Microeconomics – Université Paris 1 Panthéon–Sorbonne
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Midterm Exam (1 hour and 30 minutes)

Read and think before you write, and try to be both concise and precise

Exercise 1 (30 minutes). Draw the indifferences curves in $\mathbb{R}^2$ of the following preferences.

1. Linear preferences such that for all $x_1 = (x_{11}, x_{12})$ and $x_2 = (x_{21}, x_{22})$, one has

$$x_1 \succ x_2 \Leftrightarrow x_{11} + x_{12} > x_{21} + x_{22}$$

2. Cobb-Douglas preferences such that for all $x_1 = (x_{11}, x_{12})$ and $x_2 = (x_{21}, x_{22})$, one has

$$x_1 \succ x_2 \Leftrightarrow x_{11}x_{12} > x_{21}x_{22}$$

3. Give three exemples of utility functions representing the preferences considered in 2.

1. See class notes
2. See class notes
3. We know that if $f : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing function that u and $f \circ u$ represent the same preferences; Applying this property to (for example):
   - $f(y) = \log(y)$ one gets $v(x_1, x_2) = \log(x_1) + \log(x_2)$
   - $g(y) = y^2$ one gets $w(x_1, x_2) = x_1^2 x_2^2$
   - $h(y) = \sqrt{y}$ one gets $w(x_1, x_2) = \sqrt{x_1 x_2}$

Exercise 2 (30 minutes). Determine the demand functions associated with the following utility functions

- The utility $u : \mathbb{R}_+^n \to \mathbb{R}_+$ such that

$$u(x_1, \cdots, x_n) := \prod_{i=1}^{n} x_i^{\alpha_i}$$

  where $\alpha = (\alpha_1, \cdots, \alpha_n) \in \mathbb{R}_+^n$ is a fixed vector of parameters

- The utility $v : \mathbb{R}_+^n \to \mathbb{R}_+$ such that

$$v(x_1, \cdots, x_n) := \min(\frac{x_1}{a_1}, \cdots, \frac{x_n}{a_n})$$

  where $a = (a, \cdots, a) \in \mathbb{R}_{++}^n$ is a fixed vector of parameters

- The utility $w : \mathbb{R}_+^n \to \mathbb{R}_+$ such that

$$w(x_1, \cdots, x_n) := \sum_{i=1}^{n} a_i x_i$$

  where $a = (a_1, \cdots, a_n) \in \mathbb{R}_{++}^n$ is a fixed vector of parameters

1. See handwritten notes...
2.

Exercise 3 (30 minutes).

1. What is a rational preference relation ?
2. Give an exemple of non-rational preference relation.
3. Give an exemple of consumption set and preference relation so that no two bundles are indifferent (i.e indifferences curves are points).
4. Let us consider a preference relation on $Y = \{a_1, \cdots, a_n\}$ such that $a_1 \succ a_2 \succ \cdots \succ a_n$. Give a utility function representing $\succ$.

1. A rational preference relation is a relation that is complete and transitive
2. Consider $X = \{A, B, C\}$ the preference relation on X defined y $A \succ B \succ C \succ A$ is non rational because it is non-transitive.
3. Lexicographic preferences (see class notes)
4. One can set form example $u(a_i) = -i$, one then has $u(a_i) \geq u(a_j) \Leftrightarrow i \leq j \Leftrightarrow a_i \succeq a_j$

1. One can remark that if $x_i = 0$ for at least one i then $u(x) = 0$. Thus, unless $w = 0$, one must have $\forall i = 1 \dots n \quad x_i > 0$. Thus the problem of the consumer can be written as:

$$\max \prod_{i=1}^{n} x_i^{\alpha_i}$$

$$\sum p_i x_i = w$$

where the budget constraint is an equality because the utility is ~~strictly~~ monotonic. First-order conditions then give that $\exists \lambda \in \mathbb{R}$ such that for all $i = 1 \dots n$, one has at an optimal x:

$$\frac{\alpha_i}{x_i} \prod_{i=1}^{n} x_i^{\alpha_i} = \lambda p_1 \Leftrightarrow \frac{\alpha_i}{\alpha_1} \prod_{i=1}^{n} x_i^{\alpha_i} = \lambda p_1 x_1$$

Thus for all i, one has

$$\frac{p_1 x_1}{p_i x_i} = \frac{\alpha_1}{\alpha_i} \Rightarrow p_i x_i = \frac{\alpha_i}{\alpha_1} p_1 x_1$$

Now $\sum p_i x_i = w$, and thus $\frac{\sum \alpha_i}{\alpha_1} p_1 x_1 = w$

$$\Rightarrow x_1 = \frac{\alpha_1}{\sum \alpha_i} \frac{w}{p_1}$$

and likewise $x_j = \dfrac{\alpha_j}{\sum \alpha_i} \dfrac{w}{p_j}$

2) It is straightforward to show (see lecture notes) that at an optimum, one must have for all j:

$$\frac{x_j}{a_j} = \frac{x_1}{a_1} \Rightarrow x_j = \frac{a_j}{a_1} x_1$$

Using the monotonicity of the utility function, one further has $\sum p_i x_i = 1$

and thus $$\sum p_i a_i \frac{x_1}{a_1} = w$$

hence $$x_1 = \frac{a_1 w}{\sum p_i a_i}$$

and likewise $$x_j = \frac{a_j w}{\sum p_i a_i}$$

3) As above, the utility function is (strictly) increasing and thus one must have $\sum p_i x_i = w$

The level of utility per unit of ~~consumption~~ spending is constant for each good and equal to $\frac{a_i}{p_i}$

One can only consume goods j such that

$$\frac{a_j}{p_j} = \max_{i=1 \dots n} \frac{a_i}{p_i}$$

and eventually one gets:

$$d(p, w) = \left\{ (\lambda_1, \lambda_2, \dots, \lambda_n) \,/\, \lambda_j > 0 \iff \frac{a_j}{p_j} = \max_{i=1 \dots n} \frac{a_i}{p_i}, \ \sum \lambda_j p_j = w \right\}$$