

## Chapter 9 : Bayesian Games

1. EXERCISE 276.1 (Equilibria of a variant of BoS with imperfect information) Show that there is no pure strategy Nash equilibrium of this game in which player 1 chooses  $S$ . If you have studied mixed strategy Nash equilibrium (Chapter 4), find the mixed strategy Nash equilibria of the game. (First check whether there is an equilibrium in which both types of player 2 use pure strategies; then look for equilibria in which one or both of these types randomize.)
2. EXERCISE 282.1 (Fighting an opponent of unknown strength) Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; she assigns probability  $\alpha$  to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Each person's preferences are represented by the expected value of a Bernoulli payoff function that assigns the payoff of 0 if she yields (regardless of the other person's action) and a payoff of 1 if she fights and her opponent yields; if both people fight, then their payoffs are  $(-1, 1)$  if person 2 is strong and  $(1, -1)$  if person 2 is weak. Formulate this situation as a Bayesian game and find its Nash equilibria if  $\alpha < \frac{1}{2}$  and if  $\alpha > \frac{1}{2}$ .
3. EXERCISE 282.2 (An exchange game) Each of two individuals receives a ticket on which there is an integer from 1 to  $m$  indicating the size of a prize she may receive. The individuals' tickets are assigned randomly and independently; the probability of an individual's receiving each possible number is positive. Each individual is given the option of exchanging her prize for the other individual's prize; the individuals are given this option simultaneously. If both individuals wish to exchange, then the prizes are exchanged; otherwise each individual receives her own prize. Each individual's objective is to maximize her expected monetary payoff. Model this situation as a Bayesian game and show that in any Nash equilibrium the highest prize that either individual is willing to exchange is the smallest possible prize.
4. EXERCISE 287.1 (Cournot's duopoly game with imperfect information) Consider the game when the inverse demand function is given by  $P(Q) = \alpha - Q$  for  $Q \leq \alpha$  and  $P(Q) = 0$  for  $Q > \alpha$  (see (56.2)). For values of  $c_H$  and  $c_L$  close enough that there is a Nash equilibrium in which all outputs are positive, find this equilibrium. Compare this equilibrium with the Nash equilibrium of the game in which firm 1 knows that firm 2's unit cost is  $c_L$ , and with the Nash equilibrium of the game in which firm 1 knows that firm 2's unit cost is  $c_H$ .