Tutorial 6: The Ramsey Growth Model

Exercise 1: Comparing the Solow and the Ramsey model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function $Y_t = F(K_t, L) = K_t^\alpha L^{1-\alpha}$ with $0 < \alpha < 1$, where K_t is aggregate capital and L is the number of workers in the economy. The law of motion for aggregate capital is given by $K_{t+1} = (1-\delta)K_t + I_t, K_0 > 0$ where I_t denotes aggregate investment, and $0 < \delta < 1$ the depreciation rate. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that consumption per worker, c_t, is the same as aggregate consumption, $C_t = c_t = c_t L$.

Consider now two different models. In the Solow model, agents have a constant saving s such that $I_t = sY_t$. In the Ramsey model, the household utility is maximized

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

such that aggregate investment (savings) is endogenous, $I_t = Y_t - C_t$. Markets are competitive, thus input factors K_t and L are paid their marginal product.

1. Compute the interest rate in this Solow model's stable steady state. (Hint: you need to compute the stable aggregate steady-state capital stock first.)
2. Write up the maximization problem of the consumer is the Ramsey model and derive the optimality conditions (the Euler equation and the resource constraint).
3. Compute the interest rate in the Ramsey model in a steady-state.
4. Does the interest rate in the Solow model depend on the saving rate s in the stable steady-state? Is the interest rate increasing or decreasing in s or independent from s. Why?
5. Does the interest rate in the Ramsey model depend on the discount factor β in a steady-state? Is the interest rate increasing or decreasing in β or independent from β. Why?
6. Compute the saving rate $\overline{s}$ which gives the same steady-state capital stock in the Solow model as in the Ramsey growth model. Is this saving rate, $\overline{s}$ increasing or decreasing in the discount factor β?

Exercise 2: The Ramsey model - optimal growth and labor supply

Part 1

The production technology of the economy is given by: $Y_t = K_t^\alpha L_t^{1-\alpha}$ Y_t is the aggregate output at period t, K_t is the physical capital stock, L_t is the quantity of labor. The depreciation rate of the capital stock is full ($\delta = 1$).

One representative consumer is living in this economy. He is endowed with one unit of labor at each period, and with the following intertemporal utility function:

$$\sum_{t=0}^{\infty} \beta^t ln(C_t)$$

where β is positive and < 1. C_t is the level of consumption at period t.

1. Write the optimization program that defines the optimal state of the economy.
2. Write the optimality conditions.
3. Show that there exists a stationary state solution to the optimality conditions and and the stationary values of capital and consumption. How do these values depend on ? How can you interpret the results?

Part 2

In this part, the production function is the same, but the intertemporal utility of the consumer now depends on the time L_t he spends to work:

$$\sum_{t=0}^{\infty} \beta^t \left[ln(C_t) - L_t\right]$$

1. Write the optimization program that defines the optimal state of the economy. In this program, C_t and L_t are the command variables and K_t the state variable.
2. Write the optimality conditions.
3. Show that there exists a stationary state solution to the optimality conditions and find the stationary values of the following variables : $k = K = L$; C and L. How do these values depend on β ? How can you interpret the results ?

Exercise 3: The Ramsey model - growth and public good

Part 1

In this part, the production technology of the economy is given by: $Y_t = AK_t$. Y_t is the aggregate output at period t, K_t is the physical capital stock. The depreciation rate of the capital stock is full ($\delta = 1$). A is a constant parameter. One representative consumer is living in this economy, endowed with the following intertemporal utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[ln(C_t) - L_t\right]$$

β is positive and < 1, c_t is the level of consumption at period t.

1. Write the optimization program that define the optimal state of the economy.
2. Write the optimality conditions.
3. Find the growth rate of consumption c_t.
4. Show that K_t increases at the same rate.

Part 2

In this part, the production function is $Y_t = AK_t^{\alpha} G_t^{1-\alpha}$ with $0 < \alpha < 1$. G_t represents an amount of public good that is used in production. G_t is provided by the government, that is able to transform one unit of consumption good into one unit of public good. The new resource constraint of the economy is then:

$$Ak_t^{\alpha} G_t^{1-\alpha} = c_t + G_t + K_{t+1}$$

.

1. Write the optimization program that dene the optimal state of the economy. In this program, ct and Gt are the command variables and Kt the state variable. Find the new optimal state of this economy. What is the relation between Gt and Kt along the optimal path?