

Tutorial 5

Exercise 1: Product-variety model

We consider an economy with one single output produced in quantity Y_t with a continuum of intermediate products and labor according to the production function $Y(t) = N_Y(t)^{1-\alpha} \int_0^{A(t)} x_i(t)^\alpha di$, with $0 < \alpha < 1$. The intermediate inputs are defined over the interval $[0, A(t)]$, $x_i(t)$ denotes the quantity of intermediate product i ; $N_Y(t)$ the quantity of labor used in the industry and $A(t)$ is the number of varieties of products. Population is constant over time. We denote $p_i(t)$ the unit price of intermediate product i and $\omega(t)$ the wage. The final output industry is perfectly competitive.

1. Write the profit of the final output industry.
2. Compute the demands of the final output industry for both the intermediate product i and labor.

Intermediate products are produced using the final good (one unit of final good for one unit intermediate product) and the work of researchers N_A . Each intermediate product is produced by a monopoly. Researchers are paid r_A for their work. Producers of intermediate products get a subsidy $s \in (0; 1)$ for each unit of production of intermediate good.

3. Write and solve the profit maximization problem of the monopoly which produces an intermediate product. Determine the production level of an intermediate product $x_i(t)$ and the profit of its producer.
4. What is the impact of an increase in the subvention s on the production of intermediate products? On the profits of the intermediate good producer?

Exercise 2: Myopic Schumpeterian Model

We consider an economy with a representative final output producer, following the production function: $Y_t = N_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha$, where N_t defines the labor in the final output industry; x_{it} denotes the quantity of intermediate good of variety i . The intermediate good of variety i is produced by a monopolist in each period. Further, assume that for producing each unit of intermediate product, the monopolist must use one unit of final good as input. We denote w_t the wage and p_{it} the price of the intermediate input x_{it} at period t . Define the aggregate productivity index or the continuous average of varieties as $A_t = \int_0^1 A_{it} di$ and the total amount of final good used in producing intermediate products as $X_t = \int_0^1 x_{it} di$.

1. Write down and solve the maximization problem of the final good producer. Subsequently, find the demand for labor and intermediate inputs.
2. Write down and solve the maximization problem of the intermediate good producer. Find the equilibrium price charged by each intermediate good producer.
3. Rewrite the production function of the final good, replacing $x_{i,t}$ with its optimal value. What is the growth rate of final output in this economy?

Now, assume that before the production of Y_t and $x_{i,t}$, $\forall i \in [0, 1]$ take place, there is a randomly chosen entrepreneur among the monopolists who attempts to innovate a given variety i . If the entrepreneur is successful, the innovation will create a new version of the intermediate good, increasing the intermediate good productivity from $A_{i,t-1}$ to $A_{it} = \gamma A_{i,t-1}$. If the entrepreneur fails, there is no innovation, the entrepreneur

gets nothing, and another randomly chosen monopolist will produce the older version of the intermediate good, leading to the previous period's productivity $A_{i,t-1}$. In summary, for any $\gamma > 1$, we have:

$$A_{it} = \begin{cases} \gamma A_{i,t-1}, & \text{if the entrepreneur succeeds} \\ A_{i,t-1}, & \text{if the entrepreneur fails} \end{cases}$$

Let the probability that the entrepreneur succeeds be given by z_t . Clearly, the entrepreneur must invest into R&D to innovate, a costly activity that we assume to be $R_{it} = \delta A_{i,t-1} z_t^2 / 2$. Hence, a higher intensity of research is synonymous with a higher probability of success.

4. Maximize the firm's expected benefits from innovation and find the optimal innovation intensity z_t^* .
5. Compute the output growth rate using the expected aggregate productivity index A_t .

To look at the social optimum of this economy — assume that instead of individual firms making their respective optimal decisions, we have a myopic social planner who seeks to maximize consumption $C_t = Y_t - X_t - R_t$.

6. Suppose that the R&D investment is already made. Write and solve the social planner problem to find the optimal level of intermediate input. Compute the total output level in this economy. Compare with the case of the decentralized economy.
7. Suppose that the planner now decides to intervene in the intensity of R&D investment. Write and solve the planner maximization problem for the optimal innovation decision. Compare with the case of the decentralized economy.
8. Explain where the inefficiencies of the model come from and propose a policy to solve them.