An Introduction to Economic Growth and Macro-Models

This course focuses on **economic growth** and **long-term economic development**. While macroeconomists may have differing views on how to analyze short-term economic fluctuations and policies to implement (e.g. minimum wage), there is broad consensus on the foundational models used in long-run macroeconomic growth analysis. Among these, the Solow growth model and the Ramsey/neoclassical growth model stand out as central frameworks.

A) Economic Growth

Economic Growth *refers to the sustained increase in real aggregate income, production, and consumption, both in total and on a per capita basis. This growth leads to a continuous enhancement in the standard of living.*

Key Facts

There are substantial disparities in income per capita and output per worker across different countries. Nations at the top of the global income ladder are over 30 times wealthier than those at the bottom. For instance, in 2023, the GDP per capita in the United States exceeded $85,000. In comparison, income per capita was significantly lower in various countries: under $14,000 in Mexico, around $12,500 in China, around $2,500 in India, and only under $500 in Sierra Leone.
The significant income disparities between countries can be attributed to several key factors (for more on this, read "*Why Nations fail?*" from Daron Acemoglu & James A. Robinson):

(i) Random factors, or "luck," which can lead to different developmental paths;
(ii) Geographic differences that influence the environment where individuals live, affecting agricultural productivity, access to natural resources, and shaping individual behavior and attitudes;
(iii) Institutional differences, which determine the legal and regulatory frameworks under which individuals and firms operate, thereby shaping incentives for investment, accumulation of capital/wealth, and trade; and
(iv) Cultural differences, which shape individuals' values, preferences, and beliefs, and therefore impact economic outcomes.

Why should we be concerned about these cross-country income disparities? Because high-income levels are closely related with higher standards of living. Comparing a wealthy, developed economy to a less-developed one reveals huge contrasts in terms of quality of life, living standards, and health outcomes.

Although economic growth generally enhances welfare, it can also create *"winners"* and *"losers"*. A key concept in economics, Joseph Schumpeter's creative destruction, highlights this aspect of growth, where productive firms and jobs may be destroyed by the forces of economic progress and innovation.

Computing growth rates:

Average annual growth rate between year 0 and year t:

- *Discrete case:* g so that $Y(t) = (1+g)^t Y(0)$
- *Continuous case:* $Y(t) = Y(0)e^{gt}$

where $Y(0)$ is the GDP (or per capita GDP) of year 0, $Y(t)$ the one of year t.[1]

[1] How to go from discrete to continuous time and show that: $Y(t) = Y(0) \times (1+g)^t$ is equivalent to $Y(t) = Y(0)e^{gt}$.
To go from discrete to continuous time, we first have to reduce the time step from 1 to Δ, we have:
$Y(t+\Delta) = Y(t)(1+g\Delta)$, then as $\Delta \to 0$, we have: $\frac{Y(t+\Delta)-Y(t)}{\Delta} = \dot{Y}(t) = Y(t)g \iff \frac{\dot{Y}(t)}{Y(t)} = g$.

Integrating both sides with respect to time, we get: $\int_0^t \frac{\dot{Y}(s)}{Y(s)} ds = \int_0^t g ds \iff \ln(Y(t)) - \ln(Y(0)) = \ln\left(\frac{Y(t)}{Y(0)}\right) = gt \iff Y(t) = Y(0)e^{gt}$.

B) Macro-Models

"All models are wrong, but some are useful", George E. P. Box, British statistician.

In other words, no model perfectly captures the full complexity of reality. However, some models are still valuable because they provide insights that help us understand, predict, or influence outcomes in the real world.

This concept is highly relevant in economics. Every economic model is a simplification of the real economy. They involve assumptions and abstractions that may not hold true in every situation. For example, models often assume rational behavior, perfect markets, or constant returns to scale—conditions that are rarely, if ever, fully met in practice. Despite these limitations, economic models remain essential tools. They allow economists to analyze relationships, make forecasts, and design policies based on a structured understanding of economic processes.

Formally, a **Macro-Model** consists of:

1. a list of **agents** (their preferences, behavioral rules, endowments, ...).
2. a list of **goods**.
3. a specification of the **production technology**.
4. a definition of **time** (discrete Vs continuous, number of periods, ...).
5. a specification of **uncertainty** (perfect Vs imperfect information).
6. a **market structure** (perfect competition Vs monopoly or oligopoly).

Throughout this course, in most of the exercises we will assume the same general setting:

1. Only two agents: a **representative firm** and a **representative household**.
2. **One single final good**, produced by the representative firm, which price will be normalized to one.
3. A **Cobb-Douglas production technology** for the representative firm, that uses capital and labor to produce the final output.
4. We will work in **discrete time**, $t = 0, 1, 2, \ldots$ and will be assuming for the interpretation that each time step is a year.
5. We will work with **no uncertainty**.
6. We will assume **perfect competition**.

All these assumptions are meant to simplify the model we will work with, while keeping the model reasonable for what we study - that is long term economic growth.

"Everything should be made as simple as possible, but not simpler", Albert Einstein, German theoretical physicist.

In other words, if a model is too complex, it's hard to understand, but if it's too simple, it may miss critical aspects of the real economy. The goal is to find the right balance between simplicity and usefulness.