

Tutorial 2: The Solow Model without Technological Change

Exercise 1: The basic Solow Model

It is assumed that the aggregate good of the economy is produced by one representative firm endowed with the production technology: $Y_t = K_t^\alpha (N_t)^{1-\alpha}$ with α such that $0 < \alpha < 1$. Y_t is aggregate output, K_t the total capital stock and N_t is the quantity of labor. There is no technical progress. N_t increases at the constant rate n : $N_{t+1} = (1+n)N_t$:

1. The saving rate of agents is equal to s . The depreciation rate of the capital stock is δ . Write the accumulation equation of the capital stock.
2. The variable k_t is defined as K_t/N_t . Find the equation defining the dynamics of k_t . On a drawing, show that k_t converges toward a stable steady state k^* . Give the expression of k^* . What is the effect of s on k^* ?

Exercise 2: Growth and saving rate

The setup is the same of the one in the previous exercise.

1. Describe the paths of the main variables of the economy in the long run: k_t , K_t , Y_t and $y_t = Y_t/N_t$. What is the effect of an increase of s ?
2. From this question, it is assumed that the saving rate of each agent is an *increasing function* of his income. It is assumed that all agents earn the same income y_t . The saving rate is now denoted by a function $s = \sigma(y_t)$. Can you interpret this assumption?
3. To have simple results, it is assumed that the function σ is such that:

$$\begin{aligned}\sigma(y) &= s_1 \text{ for } y < \bar{y}, \\ \sigma(y) &= s_2 \text{ for } y > \bar{y}.\end{aligned}$$

with $s_1 < s_2$. Moreover, it is assumed that \bar{y} is such that:

$$\left(\frac{s_1}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} < \bar{y} < \left(\frac{s_2}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

Represent by a drawing how k_{t+1} depends on k_t . What is the long run value of k_t ? Is this long run value independent of the initial value k_0 ? Compare the results with the one found in question 1.

Remark: When drawing k_{t+1} as a function of k_t , notice the discontinuity at \bar{y} .

4. What can you say about the dynamics of k_t in the case where

$$\bar{y} < \left(\frac{s_1}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}.$$

What can you say if:

$$\left(\frac{s_2}{n + \delta} \right)^{\frac{\alpha}{1-\alpha}} < \bar{y}.$$

Compare the results with questions 1 and 3.