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Presidential Address

The challenge of monetary theory*

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1. Introduction

Departing from custom, I want to address a problem that needs to be solved rather than one that has been solved. I do not have much to contribute to its solution, but I believe that the problem is important and want to focus attention on it.

In general terms, the problem is to find appropriate conceptual foundations for monetary economics. I believe that we do not, as yet, have a suitable theoretical framework for studying the functioning of a monetary system. The main obstacle to the development of such a framework is our habit of thinking in terms of frictionless, organized, i.e. Walrasian markets.

The problem is partly a problem of language. The central concepts of economics, demand, supply, and price, are concepts that presuppose a system of centralized Walrasian markets. We are used to thinking in terms of these concepts, and we tend to apply them rather automatically to the 'market' for money, frequently without realizing that the 'market' for money must be a rather non-Walrasian 'market', if indeed it can be called a 'market' at all. More or less all the models of monetary economics that are being used in practice proceed in this way, using artifical adaptations of the Walrasian centralized-markets model to accommodate money without actually giving an account of what the role of money in the economy is.

The problem is one of pure economic theory. However, it bears on many important questions of applied economics and economic policy. To study the relation between money, income and prices, we need a framework which gives a satisfactory account of the relation between the monetary system and the price mechanism. To study the effects of central-bank open-market

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policies, we need a framework that gives a satisfactory account of the relation between money, goods and asset markets. To assess the relative importance of macroeconomic and microeconomic considerations for the regulation of banking, we need a framework that encompasses the role of banks in 'the payments system' as well as more straightforward considerations of tax distortions and agency costs in financial intermediation.

The challenge ahead involves not just the development of conceptual foundations for monetary economics. It also involves the development of a language that corresponds to these conceptual foundations and yet is as easily adaptable to the study of applied problems as the Walrasian language of 'demand', 'supply', and 'price'.

In the following, I will first explain why I believe that we do not as yet have a suitable theoretical framework for studying the functioning of a monetary economy. For this purpose, I discuss some problems and puzzles in monetary theory and macroeconomics. These problems and puzzles are intended to show why the Walrasian framework is unsuitable for monetary theory. Subsequently, in the second half of this lecture, I will develop a few ideas about the direction in which we have to go. The discussion will be quite tentative, involving warnings against easy shortcuts rather than a blueprint for future work. However, I hope that it will show how substantial the challenge of monetary theory is.

2. Some problems and puzzles in monetary theory and macroeconomics

I begin with a brief account of what has become known as *the Hahn problem*:

Problem 0. Why does fiat money have a positive value in exchange against goods and services even though it is not intrinsically useful?

This problem was first posed by Hahn (1965) in his famous critique of Patinkin's proposed integration of monetary theory and value theory in a Walrasian general-equilibrium model [Patinkin (1965)]. Hahn actually suggested that the problem cannot be satisfactorily addressed in Patinkin's Walrasian model. More precisely, he showed that if indeed fiat money is not intrinsically useful and is held *only* as it can be exchanged for something else, then under the usual regularity conditions on excess demand functions, Patinkin's model always has a general equilibrium in which fiat money is worthless. Knowing that a general equilibrium in Patinkin's model exists is therefore not enough if one wants to establish the existence of a *monetary* equilibrium, i.e. an equilibrium in which the real value of money is positive. To establish the latter, one needs additional arguments, which, according to Hahn, transcend Patinkin's Walrasian framework.

Subsequent research has answered Hahn's query at different levels:

- According to the theory of general temporary equilibrium, the real value of fiat money in current exchanges is positive if agents expect the real value of fiat money in future exchanges to be positive [see, e.g., Grandmont (1983)].[1]
- In rational-expectations equilibrium models, the real value of fiat money in all exchanges may be positive and bounded away from zero because there is an underlying need for fiat money as a store of value. As in the theory of temporary equilibrium, the real value of fiat money in any exchange is positive because agents expect money to have value in future exchanges; in addition, the actual value in any exchange validates past expectations. This is possible because, at real rates of return on money that correspond roughly to the growth rate of the system, in *each* exchange there is enough of a demand for fiat money as a store of value [see, e.g., Grandmont and Laroque (1973)].[2]

As a response to the Hahn problem, the rational-expectations approach is more satisfactory than the temporary-equilibrium approach because it relates the equilibrium real value of fiat money to the underlying real data of the economy and to the need for a store of value that is implied by these data. One may have doubts about the appropriateness of the rational-expectations assumption as a tool of positive analysis. In the present context though, it is reassuring to know that the use of fiat money in an economy is not necessarily based on speculative illusions but may reflect a real function of money. Throughout the following discussion, I will therefore use the concept of a rational-expectations equilibrium as the appropriate solution concept for studying the functioning of a monetary economy.

The rational-expectations equilibrium models of money require that the time horizon be infinite so that the system is open-ended, there is no last period, and in each exchange there is a future for which at least some agents want to provide by holding money as a store of value. The infinite time horizon may lead to an indeterminacy of equilibrium as the infinite tail of expected future prices may wag the dog of current events in a way which is

[1]The formulation in the text assumes that expectations about future prices are independent of current prices. While giving the essence of the economic argument, it fails to do justice to the difficult problems that arise when expectations depend on current prices. For a full treatment, the reader is referred to Grandmont (1983).

[2]The need for a store of value can be motivated in many different ways. In overlapping-generations models, the store of value provides for consumption after retirement [Grandmont and Laroque (1973)]. In precautionary money holding models, the store of value allows agents to smooth income fluctuations [Bewley (1980, 1983); Hellwig (1982)]. In transactions costs models, the store of value allows agent to bridge the gap between income payments that are spaced apart to save on transactions costs [Gale and Hellwig (1984)]. The precise details do not matter as long as the underlying structure is such that there always is a sufficient demand for fiat money as a store of value.

only partly related to the real structure of the economy [see, e.g., Grandmont (1985), Azariadis and Guesnerie (1986)]. This problem of indeterminacy is a deep one that plagues all our theorizing about money. However, it is not related to the points I want to make here. Therefore I shall leave it aside and merely refer to the reader to Guesnerie (1993) for an extensive discussion.

Even if one neglects the problem of indeterminacy of equilibrium, one may still question whether the rational-expectations equilibrium models of money as a store of value have actually solved the Hahn problem. In these models, fiat money is the *only* store of value; there are no other assets. Any demand for a store of value is automatically a demand for money; all considerations of portfolio choice have been excluded.[3]

When there are many assets, a positive real value of fiat money in equilibrium cannot simply be deduced from the existence of an underlying need for a store of value. In addition to the underlying need for a store of value, one requires a portfolio-choice argument whereby the return characteristics of fiat money are sufficiently attractive for agents to be willing to hold money along with or in preference to other assets.

Such a portfolio choice argument is easily available if, with a sufficiently high probability, the own rate of return on any alternative asset is sufficiently small relative to the own rate of return on fiat money.[4] However, this condition on alternative-asset returns is quite restrictive. It excludes, e.g., the government consol of our textbooks, i.e., it excludes any fixed-interest perpetuity that is devoid of bankruptcy risk. The gross return on a perpetuity over the holding period consists of the perpetuity itself and the interest that is paid during the holding period. The (gross) own rate of return on the perpetuity is thus always greater than one, the own rate of return on non-interest-bearing fiat money.[5] We are thus led to what I would like to call *the modified Hahn problem*:

Problem 1. Why does 'worthless' fiat money have a positive value in exchange against goods and services when there are other assets whose own rates of return in each period exceed the own rate of return on money?

At this point, we do not have a satisfactory solution to the modified Hahn

[3]The exception is the transactions costs model of Gale and Hellwig (1984); see problem 4 below.

[4]See Wallace (1981) or Chamley and Polemarchakis (1984) for examples showing how such a condition on own rates of return of alternative assets can be used to establish the existence of a monetary rational-expectations equilibrium.

[5]For finite-duration bonds, the matter is less clear because, e.g., over one period a T-period bond returns a $(T-1)$-period bond and the interest, so the own rate of return depends on the relative price of T-period and $(T-1)$-period bonds. However, if an interest-bearing bond is issued at par and eventually redeemed at par, the money rate of return on money invested in the bond exceeds the own rate of return on cash with probability one in at least one period during the lifetime of the bond.

problem. We merely have a strong negative result, namely: *If there is an asset whose own rate of return exceeds the own rate of return on money in each period with probability one and if this asset has the same marketability properties as fiat money, then there exists no rational expectations equilibrium in which fiat money has a positive real value.*[6]

This negative result undermines practically all our textbook models of closed-economy and open-economy macroeconomics. In these models we typically consider the temporary equilibrium of an economy with Walrasian markets for goods, labour, money, and interest-bearing assets in a given period such that at the current Walrasian budget constraints, agents are free to trade as they wish in all these markets. The existence of an equilibrium in which money has positive value is taken for granted. However, the preceding proposition shows that once one looks at the evolution of the economy over time, in such a model with a sequence of Walrasian markets, an equilibrium with a positive real value of money cannot exist unless expectations are systematically incompatible with experience or else, agents systematically fail to exploit the trading opportunities that the sequence of Walrasian markets provides. This observation vitiates many well-known analyses of the effects of open-market operations [see, e.g., Tobin (1963)], government deficit finance [see, e.g., Blinder and Solow (1973)], or balance-of-payments dynamics [see, e.g, Dornbusch (1980)]. In these analyses, the role of expectations is usually left implicit. If one accounted for expectations explicitly, one would see that in the given framework of a sequence of frictionless Walrasian markets with interest-bearing assets, expectations cannot be rational.

The argument is simple and very general; it does not depend on the details of a particular model. For suppose that, e.g., a consol has the same marketability properties as fiat money. Then the portfolio choice between consols and money depends *only* on rate-of-return considerations. If anybody is willing to hold fiat money at any time, he must be assigning a positive probability to the event that the money rate of return on money over the time interval between the current trading opportunity and the next one is at least as high as the money rate of return on consols over this time interval. If expectations are rational, it follows that the actual probability of this event is positive. If fiat money is held without interrruption – though not necessarily always by the same agents – over a sequence of time intervals between successive trading opportunities, the preceding conclusion must hold for each subinterval where the probability assessment concerns the *conditional* probability at the beginning of each subinterval when a decision to hold money is taken. It follows that with positive probability the (cumulative) money rate of

[6]As far as I know, the first version of this result – as well as the formulation of the modified Hahn problem – is due to Hahn (1982). Implicitly though, the problem already underlies Leontief's (1947) criticism of the relevance of Keynes's concept of the speculative demand for money.

return on money is at least as high as the money rate of return on consols over the overall period that is given by the union of all these successive subintervals. If this overall period contains T interest payment dates for the consol, there must then be a positive probability of cumulative capital losses on the consol outweighing the T interest payments.

Now suppose that we have a model of an economy with money and consols having the same marketability properties, with a rational-expectations equilibrium in which the real value of fiat money is positive. With free disposal, the price of consols cannot be negative, so cumulative capital losses on consols over any time interval cannot exceed the initial price. Given the initial price, let T be large enough so that the sum of T interest payments exceeds the initial price. For an overall period including T or more interest payments the probability that cumulative capital losses outweigh the interest on consols must then be zero, so with probability one, the money rate of return on money over this period is less than the money rate of return on consols. There must be at least one point in the overall period at which nobody is willing to hold money, contrary to the assumption of equilibrium at all dates. The assumption of a rational-expectations equilibrium with a positive real value of fiat money thus leads to a contradiction. The negative result stated above follows immediately.[7]

To solve the modified Hahn problem, we need to assume that money and interest-bearing assets have different marketability properties. Such an assumption corresponds to the notion that money serves as a medium of exchange or, in the well-known formulation of Clower (1967), 'money buys goods and goods buy money, but goods do not buy goods'; here the term 'goods' must be taken to include interest-bearing assets. The question is how the medium of exchange role of money is to be articulated in our models.

The most popular procedure nowadays seems to be to take a standard model of a sequence economy with Walrasian, centralized markets and to impose the additional constraint, first proposed by Clower (1967), that in any given period, certain purchases of goods and assets have to be financed by cash held over from previous periods [see, e.g., Grandmont and Younès (1972), Lucas (1980), Lucas and Stokey (1987)]. When this cash-in-advance constraint is imposed, fiat money is demanded and has a positive equilibrium value because it is needed to carry out certain transactions for which interest-bearing assets cannot be used.

[7]The general point is that in a rational-expectations equilibrium, capital gains and losses cannot forever compensate for differences in own rates of return. Leontief (1947) made this point with reference to the 'classical stationary state'. Tobin (1958) proposed to refute Leontief's argument by allowing for uncertainty about bond prices. The argument here, which involves no assumption about the extent of uncertainty, shows that Tobin's proposed refutation breaks down once one moves from a temporary-equilibrium framework with given expectations to a rational-expectations framework.

However, the attempt to solve the modified Hahn problem by appealing to cash-in-advance constraints raises a new problem, namely:

Problem 2: Why should cash-in-advance constraints be imposed?

Presumably cash-in-advance constraints are not just imposed to make life easier for economists who are concerned about the existence of monetary equilibria. In a context of centralized Walrasian markets, it is not clear why agents should be subjected to these constraints in addition to the usual Walrasian budget constraints. Arguments about the need to make sure that agents actually pay for what they buy [see, e.g., Lucas (1980)] are rather beside the point in the Walrasian system, because here the requisite quid-pro-quo is automatically ensured by the simultaneity of all exchanges under the Walrasian budget constraint. We may have doubts about the powers of the Walrasian auctioneer, but I am not convinced that the imposition of a cash-in-advance constraint is the appropriate way to reduce our reliance on this mythical figure.

Within the formalism of Walrasian market models, the cash-in-advance constraint gives the impression that the use of money as a medium of exchange impedes trade [Gale (1982)]. This is rather at odds with the naive notion that the use of money as a medium of exchange facilities trade and enables the economy to exploit opportunities that would not be available under barter.

The frictions that stem from the imposition of cash-in-advance constraints in a system of Walrasian markets are most apparent in the literature on fix-price equilibria that is associated with the names of Clower (1965) Patinkin (1965), Barro and Grossman (1971), Bénassy (1975) and Malinvaud (1977).[8] If we accept the view that the evils of price and wage rigidities as such had been known before – e.g., by Pigou who was criticized by Keynes on this very account – then the main substantive contribution of the fix-price-equilibrium literature lies in the observation that generalized excess supply or generalized excess demand of goods *and* labour is strictly more likely and the amounts by which markets are in generalized excess supply (generalized excess demand) disequilibrium are strictly larger in an economy that imposes a cash-in-advance constraint than in one that does not. Because of the principle of effective demand [Clower (1965)], rationing in the market for goods against money feeds back into the market for labour against money and vice versa. If the relative prices of goods against money and of labour against money are too high, the resulting excess supplies on the market for goods against money and on the market for labour against money reinforce

[8]For another example, this one with flexible prices and market clearing, see Grandmont and Younès (1972, 1973).

each other because the worker who cannot find a job has a lower effective demand for goods against money and the firm which cannot sell its goods has a lower effective demand for labour against money.

The question is why, in a situation of generalized excess supply or generalized excess demand, workers and firms would not engage in a direct exchange of labour against goods. This would not remove the disequilibrium altogether, which is due to prices being rigid, but it would reduce the problem of effective demand (or supply) and thereby improve the disequilibrium allocation.

Within the usual fix-price equilibrium model, the answer to this question is that direct exchanges of labour for goods are ruled out by the cash-in-advance constraint. But then the next question is precisely why this constraint should be imposed if it only exacerbates the consequences of price and wage rigidities and thereby makes people worse off. Within a setting of multilateral, centralized exchanges, this objection to the cash-in-advance constraint seems unanswerable.

Once we go beyond multilateral, centralized exchanges, we do of course know why the direct trade of labour against goods doesn't offer much scope for improvement. The steelworker wants to exchange his labour for bread rather than steel. The owner of the steel mill doesn't have the bread that the steelworker wants. The owner of the bakery is not interested in the special skills that the steel worker has to offer. In a world of multiple, decentralized exchanges, the scope for direct trades of labour against goods is rather limited because the requirement of a *double coincidence of (real) wants* [Jevons (1910)] is rarely satisfied.

It is of some interest to note that *the requirement of a double coincidence of want is itself a manifestation of the principle of effective demand.* In a bilateral exchange with the steelworker, the effective demand of the owner of the steel mill for the steelworker's labour is constrained by his ability to offer goods that the steelworker is willing to accept in return for his labour. The entrepreneur's notional demand for labour in return for steel is irrelevant if the steelworker doesn't want steel. Similarly the steelworker's effective demand for bread from the baker is limited by his ability to offer something that the baker is willing to accept; his notional demand for bread in return for labour is irrelevant.

We all know that the requirement of a double coincidence of wants in decentralized trading is alleviated if agents are willing to engage in indirect exchange, using money as a medium of exchange [see, e.g., Jevons (1910), Starr (1972)]. In a sense, the use of money as a medium of exchange provides a *solution to an effective-demand problem.* It is all the more paradoxical that the cash-in-advance constraint in our models makes the use of money as a medium of exchange appear to be the *source of an effective-demand problem.*

The paradox arises because our models do not account for the decentrali-

zation of exchange which underlies the need for a medium of exchange. They assume centralized markets in which agents trade multilaterally. Knowing that this assumption is unrealistic, we do not take seriously the notion that effective-demand problems might be alleviated by direct exchanges of labour against goods – even though this comes out of the formalism. But then, why should we take other implications of the centralized-markets model seriously. For instance, why should we accept the law of one price? How do we distinguish as to which implications of an unrealistic model we take seriously and which ones we don't?

The imposition of the cash-in-advance constraint is usually justified on descriptive grounds, by an appeal to realism [see, e.g., Clower (1967), Lucas (1980)]. The problem is that this piecemeal introduction of realism into an otherwise highly idealized model begs the question of what is the function of this constraint, and how does this function fit into the conceptual structure of the overall model.

The role of money as a medium of exchange also bears on another problem, which was one hotly debated and now seems almost forgotten:

Problem 3. What is the relation between stocks and flows in a monetary economy?

In my opinion, this classical problem has not really been solved. To be sure, past discussions about liquidity preference and loanable funds have been thoroughly debunked [see, e.g., Patinkin (1965)]. We understand all about the mechanics of stocks and flows within the formalism of Walrasian sequence economies [see, e.g., Foley (1975)]. However, I am not convinced that we understand the interrelation of stocks and flows in a monetary economy in reality.

In standard macroeconomic models, we treat factor services, production and consumption activities as flows, and asset demands and supplies as stocks. Flows are taken to relate to a 'period', i.e., a time interval however short; stocks are taken to relate to an 'instant', e.g. the instant at the end of the current period.[9] With some inspiration from Wall Street, we think of asset markets as being well organized, clearing instantaneously, but we are willing to allow for some sluggishness in the markets for factor services and goods [see, e.g., Dornbusch (1980)].

The question is what to do about money in this classification. Since money is an asset, the logic of the Walrasian formalism forces us to treat it as such, as though there was a demand for money to be held 'at the end of the period', which was regulated through an organized exchange. However, the

[9]Foley (1975) compares the implications of end-of-period and beginning-of-period specifications of asset markets.

use of money as a medium of exchange is related to the process of transactions in goods and services. In this process of transactions, *money is traded back and forth during the period rather than demanded to be held at the end of the period.* The formalism that we use to deal with stocks and flows, asset markets and goods markets, fails to take account of this special role of money, which is an asset, but one that is used in goods market transactions.

To understand this point more clearly, consider the famous adding-up constraints of Brainard and Tobin (1968) for the specification of asset demands in macroeconomic models. These adding-up constraints are derived from budget constraints in a model with centralized Walrasian markets in all periods. In a discrete-time model with consumption goods, consols, and money, an agent's budget constraint for 'period t' requires that

$$p_t c_t + q_t B_t + M_t = Y_t + (1 + q_t) B_{t-1} + M_{t-1};$$

here p_t and q_t are the money prices of consumption goods and consols in period t, c_t, B_t, and M_t are the demands for consumption goods, consols, and money in periods t, and Y_t is noninterest money income in period t. In a temporary-equilibrium analysis of behaviour in period t, differentiation of the budget constraint yields the adding-up condition

$$p_t \frac{\partial C_t}{\partial Y_t} + q_t \frac{\partial B_t}{\partial Y_t} + \frac{\partial M_t}{\partial Y_t} = 1$$

for the dependence of consumption demand, bond demand and money demand on money income. In a continuous-time version of the analysis, with instantaneous flows being negligible in comparison to stocks, the adding-up condition becomes

$$q(t) \frac{\partial B(t)}{\partial Y(t)} + \frac{\partial M(t)}{\partial Y(t)} = 0.$$

Thus if one considers $M(t)$ to be increasing in $Y(t)$, one must ipso facto consider $B(t)$ to be decreasing in $Y(t)$ [May (1970)].

Without doubting the internal logic of the argument – at least in a temporary-equilibrium context[10] – I note that *the Brainard–Tobin adding-up conditions are incompatible with any notion that 'the period' is to be interpreted as a Hicksian 'week', with $M(t)$ or M_t reflecting the need for money in trading during the week* [see, e.g., Patinkin (1965)]. For example, if we model the

[10]For a rational-expectations equilibrium analysis, one would need the cash-in-advance constraint or some other device to eliminate the modified Hahn problem. Given that the Walrasian budget constraints remain in place, this would not affect the validity of the adding-up conditions. However, in a rational-expectations context, it is not clear which thought experiment corresponds to the partial $\partial M_t / \partial Y_t$ with B_{t-1} and M_{t-1} remaining fixed.

need for money in trading during the 'week' by the well-known inventory approach of Baumol (1952) and Tobin (1956), we find that the increase in the volume of transactions that is associated with an increase in income entails increases in the average transactions balances of both, money *and* bonds, so the Brainard–Tobin adding-up conditions do not hold [Hellwig (1975)]. Contrary to the impression given by many macroeconomics textbooks, one cannot accommodate the transactions role of money within the Walrasian framework simply by interpreting $M(t)$ or M_t as an average of money holdings during the Hicksian 'week'. The point is that any detailed account of transactions within the Hicksian 'week' involves budget constraints that are incompatible with the imposition of a Walrasian budget constraint for the 'week' as a whole.

One response to this difficulty may be to banish the Hicksian 'week' from our vocabulary and to replace transactions costs by the cash-in-advance constraint as the main reason why people hold money. As I explained above, this approach fails to give a proper account of the role of money as a medium of exchange. If one insists that the use of money in the economy be tied to its role in transactions and that its role in transactions be tied to the need for a medium of exchange in decentralized trading, one needs to look at the details of the transactions process 'during the week' even if this entails abandoning the Walrasian budget constraint and the Brainard–Tobin adding-up condition for 'the week' as a whole. Once one proceeds in this direction, one realizes that the classical problem of stocks and flows in a monetary economy remains unsolved.

Consideration of the transaction process in detail immediately leads to a further problem, the last one I want to discuss here.

Problem 4. How can the theory of the transactions demand for money be integrated into an analysis of market equilibrium?

In the theory of the transactions demand for money, an agent's demand for money is given by a fairly irregular time path of money holdings. The question is how such irregular time paths of money holdings of different individuals fit together in the market. This question is frequently swept under the rug. Abstracting from the details of the time paths of money holdings, one is satisfied with a representation of the 'transactions demand for money' by a simple time average of money holdings along the path. As mentioned above, this procedure raises questions about the proper specification of budget constraints and adding-up conditions for such time averages. More fundamentally though, there are difficulties with the notion of a market for time averages of money holdings.

For concreteness, consider figs. 1(a) and 1(b). These figures exhibit the time patterns of money holdings of a typical household and a typical firm in the

Baumol–Tobin inventory model. They are drawn under the assumptions that (i) wages are paid once a month in cash, (ii) there are no variable transactions costs, and (iii) fixed transactions costs, transactions volumes, and the interest rate are such that the household wants to make two and the firm wants to make three transactions per month between money and interest-bearing assets.

It is important to note that the underlying theoretical analysis determines the entire time paths of money holdings in figs. 1(a) and 1(b), not just the time averages. Neither the household nor the firm would be indifferent if the time paths in figs. 1(a) and 1(b) were replaced by other time paths having the same time averages. Behaviours – planned and actual – must therefore be assessed in terms of the entire paths rather than merely the averages of these paths. *Given the irregularity of these time paths, the question then is who trades, e.g., with the household when he wants to increase or decrease his holdings of money.*

In Irving Fisher's version of the quantity theory of money, the answer to this question would be simple: as households purchase goods for money, so firms sell goods for money, and the aggregate amount of money held by households and firms jointly does not change; nor does it change when firms pay wages to households. Thus the Fisherian time paths of money holdings of households and firms in figs. 2(a) and 2(b) add up to a constant aggregate demand for money as shown in fig. 2(c).

In a Baumol–Tobin world, matters are more complicated. To be sure, household purchases of goods for money from firms again leave joint money holdings of households and firms unaffected. However, as shown in fig. 1(c), the time path of joint money holdings of households and firms is still quite irregular because the desires of households and firms to exchange money for interest-bearing assets need not match up. The question now is *who trades with the households and firms in figs. 1(a) and 1(b) when they want to buy or sell interest-bearing assets for money*? For instance, who is willing to provide the extra money that is needed for wage payments at the turn of the month? Or alternatively, given that this money is only needed for wage payments at the turn of the month, why should anybody be willing to hold it during the month when it is dominated by interest-bearing assets?

The peak-load problem that is caused by the bunching of transactions at the turn of the month is of course well known in practice. It is usually solved by central-bank interventions, i.e., by central banks providing additional liquidity to the banking system for a few days around the turn of the month. Are we to infer that such interventions are actually necessary for the monetary system to work? Such a conclusion would seriously undermine the notion that a market system can work on its own, without government intervention.

To assess whether the market would be able to work on its own or

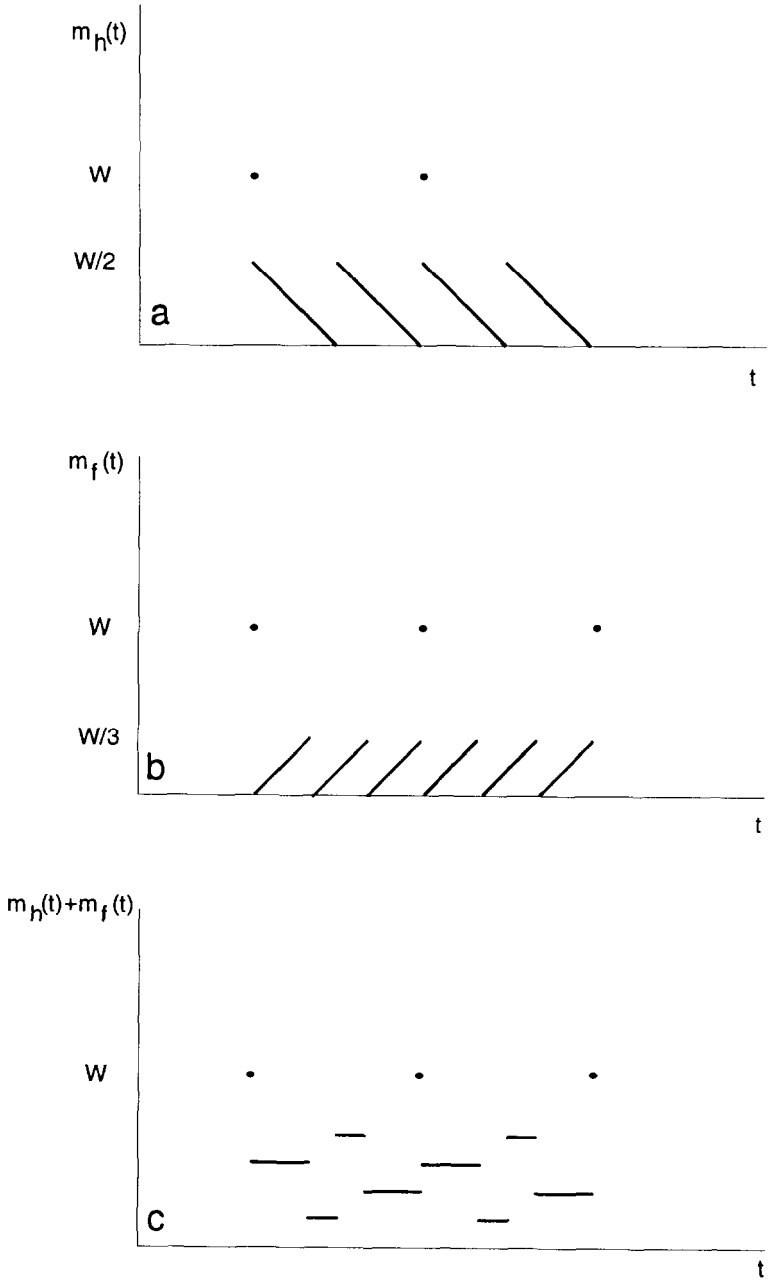

Fig. 1(a) Money holdings of a household in the Baumol–Tobin model; (b) Money holdings of a firm in the Baumol–Tobin model; (c) Aggregate money holdings of households and firms in the Baumol–Tobin model.

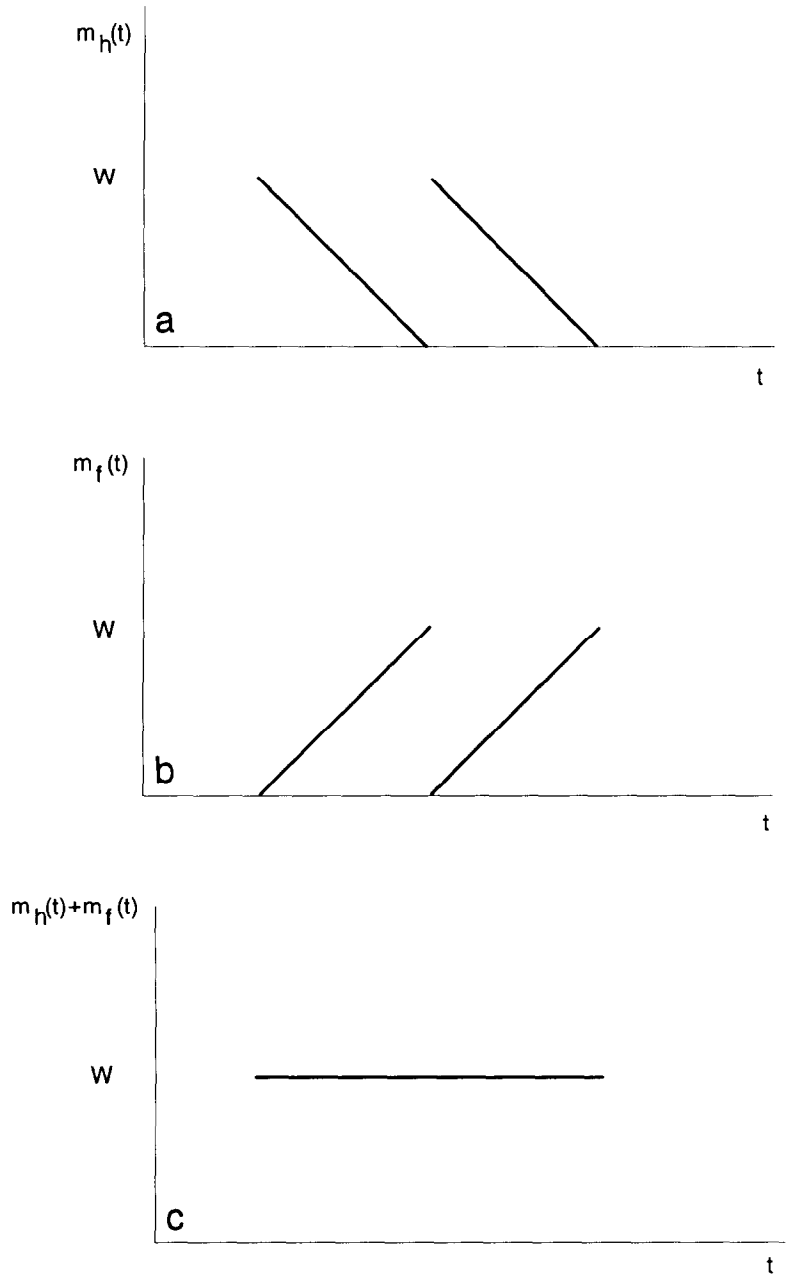

Fig. 2(a) Money holdings of a household in a Fisherian world; (b) Money holdings of a firm in a Fisherian world; (c) Aggregate money holdings of households and firms in a Fisherian world.

whether central-bank intervention is necessary, one must abandon the Baumol–Tobin assumption of a single interest rate which prevails throughout 'the period'. The notion of a single interest rate is convenient if one wants to insert results from the Baumol–Tobin analysis into the Walrasian framework of conventional macroeconomic models. However, since time paths matter in the Baumol–Tobin model, constancy of the interest rate during 'the period' would have to be derived from an analysis of equilibrium rather than assumed. In fact, it is well known that the peak-load problem associated with the bunching of transactions at the turn of the month tends to manifest itself in increases in call money rates that last just for a few days. More generally, one should expect that changes in aggregate desired money holdings of households and firms as shown in fig. 1(c) would entail changes in market interest rates, so the notion of a constant interest rate seems unwarranted. The question then is whether day-to-day changes in interest rates are enough to equilibrate the system, i.e., whether a time path of interest rates in the Baumol–Tobin model exists such that at all dates desired transactions of money for interest-bearing assets just balance.

Even if interest rates are fully flexible within 'the period', the answer to this question is unclear. In the Baumol–Tobin model, the fixed costs of transacting between money and interest-bearing assets introduce important nonconvexities into the optimization problems of households and firms: a convex combination of, e.g., a time path involving two such transactions and a time path involving three such transactions is typically a time path involving five transactions and therefore five times (rather than, e.g. two and a half times) the fixed transactions cost. Because of this nonconvexity, the simplest arguments concerning the existence of equilibrium are not available.[11]

Given the nonconvexities that are associated with fixed transactions costs, the existence of a laissez-faire market equilibrium can only be assured if there are many agents so that one can rely on smoothing or convexification by aggregation over large numbers. In the present context, this means that there must be many different households and firms with different plans involving different desired transactions dates. The above question concerning the trading partners of households or firms who want to buy or sell interest-bearing assets for money might then be answered with a reference to other households and firms which have different desired transactions patterns.

For the constellation of fig. 1, suppose for instance that there are different types of firms according to whether they pay their wages on the first, second,

[11]A few authors propose to work with convex transactions technologies [see, e.g., Niehans (1978)]. With wages paid in cash, this approach can support a demand for money during the first part of the wage payment period. However, during the latter part of the wage payment period, the demand for money will be zero, unless a cash-in-advance constraint is imposed [see, e.g., Grossman and Weiss (1983)]. Therefore a convex transactions technology seems unsuitable as a basis for studying monetary equilibrium.

third, etc. day of the month, and suppose that each firm employs the same number of workers. If the system is in a steady state, then on *each* day of the month, the cross-section distribution of money holdings across households and firms will reflect the entire time paths of figs. 1(a) and 1(b) as there are households (firms) who received (paid) their wages on this very day, households (firms) who received (paid) their wages one day before, etc. The averages of money holdings along the time paths of figs. 1(a) and 1(b) can then be interpreted as cross-section averages reflecting the aggregate demand to hold money at all instants. Market clearing at all dates is possible as, e.g., the transactions of households who want to exchange some of their wage receipts into interest-bearing assets are matched by a new cohort of firms needing cash for the next wage payment.

Suggestive though it is, this sketch cannot really be regarded as a solution to the problem of integrating the transactions demand for money into the analysis of market equilibrium. First, the staggering of wage payments needs to be explained. If we believe that in the absence of central-bank interventions, the market system induces staggering of wage payments in order to avoid the peak-load problem associated with the bunching of certain transactions, then we should derive such staggering endogenously rather than impose it exogenously. Secondly – and perhaps more importantly – there is no account of the behaviour of the system outside of a steady state. The identification of population cross-section averages with individual time averages of money holdings presumes that the system has reached a steady state in which the cross-section distribution of money holdings is constant over time. An analysis that is based on this identification of cross-section and time averages is thus inherently incapable of dealing with non-steady-state, short-run problems such as the dynamics of adjustment to a one-time open-market operation of the central bank.

Indeed, for non-steady-state settings, it is as yet not even known whether the large-numbers argument is sufficient to establish the existence of monetary equilibria in models involving goods, labour, and bond markets, with transactions patterns motivated by the Baumol–Tobin tradeoff between transactions costs and return differentials. Gale and Hellwig (1984) obtain the existence of a monetary equilibrium in a much simpler model involving neither labour nor bond markets. In this simpler model, the only 'real' allocation problem is the Ramsey problem of the division of current production between consumption and capital accumulation, modified however by the assumption that any distribution of returns from 'firms' to 'households' (as owners of the firms) involves fixed real costs. To keep transactions costs down, these distributions of returns are spaced out in time, and households use fiat money as a store of value to bridge the gap between distributions of returns and consumption. In this simple model, smoothing by aggregation across large numbers is particularly powerful because all

transactions are determined endogenously as the results of individual optimization.[12]

An application of the smoothing-by-aggregation approach to a non-steady-state model involving labour and bond markets, i.e., to the real Baumol–Tobin model, seems much more difficult. In a model with labour and bond markets, the timing of wage and interest payments is important. If the dates of wage and interest payments are treated as endogenous, they must be determined in labour contracts and bond covenants, so they will be the results of contract negotiations and market coordination rather than merely individual optimization. The logic of the smoothing-by-aggregation approach suggests that in equilibrium there should be many different labour contracts with different wage payment dates and many different bond covenants with different interest payment dates. To accommodate these different contracts one needs a model with large sets of – differentiated – labour and bond markets corresponding to the various payments provisions. The formulation and analysis of such a model seems like a formidable task, but one that is necessary if we are to know whether a Baumol–Tobin world admits a laissez-faire solution or whether indeed in such a world central bank interventions are necessary for the market system to work.

3. Decentralized trading, money and the Walrasian paradigm

The preceding critical discussion of various problems in monetary theory and macroeconomics has a simple underlying theme: the Walrasian approach to economic theory assumes an exogenously given sequence of 'periods'. In each such 'period', there is a centralized system of markets for current goods and assets in which all agents alive participate. Any one agent considers his trading opportunities in this system of markets to be determined by a given vector of prices at which he can trade with 'the market'. As a blueprint for monetary economics, this approach has two flaws: (i) In the Walrasian market system for any one 'period', agents trade multilaterally, so there is no clear role for a medium of exchange. (ii) The budget constraints faced by agents in a sequence of frictionless Walrasian markets – with or without cash-in-advance constraints – differ from the budget constraints that they face in decentralized bilateral trading, especially when they can choose the frequency and/or the timing of certain exchanges by incurring the requisite transactions costs.

To obtain a satisfactory framework for monetary economics, we must

[12]Steady-state analyses of closely related models are presented by Jovanovich (1982) and Romer (1986).

abandon the Walrasian approach with its assumption that in any period all agents trade simultaneously and multilaterally with each other. We must replace the Walrasian multilateral exchanges by a decentralized system of multiple bilateral exchanges in which there is a need for a medium of exchange and the role of money in transactions can be made explicit.

The criticism of the Walrasian approach is of concern not just to monetary economics. We have a tradition of separating monetary theory and value theory, studying value theory without much concern for the way exchange is organized. From the perspective of this tradition, the objections raised here may appear as a whim of the specialist in monetary theory. However, these objections undermine the Walrasian paradigm altogether – for value theory as well as monetary theory. The central concepts of microeconomics, demand, supply, and price, presume a Walrasian organization of exchanges in which the law of one price holds as a matter of course. Once we accept the fundamentally non-Walrasian nature of the organization of exchanges in a monetary economy, we are led to question the relevance of these concepts – and with them the foundations of a good part of microeconomic theory and welfare economics. *Thus the question is not only what is a suitable framework for monetary theory, but also what is the domain of validity of traditional Walrasian microeconomics.*

To fix ideas, consider the model of monetary exchange in random pairwise meetings that is developed in Hellwig (1976).[13] This is an infinite-horizon, continuous-time model of a large exchange economy with n different commodities and n types of agents. An agent of type k is one who would like to supply commodity k in order to obtain commodity $k+1 \pmod n$; however, in pairwise meetings there never is a double coincidence of wants. More precisely, an agent of type k is assumed to have an exogenously given constant endowment flow $a_k=(a_{k1},\dots,a_{kn})$, where $a_{kk}>0$ and, for $i\neq k$, $a_{ki}=0$; k also has a time-independent instantaneous utility of consumption which depends only on commodities k and $k+1$, i.e., for $c(t)=(c_1(t),\dots,c_n(t))$,

$$u_k(c(t))=v(c_k(t)),\ c_{k+1}(t)).$$

In the absence of trade, his commodity inventory $y(t)\in R^n_+$ evolves according to

$$\dot{y}(t)=a_k-c(t),$$

while his holdings $y_0(t)$ of fiat money do not change. However, at random times, the agent meets somebody else with whom he can trade. Each time anew, his partner is drawn at random from the rest of the population. If the

[13]For related models, see Diamond (1982), Kiyotaki and Wright (1989), Diamond and Yellin (1990).

agent he meets is of type $k-1$, the agent of type k will sell some of his inventory of commodity k in return for money; if the agent he meets is of type $k+1$ (mod n), the agent of type k will use money to buy some of the other agent's inventory of commodity $k+1$ (mod n). The prices at which these trades occur are assumed to be the 'competitive' prices corresponding to the two-person exchange economy consisting of the two agents who meet with their current inventories of goods and money and with preferences given by the dynamic-programming value functions for inventories and money.[14]

An equilibrium in this model is characterized by the vectors of distributions indicating for each type k a cross-section distribution F_k of inventories of goods and money holdings and a probability distribution G_k of prices to be encountered in different meetings. For $(F_1, G_1, \ldots, F_n, G_n)$ to be an equilibrium, it must be the case that if the agents of types $1, \ldots, n$ expect future trading opportunities to be given by the price distributions $G_1, \ldots, G_n$, then (i) for each k, F_k is a stationary distribution under the induced process of consumption, inventory management and trading, and (ii) for each k, G_k is the actual distribution of prices encountered by agents of type k as they draw their trading partners at random from the rest of the population.

The equilibrium as well as the model is thoroughly non-Walrasian. In the absence of an organized market, trading opportunities of any agent are represented by a price distribution rather than a single price. In any one meeting, the price at which agents trade depends on their inventories of goods and money balances. These inventories in turn depend on the agents' histories, e.g., how long it has been since the last trade. The inventory distributions F_k are therefore nondegenerate, and so are the price distributions G_k, i.e., the law of one price does not hold.

Similarly, the 'demand for money' is now a non-Walrasian concept; it does not designate a demand which is expressed in a market or indeed in any one trading opportunity. An individual agent's money holdings at any one instant are the result of his history, reflecting a considerable element of chance. To abstract from this element of chance, one must consider cross-section distributions of money holdings, e.g., under the stationary distributions $F_1, \ldots, F_n$. The 'real value' of the 'demand for money in the economy' is then obtained from a joint consideration of the stationary distributions $F_1, \ldots, F_n$ and the price distributions $G_1, \ldots, G_n$; this is again a distributional concept. In a monetary equilibrium, this 'real value' of the 'demand for money' is positive because for each k, agents of type k accept fiat money in payment for commodity k in order to eventually obtain commodity $k+1$. This use of money as a medium of exchange facilitates the indirect exchange that is necessary if trade is to get off the ground at all.

[14]Clearly, this was developed before Rubinstein (1982) and the subsequent literature.

Although the model is non-Walrasian, yet the Walrasian paradigm retains some relevance. If the frequency with which agents meet is large, the monetary equilibria of the model with random bilateral meetings must be close to the Walrasian equilibria which would be obtained if there was at each instant a centralized Walrasian market in which agents could trade multilaterally without any frictions or transactions costs. More precisely, if one considers a sequence of model specifications with different frequencies of meetings, one finds that as the frequency of meetings goes out of bounds, the monetary equilibria of the random-meetings model approach the competitive equilibria of a Walrasian model in the sense that (i) the distributions of *relative* prices of goods $1,\ldots,n$ that are implied by the equilibrium distributions $G_1,\ldots,G_n$ converge to a degenerate distribution which assigns all mass to a single Walrasian-equilibrium price; (ii) for each k, the distributions of consumption flows that are induced by type k's optimal policy in conjunction with the inventory distribution F_k converges to a degenerate distribution which assigns unit mass to the Walrasian-equilibrium consumption of type k; (iii) for each k, the stationary distributions F_k of inventories of goods and money balances converge to a limit distribution under which all goods inventories are zero; (iv) for each k, the distribution of *real* money holdings that is implied by the inventory distribution F_k and the price distribution G_k converges to a degenerate distribution which assigns all mass to the value zero.

The logic underlying this result is straightforward. If the frequency of meetings is large, there is little scope for price dispersion because an agent who is faced with an unfavourable price will refuse to trade, estimating that the next trading opportunity with a more favourable price is waiting just round the corner. Further, if trades occur frequently, agents have little time to accumulate inventories of the goods they want to sell; these inventories are almost immediately depleted by sales. At the same time they have little incentive to maintain inventories of the goods they want to consume; they can always expect to be able to buy more of these goods soon. This explains why equilibrium inventories must be low if the frequency of meetings is large. The equilibrium real value of money holdings must then also be low because in any one meeting, this value must be commensurate with the values of the inventories held by the trading partners. Finally, the Walrasian nature of the limits of equilibrium consumption flows follows from simple considerations of feasibility at the aggregate level and budget balance at the individual level.

These results lend some support to the contention that the Walrasian paradigm may provide a good approximation to reality even though the centralized market of the Walrasian paradigm has little to do with trading institutions in reality. However, they also show that the usefulness of the approximation may depend on the questions that one is interested in. Specifically, the approximation may be more useful for the traditional

concerns of microeconomics, relative commodity prices and the allocation of consumption and production flows, than for monetary economics. In the random-meetings model, the law of one price does not hold, but if the frequency of meetings is large, deviations from this law will be insignificant. Similarly, the distributions of relative prices and of consumption flows are non-Walrasian, but if the frequency of meetings is large, deviations from the predictions of the Walrasian model will be insignificant. If one believes that indeed the level of frictions in the economy is low, such results may justify the reliance of microeconomic theory on the Walrasian paradigm.

However, from the perspective of monetary theory, the Walrasian approximation is less useful. In the random-meetings model we only find that if the frequency of meetings is high, i.e., if the level of frictions is low, equilibrium real money balances and inventories will be close to zero as they are in a frictionless Walrasian world with continuous trading.[15] This result tells us nothing about how the use of money and inventories makes the system work at a given frequency of meetings. Presumably though, this is the interesting question for monetary economics.

More generally, I suspect that the Walrasian paradigm has little to offer if we are interested in trading institutions – not only the use of money and inventories, but also, e.g., the role of banks in the payments system – which are there to deal with the very frictions that make for a departure from the Walrasian market system. This objection need not affect the usefulness of the Walrasian paradigm if one is interested in questions of traditional allocation theory. However, one needs to be careful to distinguish between cases where the Walrasian approximation is appropriate and cases where it is not.

For instance, it is not clear that traditional results about the welfare properties of market equilibria are relevant for the assessment of monetary policy. If monetary policy has an effect on the way in which the monetary system deals with the frictions inherent in decentralized exchange, perhaps the non-Walrasian features of the system might be more relevant. A case in point is Diamond's (1982) assessment of aggregate-demand management in a random-meetings model: he finds that aggregate-demand management can correct for certain externalities that are caused by the principle of effective demand. Being 'pecuniary' in nature, these effective-demand externalities would be irrelevant for a welfare assessment of Walrasian markets; in a non-Walrasian random-meetings model they do matter. More generally, we need

[15]In the related model of Diamond and Yellin (1990), the equilibrium real value of money does *not* converge to zero as one approximates the Walrasian solution. This is because an indivisibility in consumption forces households to space their consumption goods purchases out in time. In the interval between these purchases they have a demand for a store of value which in the model can only be satisfied by money. As in the discussion of the Hahn problem above, this demand for money as a store of value supports a positive real value of money even in the Walrasian setting.

to go beyond such examples and develop a framework for studying the role of policy designed to affect the non-Walrasian features of the economy.

4. Monetary equilibrium and the cash-in-advance constraint in decentralized trading

The preceding discussion of decentralized trading involves a cash-in-advance constraint. When I suggested that in a meeting of agents of types k and $k-1$, they exchange commodity k for fiat money, I implicitly assumed that the agent of type k accepts no other good in return for commodity k; in particular, he does not accept commodity $k-1$, of which the agent of type $k-1$ presumably has a large stock. As in the discussion of Walrasian models in section 2, the question is why this constraint should be imposed. To be sure, the agent of type k has no direct use for commodity $k-1$. However, he might be prepared to accept commodity $k-1$ because he hopes to resell this commodity so that – possibly after a few further exchanges – he would eventually get commodity $k+1$, which he desires, just as he does when he accepts money in return for commodity k. In a sense, the question is why agents engage in indirect exchange through money rather than general indirect barter.

Ideally, we should like to have a result whereby the cash-in-advance constraint in decentalized trading is imposed endogenously. For instance, suppose that we modify the random-meetings model sketched above so that agents are free to choose whatever they accept as means of payment for the commodities they supply. We would like to argue that in such a setting, agents choose to insist on being paid in money, e.g., because they expect money to be more acceptable than goods in further exchanges [see, e.g., Brunner and Meltzer (1971)].

Unfortunately, such a result is not available. Instead one obtains the strong negative conclusion that if agents are free to exchange all goods and money, a monetary equilibrium fails to exist in the random-meetings model, i.e., in all equilibria, the value of fiat money in all bilateral exchanges is zero. This result is closely related to the modified Hahn problem in section 2: under the given inventory accumulation equation, goods inventories have no carrying costs; hence they have the same own rate of return as fiat money. If they also have the same marketability properties, the relative acceptability of goods and fiat money in indirect exchange depends only on agents' expectations of their relative prices in future exchanges. Under the rational-expectations condition that expected price distributions and realized price distributions coincide, this implies that the distribution of relative prices of goods and fiat money must be degenerate, i.e., the law of one price must hold: Otherwise there would be prices near the boundary of the support

of the distribution where fiat money would be relatively expensive, so agents could expect to make a speculative profit by selling money for goods; since *all* agents would be doing this, such prices could not equilibrate their trading wishes, contrary to the assumption that they are in the support of the actual price distribution. However, in view of the underlying trading uncertainty, the proposition that the distribution of money/goods prices are degenerate leads to the further conclusion that the value of fiat money must be zero. To see this, we note that the value of money relative to commodity $k+1$ in an exchange between agents of types k and $k-1$ must be close to zero if neither agent has a significant inventory of commodity $k+1$. The supports of the equilibrium price distributions must therefore contain prices involving a zero value of money, and if these distributions are degenerate, the value of money in *all* exchanges must be zero.

To avoid this negative conclusion, one must assume either that goods inventories have a lower own rate of return than fiat money, e.g., because of carrying costs, or that goods inventories are less easily marketable than fiat money, e.g., because of transportation costs. Thus Kiyotaki and Wright (1989) show that if goods inventories have higher storage costs than fiat money, one can have monetary equilibria without exogenously imposing the constraint that all indirect exchange use fiat money; indeed in some of these equilibria, the restriction of indirect exchange to fiat money arises endogenously, as a feature of the equilibrium. Banerjee and Maskin (1992) obtain the existence of a monetary equilibrium in a model in which the restriction of indirect exchange to fiat money is derived from the poor marketability of goods.[16] In general, some such assumption is needed to endogenize the cash-in-advance constraint even when trading is decentralized.

The same considerations apply if one allows for real or financial assets as well as commodity inventories. To deal with the modified Hahn problem, one must assume that either with a sufficiently high probability, the own rate of return on assets is smaller than the own rate of return on fiat money or the assets are less marketable than money. In the case of claims issued by unknown private individuals, marketability problems arise naturally from considerations of moral hazard and/or asymmetric information [see, e.g., Gale (1978), Townsend (1980, 1989)].[17] In the case of interest-bearing government debt, the matter is not so clear; nor is it in the case of claims issued by well-known private institutions where problems of moral hazard

[16]Interestingly, this poor marketability arises from quality heterogeneity rather than outright marketing costs. Quality heterogeneity gives rise to a lemons problem, which reduces the marketability of real goods.

[17]However, these information problems may be less significant than we like to believe. Assessing the experience of Ireland in the 1970 bank strike, Murphy (1978) suggests that the disruption of the payment mechanism was largely compensated by an enhanced role of private credit between partners who knew each other well.

and/or asymmetric information are reduced by reputational considerations. In these cases one must presumably rely on marketability problems caused by indivisibilities, e.g. when government bonds are only issued in large units. The restriction to large units may be a tool of price discrimination [Bryant and Wallace (1984)], or it may arise from costs associated with keeping track of who should receive interest payments [see, however, Goodhart (1993)]. In either case, the lack of divisibility ensures that in at least some transactions these assets will not serve as media of exchange.

In summary, even when there is a significant need for indirect exchange, the decentralization of exchange as such is *not* enough to support the use of fiat money as a medium of exchange. *The modified Hahn problem arises in decentralized exchange as well as in centralized Walrasian markets.* To avoid it, one must assume that money has certain unique properties which make it more suitable than other assets or goods as a medium of exchange in at least some transactions. Given that such properties are assumed rather than derived, this is perhaps a less than satisfactory solution to the problem of endogenizing cash-in-advance constraints.

The discussion here is closely related to the question of what distinguishes monetary exchange from barter. Although this is not always clear from the literature, the answer to this question depends very much on whether one compares monetary exchange to direct barter or to indirect barter. The superiority of monetary exchange – with a cash-in-advance constraint imposed – over direct barter depends merely on the restrictiveness of the requirement of a double coincidence of wants in direct barter [Jevons (1910), Starr (1972)]. In contrast, there is no clear presumption that monetary exchange is superior to general indirect barter. The use of fiat money as a medium of exchange in competition with other assets and goods depends on special assumptions about storage or marketing costs of all assets and goods other than fiat money.[18]

Even accepting that certain factors do effectively lead to the imposition of cash-in-advance constraints, I am uneasy about the way in which the demand for fiat money and the equilibrium real value of money are tied to these constraints. *In the model of decentralized trade through random bilateral meetings that I have sketched above, as in the Walrasian model, all transactions dates are exogenous*; they may be random, but nonetheless they are not subject to individual choice. There is no room for the notion that

[18]Note, however, that we have no clearcut welfare comparison between nonmonetary equilibria with general indirect barter and monetary equilibria under a cash-in-advance constraint. The nonexistence of monetary equilibria in the absence of cash-in-advance constraints may well be due to the suboptimality of equilibrium in a monetary economy that was pointed out by Friedman (1969). If so, it might still be possible to confirm the idea of Ostroy and Starr (1974) whereby the concentration of indirect exchange in one medium is socially beneficial because it economizes on information.

an agent might have a choice about the times when he goes to a store or to a bank.

The assumption that transactions dates are exogenous is important because it ties the demand for money to the cash-in-advance constraint. With exogenous transactions dates, the time interval between any two subsequent transactions is exogenous. The desire to hold money for 'the next transaction' involves holding this money over this exogenously given time interval. In contrast, if transactions dates are endogenous, the (random) time interval over which money is held is a matter of choice. If it was possible and advantageous to convert interest-bearing assets into money just five minutes before purchasing goods from a different partner, the holding period would be reduced to these five minutes and the impact of the cash-in-advance constraint on the aggregate holding of money would be fairly insignificant.

From this perspective, the desire to hold money depends not so much on cash-in-advance constraints as such as on the considerations which make it desirable to hold money for more than merely five minutes – or five nano-seconds! – before the transaction in which the cash-in-advance constraint has to be met. Some of these considerations are tied to the difficulties of foreseeing when and where exactly the money will be needed; these difficulties are adequately represented by the randomness with which trading opportunities are taken to arise in the above analysis. Other considerations though will be linked to the different costs and returns that are associated with different frequencies of certain transactions. Indeed if the stakes involved in return comparisons are sufficiently high, even a five-minute holding period for money prior to a transaction need not be out of the question. These considerations cannot be captured in a framework in which the timing of all transactions is exogenous.

At this point, the discussion of money in decentralized trading links up with the earlier discussion of monetary equilibrium in the Baumol–Tobin inventory model. In a Baumol–Tobin world, the timing of transactions is at least partly endogenous; the demand to hold money is determined by the endogenously chosen frequency or money/bond transactions, not just by the requirement to pay in cash for certain purchases. The task then must be to introduce inventory-theoretic considertions à la Baumol–Tobin into the analysis of decentralized trading. Technically, this should not involve any new difficulties, i.e., as in the Walrasian setting, one must rely on large numbers to avoid the discontinuities caused by nonconvexities in transactions technologies. Conceptually though, the coordination problem involved in an endogenization of transactions dates seems even more formidable in the context of decentralized trading than in a central-market model. Perhaps this problem can eventually be handled by allowing for intermediaries as agents who set themselves up with a certain permanence, leaving the choice of timing of transactions to others.

These considerations lead rather far into the abstract world of non-Walrasian exchange systems. As we proceed to explore this world, we must not forget to operationalize the theories that we develop. As in the case of Walrasian general-equilibrium analysis, such operationalization will necessarily involve simplifications. The problem will be to find simplifications that are germane to monetary economics and yet as easy to wield as the Walrasian language of 'demand', 'supply', and 'price'. Given the abstractness of the notion of the 'demand for money' as a process (at the individual level) or a cross-section distribution (at the aggregate level), such simplifications will be difficult to find. However, we need to come to terms with the fact that the 'demand for money' may sometimes find its expression in the speed with which agents move from one transaction to the next rather than anything they announce in 'the market'.

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