Chapter 5 and 6 : Extensive Games with Perfect Information

1. (a) EXERCISE 156.2 (Examples of extensive games with perfect information) Represent in a diagram the two-player extensive game with perfect information in which the terminal histories are $(C, E), (C, F), (D, G)$, and (D, H), the player function is given by $P(\varnothing) = 1$ and $P(C) = P(D) = 2$, player 1 prefers (C, F) to (D, G) to (C, E) to (D, H), and player 2 prefers (D, G) to (C, F) to (D, H) to (C, E).

   (b) Write down the set of players, set of terminal histories, player function, and players' preferences for the game in Figure 160.1.

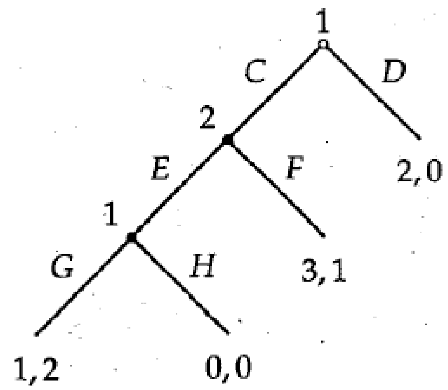

Figure 160.1 An extensive game in which player 1 moves both before and after player 2.

   (c) The political figures Rosa and Ernesto have to choose either Berlin (B) or Havana (H) as the location for a party congress. They choose sequentially. A third person, Karl, determines who chooses first. Both Rosa and Ernesto care only about the actions they choose, not about who chooses first. Rosa prefers the outcome in which both she and Ernesto choose B to that in which they both choose H, and prefers this outcome to either of the ones in which she and Ernesto choose different actions; she is indifferent between these last two outcomes. Ernesto's preferences differ from Rosa's in that the roles of B and H are reversed. Karl's preferences are the same as Ernesto's. Model this situation as an extensive game with perfect information. (Specify the components of the game and represent the game in a diagram.)

2. Exercise 161.1 (Strategies in extensive games) What are Rosa's strategies in the game in Exercise 156.2 (c)?

3. Exercise 163.1 (Nash equilibria of extensive games) Find the Nash equilibria of the games in Exercise 156.2 (a) and Figure 160.1. (When constructing the strategic form of each game, be sure to include all the strategies of each player.)

4. EXERCISE 164.2 (Subgames) Find all the subgames of the game in Exercise 156.2 (c).

5. EXERCISE 168.1 (Checking for subgame perfect equilibria) Which of the Nash equilibria of the game in Figure 160.1 are subgame perfect?

6. Exercise 173.2 (Finding subgame perfect equilibria) Find the subgame perfect equilibria of the games in parts (a) and (c) of Exercise 156.2, and in Figure 173.1.

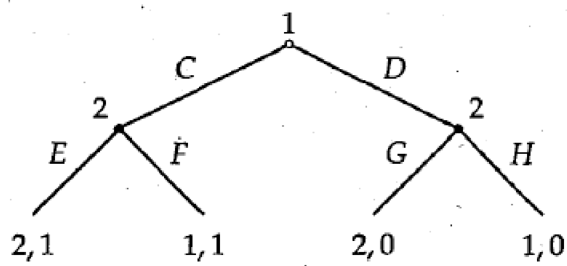

Figure 173.1 One of the games for Exercise 173.2.

7. EXERCISE 183.1 (Nash equilibria of the ultimatum game) Find the values of x for which there is a Nash equilibrium of the ultimatum game in which person 1 offers x.

8. Exercise 183.2 (Subgame perfect equilibria of the ultimatum game with indivisible units) Find the subgame perfect equilibria of the variant of the ultimatum game in which the amount of money is available only in multiples of a cent.

9. EXERCISE 183.3 (Dictator game and impunity game) The "dictator game" differs from the ultimatum game only in that person 2 does not have the option to reject person 1's offer (and thus has no strategic role in the game). The "impunity game" differs from the ultimatum game only in that person 1's payoff when person 2 rejects any offer x is $c - x$, rather than 0. (The game is named for the fact that person 2 is unable to "punish" person 1 for making a low offer.) Find the subgame perfect equilibria of each game.

10. EXERCISE 183.4 (Variants of ultimatum game and impunity game with equity-conscious players) Consider variants of the ultimatum game and impunity game in which each person cares not only about the amount of money she receives, but also about the equity of the allocation. Specifically, suppose that person i's preferences are represented by the payoff function given by $u_i(x_1, x_2) = x_1 - \beta_i|x_1 - x_2|$,, where x_i is the amount of money person i receives, $\beta_i > 0$, and, for any number z, $|z|$ denotes the absolute value of z (i.e. $|z| = z$ if $z > 0$ and $|z| = {}^{-}z$ if $z < 0$). Assume $c = 1$. Find the set of subgame perfect equilibria of each game and compare them. Are there any values of β_1 and β_2 for which an offer is rejected in equilibrium?

11. Exercise 185.1 (Bargaining over two indivisible objects) Consider a variant of the ultimatum game, with indivisible units. Two people use the following procedure to allocate two desirable identical indivisible objects. One person proposes an allocation (both objects go to person 1, both go to person 2, one goes to each person), which the other person then either accepts or rejects. In the event of rejection, neither person receives either object. Each person cares only about the number of objects she obtains. Construct an extensive game that models this situation and find its

subgame perfect equilibria. Does the game have any Nash equilibrium that is not a subgame perfect equilibrium? Is there any outcome that is generated by a Nash equilibrium but not by any subgame perfect equilibrium?

12. Exercise 185.2 (Dividing a cake fairly) Two players use the following procedure to divide a cake. Player 1 divides the cake into two pieces, and then player 2 chooses one of the pieces; player 1 obtains the remaining piece. The cake is continuously divisible (no lumps!), and each player likes all parts of it. Suppose that the cake is perfectly homogeneous, so that each player cares only about the size of the piece of cake she obtains. How is the cake divided in a subgame perfect equilibrium?