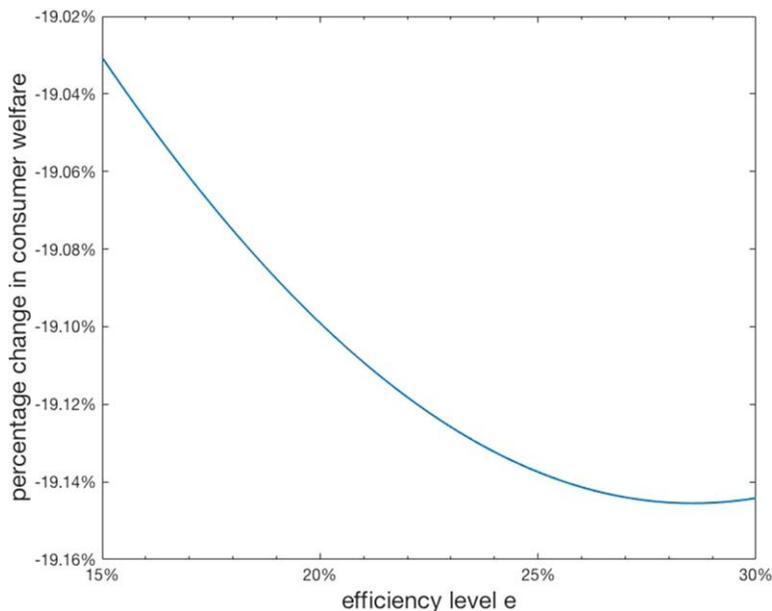


Figure 2  
Impact of Merger on Consumer Welfare  
[Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

on the substitutability pattern among products. In Sections V and VI, we have shown that this effect can be so strong that the merger leads to higher prices. Note that even if the merger lowers prices, this effect can still be present. In such a case, recognizing this effect is still useful as then prices are not expected to fall as much as would be the case with an analysis that failed to consider the effect.

For merger enforcement, it is instructive to highlight the circumstances under which this effect may arise. The key characteristics of the product market analyzed in this paper are (i) product differentiation and (ii) Cournot competition. While differentiated goods are ubiquitous in reality, examples of Cournot competition in its pure form are harder to find. However, it has been shown in the literature that the Cournot outcome can be generated from a two-stage model whereby firms commit to quantities (capacity) in the first stage and choose prices in the second stage.<sup>22</sup>

<sup>22</sup> This equivalence was first established in a model of a homogeneous good by Kreps and Scheinkman [1983]. Their result, however, depends on a particular rationing rule. Using models of differentiated goods, Yin and Ng [1997] and [2000] and Martin [forthcoming] demonstrate that the equilibrium of the two-stage game reproduces the Cournot outcome independent of rationing rules.

Therefore, our findings may be applicable to mergers in markets where firms produce differentiated goods and they pre-commit to quantities before setting prices.

One example of such markets is the production and sale of beer. First, product differentiation is a prominent feature of beer products. Beer is manufactured and sold under different brand names, and different brands of beer are differentiated in terms of product characteristics such as alcoholic content, aroma, color, flavor, foam, and packaging.

Second, the two-stage game of quantity pre-commitment before price-setting appears to be a reasonable representation of the competition among beer producers because the process of brewing beer takes several weeks.<sup>23</sup> This means that a beer producer has to commit to the quantity of output several weeks before the product is ready for sale. The Bertrand model, in which each firm sets the price and lets quantity adjust to clear the market, does not fit this feature of the beer industry. The two-stage game, on the other hand, is a better approximation for the strategic interactions among beer producers.

As an example of actual cases in which our model is potentially applicable, consider the following merger between two brewing companies in the United States. In October, 2007, SABMiller plc (Miller) and Molson Coors Brewing Company (Coors) announced their plan to establish a joint-venture, named MillerCoors, under which the companies would combine their beer operations in the United States and Puerto Rico. After an eight-month investigation, the Department of Justice's Antitrust Division announced that the proposed transaction was not likely to lessen competition substantially. In the announcement, the DOJ states, 'In one of the key parts of the investigation, the Division verified that the joint venture is likely to produce substantial and credible savings that will significantly reduce the companies' costs of producing and distributing beer. These savings... include large reductions in variable costs of the type that are likely to have a beneficial effect on prices' [USDOJ 2008]. This suggests that efficiencies were a key reason behind the DOJ's approval of this merger.

Retrospective studies of the Miller/Coors joint venture confirm that the merger indeed generated significant efficiencies that included reductions in marginal costs (Ashenfelter *et al.* [2015], Miller and Weinberg [2017]).<sup>24</sup> Retail prices of MillerCoors' beers, on the other hand, rose after the consummation of the merger. Quantitative analysis by Miller and Weinberg [2017] shows that the magnitude of the price increase was approximately 6

<sup>23</sup> The amount of time it takes beer to ferment and condition depends on beer style. For example, commercial lagers brewed in the United States typically spend a week in fermentation and three to six weeks in conditioning (BYO Staff [1998]). Incidentally, lagers account for more than 90 per cent of beer volume sold in the United States (Ashenfelter *et al.* [2015]).

<sup>24</sup> For example, estimates by Miller and Weinberg [2017] p. 25 imply that the marginal cost of Coors Light, one of MillerCoors' best-selling products, was reduced by approximately 14 per cent.

per cent, 2 per cent of which can be explained by increased market concentration due to the merger. The remaining 4 per cent of the price increase, according to Miller and Weinberg [2017], can be attributed to the coordinated effects of the merger. This conclusion, however, is obtained from a framework based on Bertrand price competition.

Our theory, on the other hand, suggests another possible explanation for the 4-per cent price hike not explained by the increase in market concentration. That is, it might be the result of efficiencies: instead of being a mitigating factor, efficiencies in the Miller/Coors case might actually have contributed to the price increase. As we argued above, the two-stage game of quantity pre-commitment before price-setting is a better representation of strategic interactions among beer producers than the Bertrand model of price competition. Regarding the degrees of substitutability among the competitors, Miller and Coors (the insiders of this merger) faced competition from many domestic and foreign brewers, the most notable of which was Anheuser-Busch. With its size exceeding that of Miller and Coors combined, Anheuser-Busch was the largest brewing company in the U.S. The investigation by DOJ's Antitrust Division in this case determined that 'by far, the greatest competition faced by both Miller and Coors came from Anheuser-Busch, not from one another' (Heyer *et al.* [2009 p351]). This seems to suggest that the degree of substitutability is lower between the two insiders than that between Anheuser-Busch and each insider. Therefore, it seems plausible to conjecture that our theory might be applicable to the Miller/Coors case.

Finally, even if our theory turns out not to be applicable to the Miller/Coors case or any other merger cases in the past, it is still useful for future merger enforcement as it cautions antitrust authorities against the presumption that efficiencies will necessarily counteract the anticompetitive effects of a merger. It suggests that they should not always treat efficiencies as a plus factor in merger assessment.<sup>25</sup>

## VIII. CONCLUSIONS

To answer the question posed in the title of this paper, merger efficiencies do not necessarily mitigate the price increases arising from the loss of competition. Our analysis demonstrates that when firms compete in quantity, lower marginal costs after a merger can exert upward rather than downward pressure on the prices of the merging firms. A policy implication of our results is that in cases where firms engage in quantity competition or where firms commit to quantities before setting prices, a competition authority can no longer presume that efficiencies will necessarily offset the

<sup>25</sup> Cabral [2003] shows that efficiencies in the form of lower marginal costs may be a weak defense in merger cases because it reduces the likelihood of post-merger entry. In this paper, we identify a different reason for which efficiencies may be a poor defense.

anticompetitive effects of the merger. The merger can lead to higher prices *because of*, rather than *despite of*, large efficiencies.

Finally, it is important to recognize that our results are obtained under a particular set of assumptions in a particular theoretical framework. Among the assumptions is that firms compete in quantity. If, instead, firms choose prices, the economic force that causes the insiders' prices to rise with efficiencies may not be present. While a formal analysis is beyond the scope of the present paper, it is our belief that efficiencies will not lead to higher prices when firms compete in prices. This is because prices are, in a standard model of oligopoly, strategic complements. Strategic complementarity implies that when the insiders reduce their prices in response to lower marginal costs, the outsiders will follow suit. This, in turn, will reinforce the insiders' incentives to cut prices. Consequently, it appears unlikely that larger efficiencies would lead to higher prices when firms compete in price. Therefore, a careful consideration of the nature of strategic interactions among firms is needed when applying our theory to a merger analysis.

#### APPENDIX A

##### LINEAR DEMAND SYSTEM

In this appendix, we present technical details associated with the special case of linear demand functions. To simplify presentation, we define  $A \equiv (n-3)\varphi+2$  and  $B \equiv (n-2)\delta$ . As will become clear below, we assume  $A > B$  to ensure that the equilibrium quantity of an insider is positive. It is easy to verify that this assumption is satisfied in the case  $\varphi = \delta$ .

It is clear that the linear demand functions (14)-(15) satisfy Assumptions 1 and 2. Regarding Assumption 3, we apply the linear demand functions to the definition of  $J$  to find

$$(A1) \quad J = 2(1+\gamma)A - 2\delta B,$$

which is positive because  $A > B$  and  $2(1+\gamma) > 2 > 2\delta$ .

Applying the linear demand functions to (4), we derive the pre-merger equilibrium quantity of each insider and each outsider:

$$(A2) \quad q_I^C = \frac{(\alpha-c)(A-B)}{(2+\gamma)A-2\delta B}, \quad q_O^C = \frac{(\alpha-c)(2+\gamma-2\delta)}{(2+\gamma)A-2\delta B}.$$

Note that the assumption  $A > B$  guarantees that  $q_I^C > 0$ . Using (A2) and the demand functions (14)-(15), we obtain the equilibrium prices before the merger:

$$(A3) \quad p_I^C = \frac{(\alpha-c)(A-B)}{(2+\gamma)A-2\delta B} + c, \quad p_O^C = \frac{(\alpha-c)[2+\gamma-2\delta]}{(2+\gamma)A-2\delta B} + c.$$

Solving (7)-(8) for the case of linear demand functions, we find post-merger equilibrium quantities and prices:

$$(A4) \quad q_I^M = \frac{(\alpha-c)(A-B)+ecA}{2[(1+\gamma)A-\delta B]}, \quad q_O^M = \frac{(\alpha-c)(1+\gamma-\delta)-\delta ec}{(1+\gamma)A-\delta B},$$

$$(A5) \quad p_I^M = \frac{(\alpha-c)(1+\gamma)(A-B)-ec[(1+\gamma)A-2\delta B]}{2[(1+\gamma)A-\delta B]} + c, \quad p_O^M = \frac{(\alpha-c)(1+\gamma-\delta)-\delta ec}{(1+\gamma)A-\delta B} + c.$$

It is easy to verify that  $\partial q_I^M / \partial e > 0$ ,  $\partial q_O^M / \partial e < 0$ , and  $\partial p_O^M / \partial e < 0$ . Moreover,  $\partial p_I^M / \partial e > 0$  if and only if  $(1+\gamma)A < 2\delta B$ , i.e., if and only if (16) holds.

Denote the right-hand side of (17) by  $F(\delta, \varphi)$ . To show that its value is greater than 3 for  $\delta \in (0, 1)$ , differentiate it with respect to  $\delta$ :

$$(A6) \quad \frac{\partial F}{\partial \delta} = -\frac{4\delta(2-\varphi)}{[2\delta^2-\varphi]^2} < 0.$$

Note that  $F(1, \varphi) = 3$ . Then (A6) implies that  $F(\delta, \varphi) > 3$  for  $\delta \in (0, 1)$ .

To prove that (16) implies  $\gamma < \delta$  in the case  $\varphi = \delta$ , it is sufficient to show that the right-hand side of (16) is less than  $\delta$  under the same condition. That is, we want to demonstrate that

$$(A7) \quad \frac{2(n-2)\delta^2}{(n-3)\delta+2} - 1 < \delta.$$

Multiplying both sides of (A7) by  $(n-3)\delta+2$  and combining like terms, we obtain  $(n-1)\delta^2 < (n-1)\delta+2$ , which is true because  $\delta^2 < \delta$ . Then (16) and (A7) imply that  $\gamma < \delta$  when  $\varphi = \delta$ . This, incidentally, also implies that (16) cannot be satisfied by  $\varphi = \delta = \gamma$ .

To show that (16) and  $\gamma > \delta$  may hold simultaneously under certain conditions, we rewrite (16) as:

$$(A8) \quad \delta^2 > \frac{(1+\gamma)[(n-3)\varphi+2]}{2(n-2)}.$$

In order to satisfy both (A8) and  $\gamma > \delta$ , it is necessary that

$$(A9) \quad \gamma^2 > \frac{(1+\gamma)[(n-3)\varphi+2]}{2(n-2)}.$$

Noting that (A9) contains a quadratic function of  $\gamma$ , we rewrite (A9) in the form  $(\gamma-\gamma_1)(\gamma-\gamma_2) > 0$ , where

$$(A10) \quad \gamma_1 = \frac{A + \sqrt{A^2 + 8(n-2)A}}{4(n-2)}, \quad \gamma_2 = \frac{A - \sqrt{A^2 + 8(n-2)A}}{4(n-2)},$$

are the roots of the quadratic function. Noting that  $\gamma_2 < 0$ , we conclude that (A9) is satisfied if  $\gamma > \gamma_1$ . Since  $\gamma < 1$ , such values of  $\gamma$  exist as long as  $\gamma_1 < 1$ . Using (A10) to rearrange the latter, we find  $\varphi < (n-4)/(n-3)$ , which is feasible as long as  $n > 4$ . Therefore, both (16) and  $\gamma > \delta$  may hold simultaneously if  $\varphi$  and  $n$  satisfy these two conditions.

Regarding Assumption 4, we use (A2) and (A4) to find that  $q_I^M > q_I^C$  at  $e=1$  if and only if

$$(A11) \quad c > \frac{(A-B)\gamma\alpha}{(A-B)\gamma + [(2+\gamma)A - 2\delta B]}.$$

Note that the right-hand side of (A11) is less than  $\alpha$ . Thus, (A11) does not contradict the assumption  $\alpha > c$ .

To understand the role of  $\varphi$  in (16), we differentiate  $q_O^M$  in (A4) to obtain

$$(A12) \quad \frac{\partial q_O^M}{\partial e} = -\frac{\delta c}{(1+\gamma)A - \delta B} < 0,$$

$$(A13) \quad \frac{\partial^2 q_O^M}{\partial \varphi \partial e} = \frac{\delta(1+\gamma)(n-3)c}{[(1+\gamma)A - \delta B]^2} > 0$$

for  $n > 3$ . Equations (A12) and (A13) imply that the magnitude of  $|\partial q_O^M / \partial e|$  decreases with  $\varphi$ . In other words, the strategic effect in (20) is larger when  $\varphi$  is smaller.

For completeness, we note that  $\varphi$  also affects the equilibrium quantity of each insider. From (A4), we find

$$(A14) \quad \frac{\partial q_I^M}{\partial e} = \frac{[(n-3)\varphi + 2]c}{2[(1+\gamma)A - \delta B]} > 0,$$

$$(A15) \quad \frac{\partial^2 q_I^M}{\partial \varphi \partial e} = -\frac{\delta^2(n-2)(n-3)c}{2[(1+\gamma)A - \delta B]^2} < 0$$

for  $n > 3$ . Intuitively, the reason  $\varphi$  plays a role in (A14) is that the equilibrium quantity of an insider is affected by the quantities of the outsiders. When each insider expands output due to its lower marginal cost, all outsiders reduce their quantities. This reduction in outsiders' quantities has an 'echo' effect on the insiders' quantities (in the sense that the insiders will expand output further). However, we expect this echo effect on the insiders to be smaller than the strategic effect on the outsiders. Indeed, using (A13) and (A15) we find that

$$(A16) \quad (n-2) \frac{\partial^2 q_O^M}{\partial \varphi \partial e} > 2 \left| \frac{\partial^2 q_I^M}{\partial \varphi \partial e} \right|.$$

When interpreting (A16), keep in mind that there are 2 insiders and  $n-2$  outsiders. Therefore, (A16) shows that at the aggregate level the magnitude of the strategic effect is larger than that of the echo effect.

Finally, we confirm that the aggregate output indeed increases with the size of efficiencies. Using (A12) and (A14), we obtain

$$(A17) \quad \frac{\partial(\sum_{i=1}^n q_i^M)}{\partial e} = 2 \frac{\partial q_I^M}{\partial e} + (n-2) \frac{\partial q_O^M}{\partial e} = \frac{c(A-B)}{(1+\gamma)A - \delta B} > 0.$$

APPENDIX B  
NONLINEAR DEMAND SYSTEM

In this appendix, we present technical details associated with the special case of the non-linear demand system derived from the utility function (26). First, we present a condition that ensures the concavity of  $u(\mathbf{q}_I, \mathbf{q}_O)$ . It is easy to find

$$(A18) \quad \frac{\partial^2 u}{\partial q_i^2} = -1 < 0, \quad \frac{\partial^2 u}{\partial q_i \partial q_j} = -\gamma, \quad \frac{\partial^2 u}{\partial q_i \partial q_k} = -\delta, \quad (i, j=1, 2, k=3, \dots, n),$$

$$(A19) \quad \frac{\partial^2 u}{\partial q_i^2} = -(1-\tau)\beta q_i^{\tau-2} < 0, \quad \frac{\partial^2 u}{\partial q_i \partial q_j} = -\varphi, \quad (i, j=3, \dots, n).$$

From (A18)–(A19), we see that  $\partial^2 u / \partial q_i^2 < 0$  for all  $i=1, 2, \dots, n$ . To obtain the condition that ensures the concavity of  $u(\mathbf{q}_I, \mathbf{q}_O)$ , we calculate the pivots of the Hessian matrix associated with the utility function (26) without row exchanges or scalar multiplications of rows. To simplify the calculations, we evaluate the derivative  $\partial^2 u / \partial q_i^2$  at a symmetric equilibrium, *i.e.*, at  $q_i = q_O$  for  $i=3, \dots, n$ . After straightforward but tedious calculations, we can show that all pivots of the Hessian matrix are negative if each outsider's quantity satisfies

$$(A20) \quad q_O < \left[ \frac{(1+\gamma)(1-\tau)\beta}{2(n-2)\delta^2 - (1+\gamma)(n-3)\varphi} \right]^{1/(2-\tau)}.$$

The parameter values used in the numerical simulations are chosen to ensure that they satisfy (A20) as well as Assumptions 1–3.

To derive condition (31), observe in (30) that

$$(A21) \quad \frac{2(n-2)\delta^2}{(n-3)\varphi + \tau(1-\tau)\beta(q_O^M)^{\tau-2}} < \frac{2(n-2)\delta^2}{(n-3)\varphi}.$$

Then (30) and (A21) imply (31).

The equilibrium quantities before the merger are determined by

$$(A22) \quad \alpha_I - (2+\gamma)q_I - (n-2)\delta q_O = c,$$

$$(A23) \quad \alpha_O + \tau\beta q_O^{\tau-1} - (n-3)\varphi q_O - 2\delta q_I = c.$$

The equilibrium quantities after the merger are solved from the equation system consisting of (A23) and

$$(A24) \quad \alpha_I - (2+2\gamma)q_I - (n-2)\delta q_O = (1-e)c.$$

To aid the determination of the conditions that guarantee an interior equilibrium (where all firms produce positive quantities), we plot equations (A22)–(A24) in Figure 3. Let  $q_a$  and  $q_b$  denote the horizontal intercept of (A23) and (A22), respectively. It is straightforward to verify that (i) the curve representing (A23) is convex while the curves representing (A22) and (A24) are straight lines, (ii) all three curves are downward sloping, (iii) the

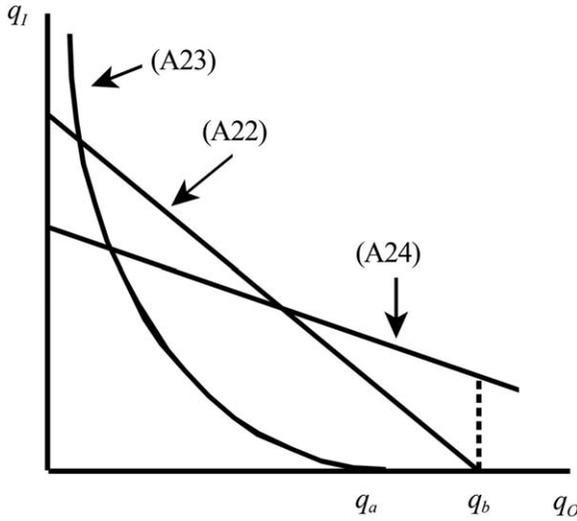


Figure 3  
Conditions for Interior Equilibria

slope of (A23) approaches (negative) infinity as  $q_O$  approaches 0, (iv) the slope of (A22) is steeper than that of (A24), and (v) for  $e > 0$ , (A24) lies above (A22) at  $q_O = q_b$ .

Given the above observations, we can be assured there is one and only one intersection between (A22) and (A23) (in the first quadrant) if  $q_a < q_b$ ; in other words, there exists a unique interior solution to the equation system (A22)-(A23) if  $q_a < q_b$ . Given the relative positions of (A22) and (A24), the condition  $q_a < q_b$  also guarantees the existence of a unique interior solution to the system (A23)-(A24).

Rewriting (A23), we obtain

$$(A25) \quad q_I = \frac{\alpha_o + \tau\beta q_o^{\tau-1} - (n-3)\phi q_o - c}{2\delta}.$$

Note that the right-hand side of (A25) decreases in  $q_O$  and is equal to 0 at  $q_O = q_a$ . Under the condition  $q_a < q_b$ , the right-hand side of (A25) must be negative at  $q_O = q_b$ , i.e.,

$$(A26) \quad \frac{\alpha_o + \tau\beta q_b^{\tau-1} - (n-3)\phi q_b - c}{2\delta} < 0.$$

Setting  $q_I = 0$  in (A22), we find

$$(A27) \quad q_b = \frac{\alpha_I - c}{(n-2)\delta}.$$

Substituting (A27) into (A26) and rearrange terms, we obtain the following conditions that ensures  $q_a < q_b$ :

$$(A28) \quad \beta < \frac{1}{\tau} \left[ \frac{\alpha_I - c}{(n-2)\delta} \right]^{1-\tau} \left[ \frac{(n-3)\varphi}{(n-2)\delta} (\alpha_I - c) - (\alpha_O - c) \right].$$

Since  $\beta > 0$ , the right-hand side of (A28) has to be positive. Hence, a necessary condition for (A28) to hold is

$$(A29) \quad \frac{(n-3)\varphi}{(n-2)\delta} (\alpha_I - c) > \alpha_O - c.$$

Conditions (A28) and (A29) are restrictions on the parameters of this model that guarantee the existence of interior equilibria before and after the merger.

The numerical simulations are conducted using the following two-step procedure. First, we choose a set of parameter values that satisfy (A28)-(A29) and solve (A22)-(A24) to find the equilibrium quantities. Then we use these quantities to verify whether Assumptions 1-3 and conditions (A20) and (30) are satisfied. The examples presented in Section VI are chosen from the sets of parameters that meet all of these restrictions.

#### REFERENCES

- Amir, R.; Diamantoudi, E. and Xue, L., 2009, 'Merger Performance under Uncertain Efficiency Gains,' *International Journal of Industrial Organization*, 27, pp. 264-273.
- Ashenfelter, O.; Hosken, D. and Weinberg, M., 2015, 'Efficiencies Brewed: Pricing and Consolidation in the U.S. Beer Industry,' *RAND Journal of Economics*, 46, pp. 328-361.
- Banal-Estañol, A.; Macho-Stadler, I. and Seldeslachts, J., 2008, 'Endogenous Mergers and Endogenous Efficiency Gains: The Efficiency Defense Revisited,' *International Journal of Industrial Organization*, 26, pp. 69-91.
- BYO Staff, 1998, 'How Do Commercial Breweries Lager So Quickly? Brew Your Own,' December, 1998. See <https://byo.com/stories/lissuelitem1870-how-do-commercial-breweries-lager-so-quickly/>
- Cabral, L., 2003, 'Horizontal Mergers with Free-Entry: Why Cost Efficiencies May Be a Weak Defense and Asset Sales a Poor Remedy,' *International Journal of Industrial Organization*, 21, pp. 607-623.
- Cheung, F.K., 1992, 'Two Remarks on the Equilibrium Analysis of Horizontal Merger,' *Economics Letters*, 40, pp. 119-123.
- Cunha, M. and Vasconcelos, H., 2015, 'Mergers in Stackelberg Markets with Efficiency Gains,' *Journal of Industry, Competition and Trade*, 15, pp. 105-134.
- Eckbo, E.B., 1983, 'Horizontal Mergers, Collusion and Stockholder Wealth,' *Journal of Financial Economics*, 11, pp. 241-273.
- Farrell, J. and Shapiro, C., 1990, 'Horizontal Mergers: An Equilibrium Analysis,' *American Economic Review*, 80, pp. 107-126.
- Fisher, A.; Johnson, F. and Lande, R., 1989, 'Price Effects of Horizontal Mergers,' *California Law Review*, 77, pp. 777-827.
- Froeb, L.M. and Werden, G.J., 1998, 'A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of a Homogeneous Product,' *Economics Letters*, 58, pp. 367-69.
- Heyer, K.; Shapiro, C. and Wilder, J., 2009, 'The Year in Review: Economics at the Antitrust Division, 2008-2009,' *Review of Industrial Organization*, 35, pp. 349-367.

- Heywood, J. and McGinty, M., 2007, 'Convex Costs and the Merger Paradox,' *Economic Inquiry*, 45, pp. 342–349.
- Heywood, J. and McGinty, M., 2008, 'Leading and Merging: Convex Costs, Stackelberg and the Merger Paradox,' *Southern Economic Journal*, 74, pp. 879–893.
- Jovanovic, D. and Wey, C., 2012, 'An Equilibrium Analysis of Efficiency Gains from Mergers,' *DICE Discussion Paper*, No. 64.
- Kreps, D. M. and Scheinkman, J. A., 1983, 'Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes,' *The Bell Journal of Economics*, 14, pp. 326–337.
- Levin, D., 1990, 'Horizontal Mergers: The 50-per cent Benchmark,' *American Economic Review*, 80, pp. 1238–1245.
- Lommerud, K. E. and Sørgaard, L., 1997, 'Merger and Product Range Rivalry,' *International Journal of Industrial Organization*, 16, pp. 21–42.
- Martin, S., forthcoming, 'Kreps & Scheinkman with Product Differentiation: An Expository Note,' *Frontiers of Economics in China*.
- Miller, N.H. and Weinberg, M.C., 2017, 'Understanding the Price Effects of the MillerCoors Joint Venture,' *Econometrica*, 85(6) pp. 1763–1791.
- Motta, M. and Vasconcelos, H., 2005, 'Efficiency Gains and Myopic Antitrust Authority in a Dynamic Merger Game,' *International Journal of Industrial Organization*, 23, pp. 777–801.
- Perry, M.K. and Porter, R.H., 1985, 'Oligopoly and the Incentive for Horizontal Merger,' *The American Economic Review*, 75, pp. 219–227.
- Salant, S.W.; Switzer, S. and Reynolds, R.J., 1983, 'Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium,' *Quarterly Journal of Economics*, 98, pp. 185–199.
- Salop, S., 1987, 'Symposium on Mergers and Antitrust,' *Journal of Economic Perspectives*, 1, pp. 3–12.
- Schumann, L., 1993, 'Patterns of Abnormal Returns and the Competitive Effects of Horizontal Mergers,' *Review of Industrial Organization*, 8, pp. 679–696.
- Stigler, G.J., 1950, 'Monopoly and Oligopoly by Merger,' *American Economic Review Papers and Proceedings*, 40, pp. 23–34.
- U.S. Department of Justice, 2008, *Statement of the Department of Justice's Antitrust Division on its Decision to Close its Investigation of the Joint Venture between SAB-Miller plc and Molson Coors Brewing Company*, June 5, 2008.
- Yin, X. and Ng, Y., 1997, 'Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes: A Case with Product Differentiation,' *Australian Economic Papers*, 36, pp. 14–22.
- Yin, X. and Ng, Y., 2000, 'Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes: A Case with Product Differentiation: Reply,' *Australian Economic Papers*, 39, pp. 113–119.
- Werden, G., 1996, 'A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of Differentiated Products,' *Journal of Industrial Economics*, 44, pp. 409–413.