Tutorial 4

Exercise 1

We consider an economy with one representative firm endowed with the following production function: $Y = K^{\alpha}(N)^{1-\alpha}G^{\gamma}$ where $0 < \alpha < 1$ and $0 < \gamma < 1$. Y is the aggregate output, K is the physical capital stock and N is the quantity of labor. G is the amount of productive public expenses (education, investments in infrastructure etc...). The depreciation rate of the capital stock is equal to δ. This assumption is taken from Barro (1990). N is assumed to be constant

1. What can you say about the production function?

2. G is financed by a constant tax rate τ on the total income Y. Show that production can be expressed as a function of the only factors K and N. What is the value of γ such that the income is expected to grow at a constant rate in the long run without technical progress? We will keep this value in the sequel.

3. s is the share of the output invested in physical capital. What is the accumulation process of the capital stock?

4. What is the impact of an increase of τ? Show that there exists a value of τ that maximizes the growth rate of K_t.

Exercise 2: Growth and efficiency of labor

It is assumed that the aggregate good of the economy is produced by one representative firm endowed with the production technology: $Y_t = K_t^{\alpha} L_t^{1-\alpha}$ with α such that: $0 < \alpha < 1$. Y_t is aggregate output, K_t the total capital stock and L_t is the quantity of labor. There is no technical progress. Labor supply is assumed to be constant $L_t = N$ The saving rate of agents is equal to s. The depreciation rate of the capital stock is δ.

1. Write the accumulation equation of the capital stock.

2. The variable k_t is defined as K_t/N. Find the equation defining the dynamics of k_t? On a figure, show that k_t converges toward a stable steady state k^*. Give the expression of k^*. What is the effect of s on k^*?

3. What are the paths of the main variables of the economy in the long run: k_t; K_t; Y_t and $y_t = Y_t/N$. What is the effect of an increase of s?

4. From this question, it is assumed that the efficient quantity of labor L_t that enters in the production function depends on the quantity of workers N and on the efficiency of labor e_t. Therefore, one can write: $L_t = Ne_t$. The efficiency variable e_t is assumed to be an increasing function of output per worker $y_t = Y_t/N$: $e_t = E(y_t)$. The function E is such that:

$$e_t = E(y_t) = (y_t)^{\gamma} \text{ for } y_t < \overline{y}, \tag{1}$$
$$e_t = E(y_t) = (\overline{y})^{\gamma} \text{ for } y_t \geq \overline{y}. \tag{2}$$

with $\gamma > 1$ and $\overline{y}$ a (positive) constant parameter. Represent the function E and interpret this assumption.

5. To have simple results, it is assumed that

$$\gamma = \frac{1}{1-\alpha^2} \Leftrightarrow \gamma(1-\alpha) = \frac{1}{1+\alpha}$$

Using the production function and equations (1) and (2), show that y_t can be written:

$$y_t = k_t^{1+\alpha} \text{ for } k_t < \overline{k},$$
$$y_t = \overline{k} k_t^{\alpha} \text{ for } k_t \geq \overline{k}.$$

6. Give the new expression of the dynamics of k_t. It is assumed that

$$\left(\frac{\delta}{s}\right)^{\frac{1}{\alpha}} < \overline{k} \ .$$

Represent by a drawing how k_{t+1} depends on k_t. What are the steady states of the dynamics? Does the long run value of k_t depend on the initial value k_0? How can you interpret this situation?

7. What can you say on the dynamics in the case $\left(\frac{\delta}{s}\right)^{\frac{1}{\alpha}} > \overline{k}$?